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Volatility modeling to identify & forewarn an asset bubble

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Financial bubble

What is an asset/ financial bubble?

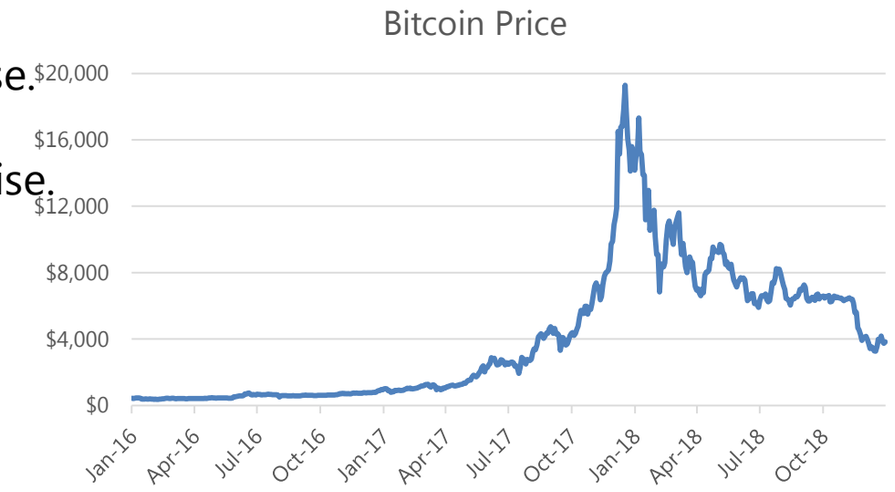
<u>Definition</u>: Quick rise in asset price due to sustained mispricing and abnormally high investors' expectations.

During bubble: -

- extremely high market price
- extremely high valuations
- dramatic price rise
- Market price >> fair value (fundamentals)

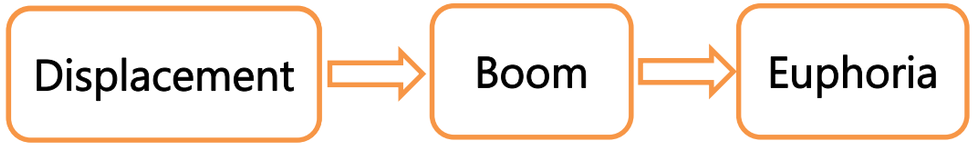
Dramatic price rise is followed by collapse.

Often the fall in prices is as swift as the rise.



Financial bubble build-up

Stages of a financial bubble (1/2)



i) displacement

- paradigm shift in the economy due to new technology or financial innovation

ii) boom

- consistent rise in asset price, due to increase in investment
- characterized by low volatility, credit expansion

iii) euphoria

- more and more investors join the bandwagon
- explosion in asset price
- unrealistically high valuations
- high volatility, trading volume

Financial bubble build-up

Stages of a financial bubble (2/2)

iv) profit taking

- astute investors start seeing an imminent fall in market sentiments
- book partial or complete profit
- but new and novice investors keep joining and sustain high prices

v) panic

- a minor event leads to bubble burst
- sudden fall in asset price
- often like a free fall, due to massive selling of the asset

Financial bubble build-up

Reasons of financial bubble

Some reasons of bubble building: -

(1) A business/ financial innovation; e.g. e-commerce bubble of late ‘90s

(2) Irrational expectations of investors

(3) Incomplete information in the market on an asset

(4) Sudden euphoria after a big change; e.g. a new infrastructure

(5) Anticipation of quick profit and fear of losing out in the ongoing rally
- results in large trading volume
- more investors further fuel the rally

(6) Durable and saleable asset that can be sold as many times for quick profit

(7) Investors/ traders who buy due to replacement price rather than fundamentals
- traders who buy and resale the asset in short time

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Mathematical treatment of bubble theory (1/2)

Stochastic calculus helps in understanding, analysis and modeling of financial bubbles.

Martingale process (random walk): -

- A discrete time-dependent stochastic process (discrete points $X_1, X_2, X_3, \ldots\ldots, X_{n-1}, X_n$).
- $E(|X|) < \infty$, and $E(X_n) = X_{n-1}$.

Strict Local martingale process: -

- A discrete time-dependent stochastic process (discrete +ve points $X_1, X_2, X_3, \ldots\ldots, X_{n-1}, X_n$)
- $E(|X|) < \infty$, and $E(X_n) > X_{n-1}$.
- Mutually exclusive with martingale.

Strict local martingale process:

- first quickly shoots to high values
- then decrease to small values and stays there
- Mirrors typical behavior of bubble

Hence, bubbles are modeled using strict local martingale process.

Statistical model

Mathematical treatment of bubble theory (2/2)

(1) Stochastic asset pricing model :

$$dS(t)/S(t) = \mu dt + \sigma(t)*dW(t)$$

(2) Asset price S(t) and price volatility σ(t) are stochastic (dependent on process path and time)

(3) S(t) for a given time interval is strict local martingale if $\int_{\epsilon}^{\infty} \frac{x}{\sigma(x)^2} dx < \infty$.

where x: price path, σ(x): [illegible]endent volatility and ε: *minimum* asset price in the interval $\int_{\epsilon}^{\infty} \frac{x}{\sigma(x)^2} dx$

(4) If the definite integral is finite/ convergent, then there is a bubble. If the integral is divergent, there is no bubble.

(5) In case of bubble, volatility becomes extremely high for very high values of x (asset price).

(6) For discrete points, integral can be approximated with: $\Sigma(x / \sigma(x)^2)$.

Statistical test

Methodology for the statistical test

(1) High Frequency (per minute, at xx : xx : 00) trade data used

(2) Periods with high daily price movement (5% , 7% , 10%) considered

(3) Price volatility estimated using stochastic volatility estimator (e.g. Florens-Zmirou estimator) at each price point

(4) Graphs plotted: (I) volatility ($\sigma(x)$) vs. price (x), (II) $\Sigma\ (x / \sigma(x)^2)$ vs. price (x)

(5) Asymptotic value of the integral $\int_{\epsilon}^{\infty} \frac{x}{\sigma(x)^2} dx$ inferred from both plots

(6) A finite asymptotic value of integral => strictly local martingale => a bubble.

Statistical test

Applications of the statistical test

This highly useful statistical test can be applied to assets like: -

- individual stocks
- stock ETFs
- bond ETFs
- commodities
- currencies
- cryptocurrencies
- stock indices
- even the complete stock market

Mandatory condition: Test can be conducted only on HFT data for a frequently traded asset.

The test cannot be applied to infrequently traded assets like: -

- corporate bonds (high yield/ illiquid bonds)
- real estate transactions

HFT: High Frequency Trade (data)

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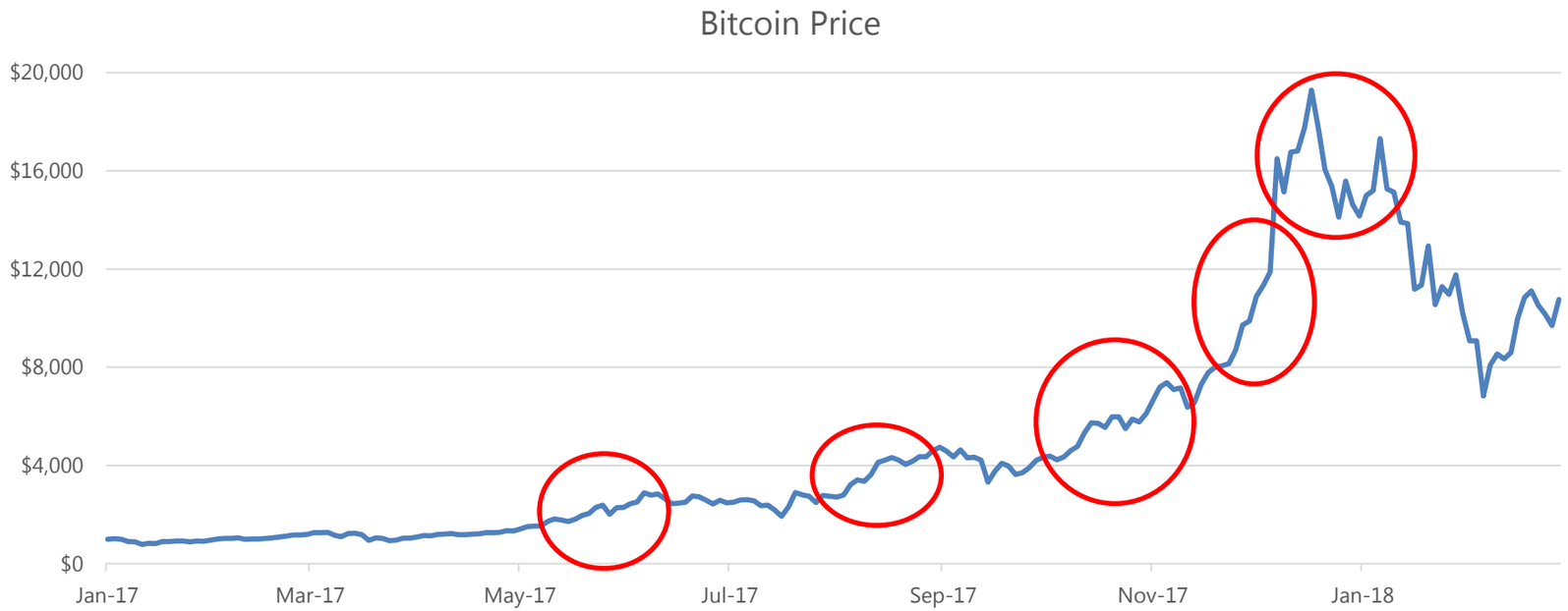
Backtesting the methodology on Bitcoin

Bitcoin price behavior in 2017

In 2017, Bitcoin had risen from $998.05 (Jan 1) to $20,000 (Dec 17): a rise of 20x!

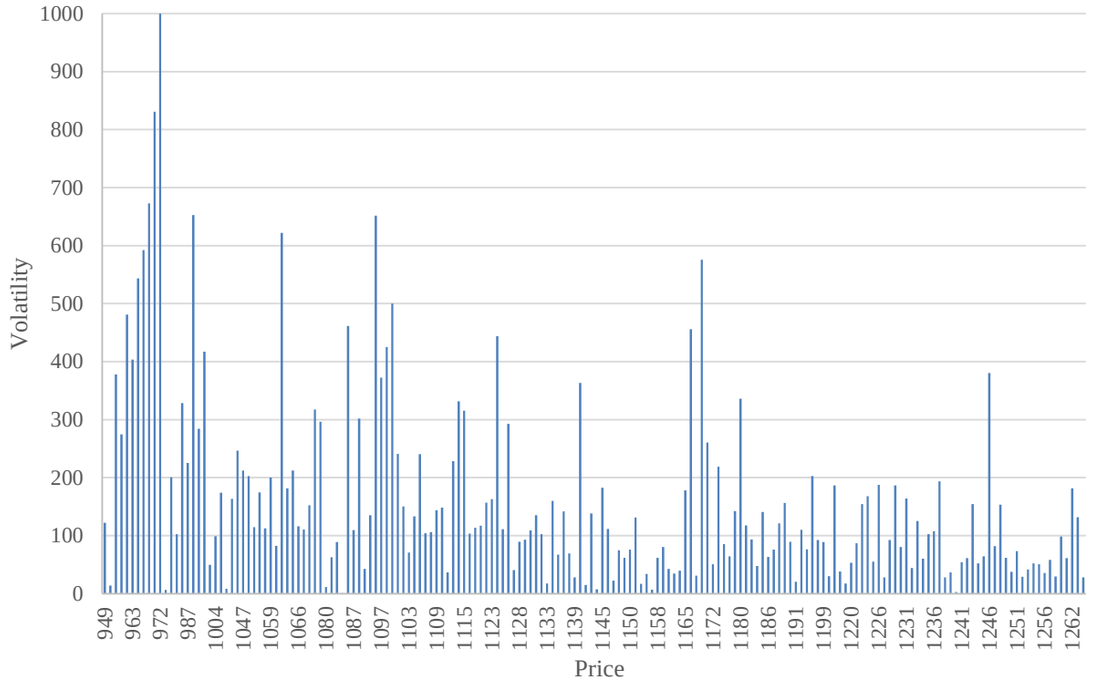
Analysis of volatility and the integr$\int_{\epsilon}^{\infty} \frac{x}{\sigma(x)^2} dx$ proves that Bitcoin was in bubble during 2017 and Jan, Feb 2018.



Bubble testing

Representative period 1

Data characteristics	
Period start	11-Mar-17
Period end	18-Mar-17
Days	8
Minimum price	$948.6
Maximum price	$1,264.9
Daily volatility	5.82%

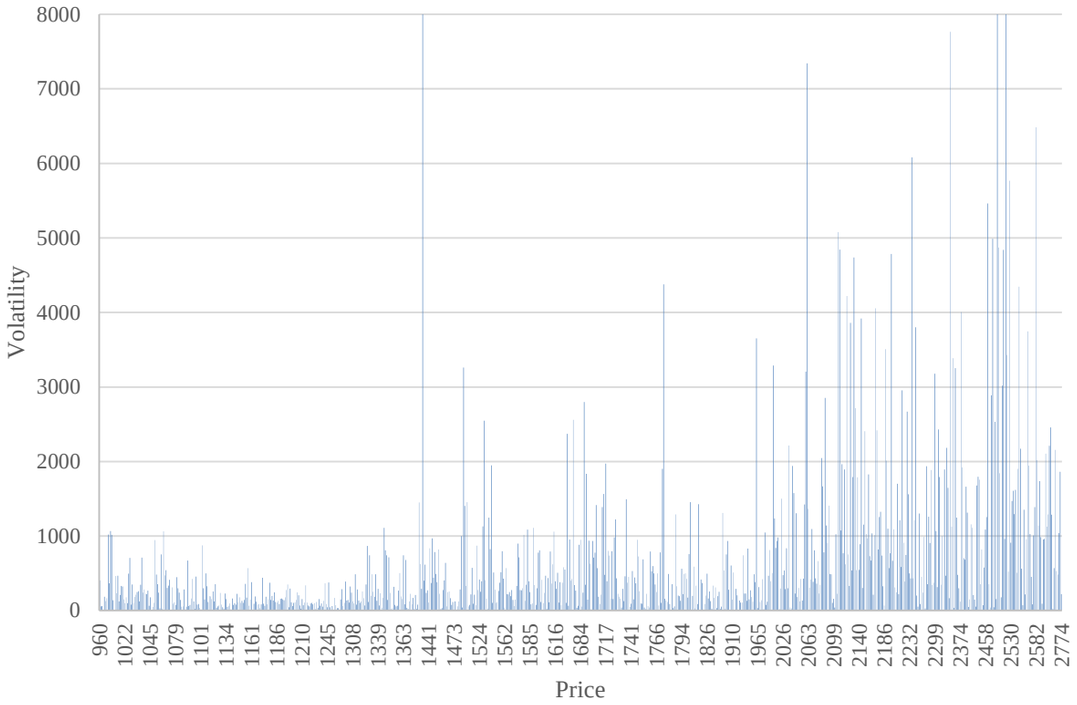


- Volatility modeling on tick data reveals low volatility, decreasing with price
- Inference: NO BUBBLE

Bubble testing

Representative period 2

Data characteristics	
Period start	27-Mar-17
Period end	27-May-17
Days	62
Minimum price	$960.2
Maximum price	$2,779.0
Daily volatility	3.16%

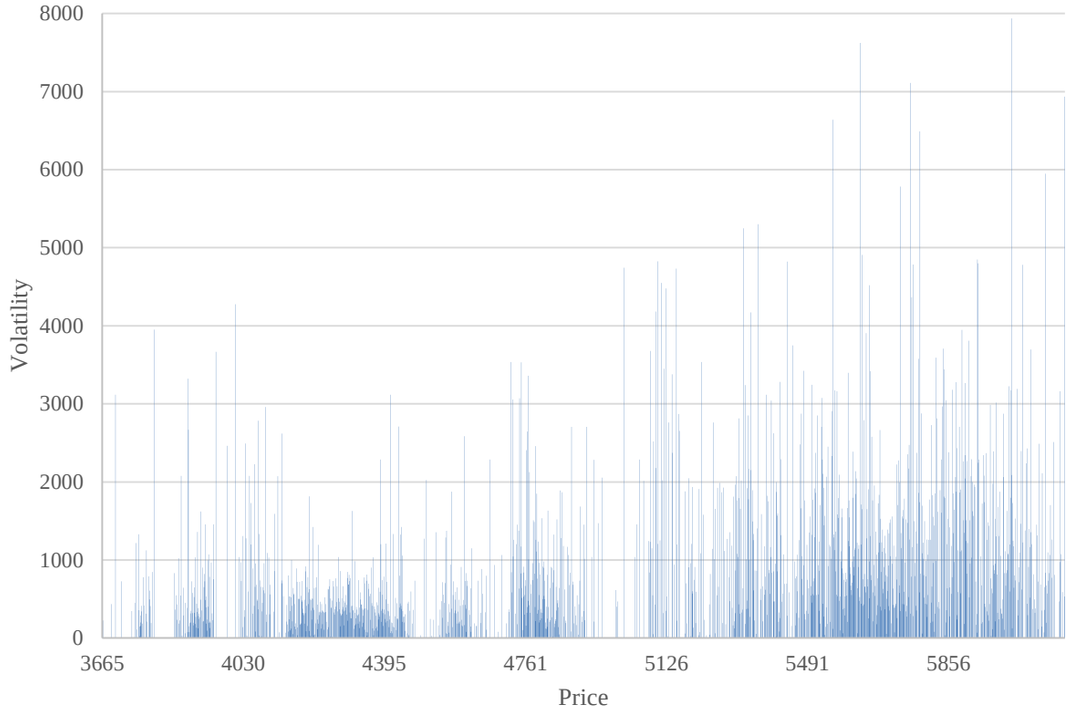


- Volatility modeling on tick data reveals high volatility, increasing with price
- Inference: A BUBBLE

Bubble testing

Representative period 3

Data characteristics	
Period start	25-Sep-17
Period end	24-Oct-17
Days	30
Minimum price	$3,666.0
Maximum price	$6,178.5
Daily volatility	3.79%

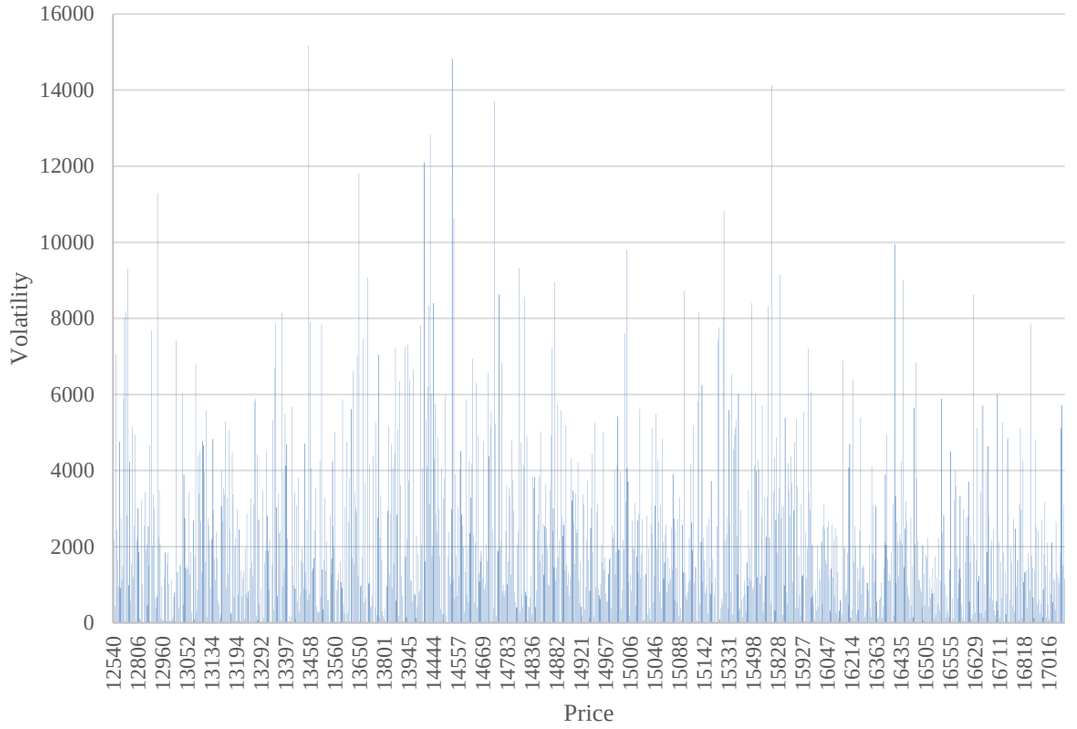


- Volatility modeling on tick data reveals high volatility, increasing with price
- Inference: A BUBBLE

Bubble testing

Representative period 4

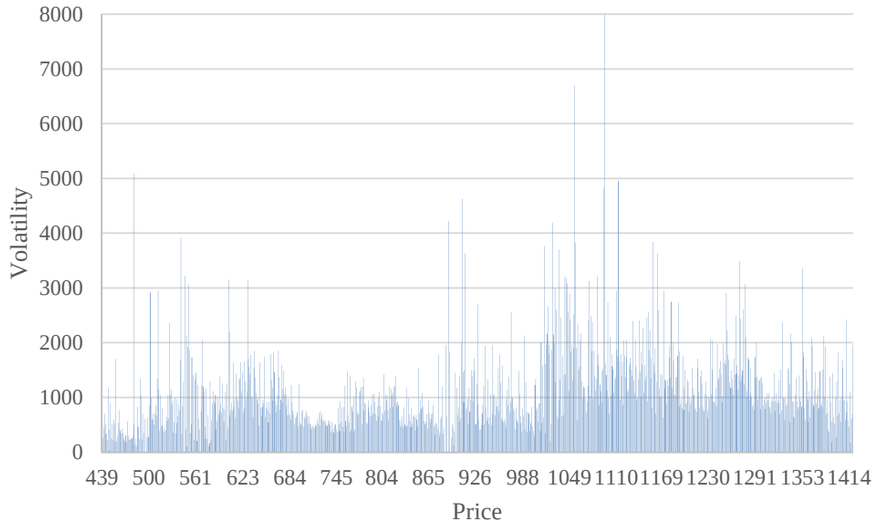
Data characteristics	
Period start	31-Dec-17
Period end	7-Jan-18
Days	8
Minimum price	$12,540.0
Maximum price	$17,222.0
Daily volatility	6.75%

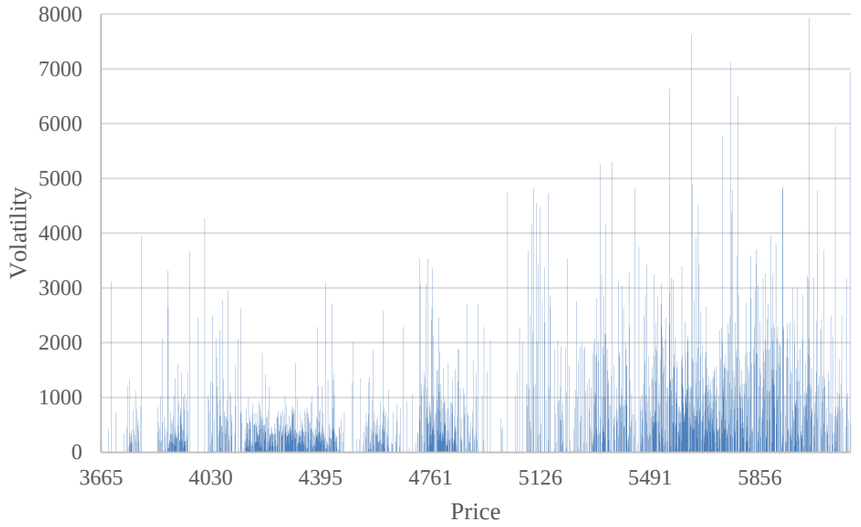


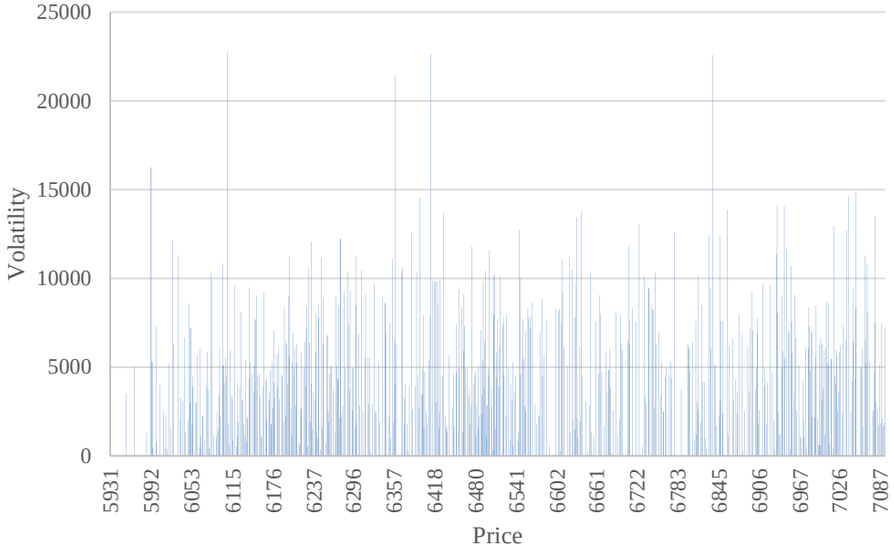
- Volatility modeling on tick data reveals very high volatility across the period
- Inference: A BUBBLE

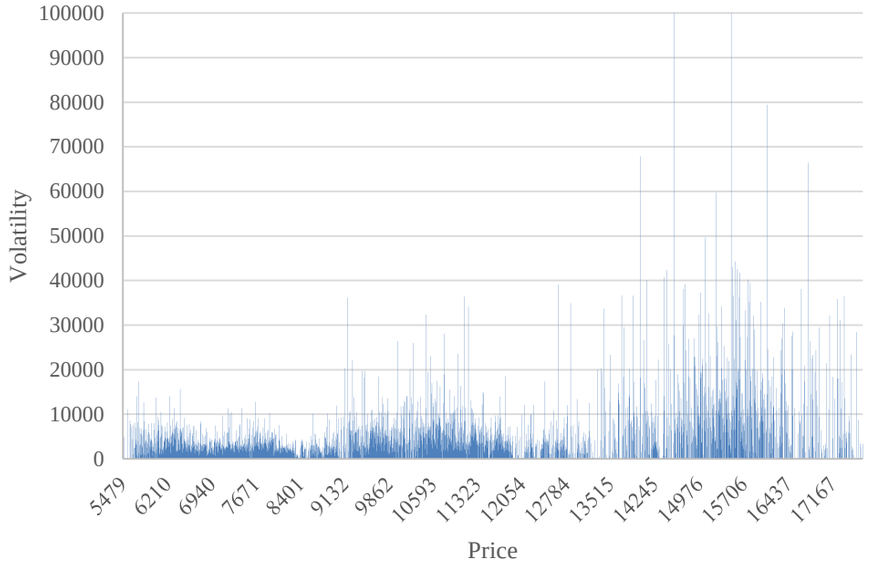
Illustrative volatility graphs

Bubble is present



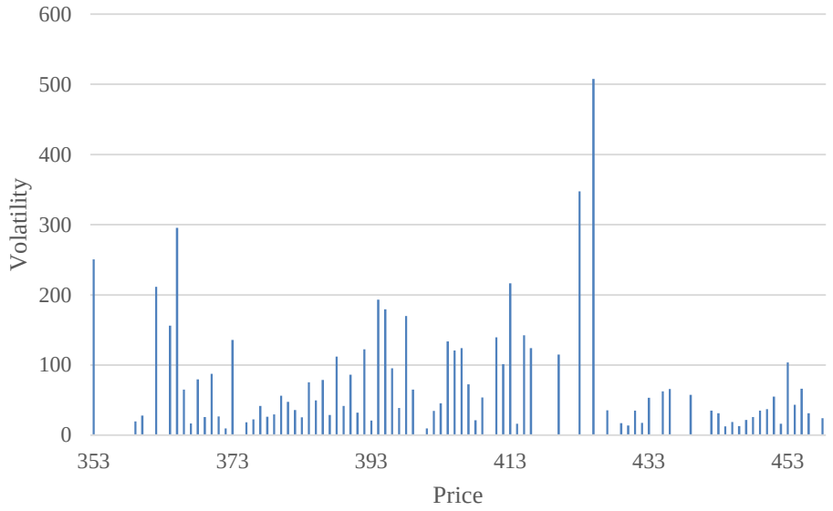


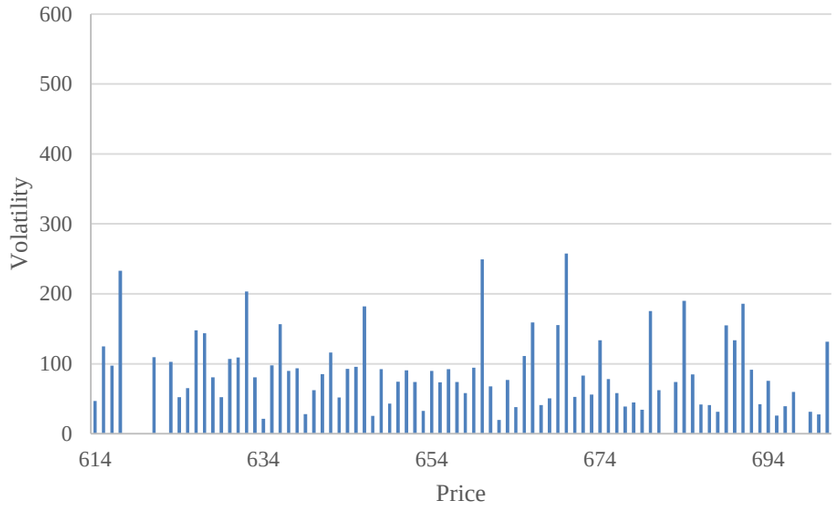


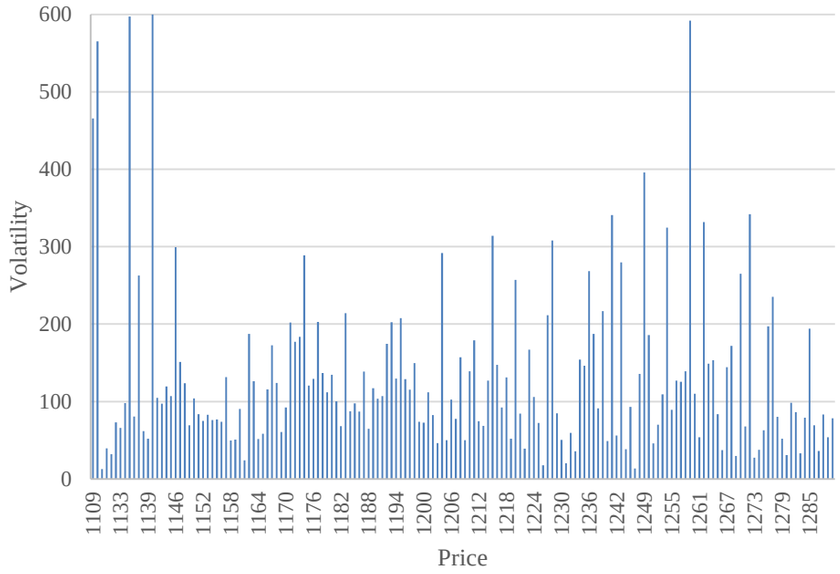


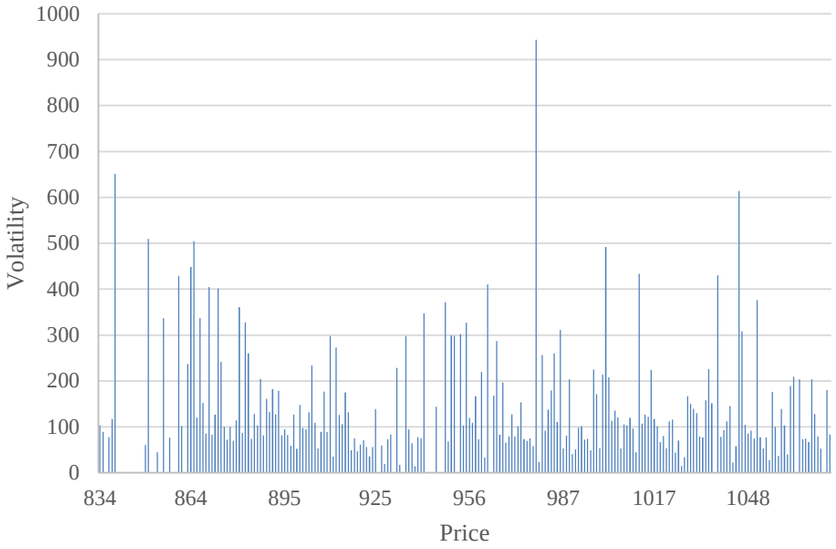
Illustrative volatility graphs

Bubble is absent









Backtesting the methodology on Bitcoin

Summary of test results (2017)

Start date	Daily change	End date	Daily change	Days	Min Price	Max Price	Daily volatility	Test conclusive?	Bubble?
17-Jan-17	8.90%	18-Mar-17	-9.12%	61	$ 834.0	$ 1,322.7	3.37%	N	
27-Mar-17	7.61%	27-May-17	-7.29%	62	$ 960.2	$ 2,779.0	3.16%	Y	Y
29-May-17	7.33%	12-Jun-17	-12.41%	15	$ 2,150.0	$ 2,975.0	5.54%	Y	Y
17-Jun-17	7.10%	15-Jul-17	-10.27%	29	$ 1,967.3	$ 2,785.2	4.11%	N	
17-Jul-17	15.28%	25-Jul-17	-7.54%	9	$ 1,897.9	$ 2,950.0	11.21%	Y	Y
5-Aug-17	13.86%	2-Sep-17	-7.55%	29	$ 2,840.3	$ 4,980.0	4.28%	N	
15-Sep-17	14.20%	21-Sep-17	-6.97%	7	$ 2,960.6	$ 4,117.0	7.95%	Y	Y
25-Sep-17	7.59%	24-Oct-17	-6.73%	30	$ 3,666.0	$ 6,178.5	3.79%	Y	Y
29-Oct-17	7.61%	9-Dec-17	-7.62%	42	$ 5,480.0	$ 17,685.0	6.18%	Y	Y
11-Dec-17	11.72%	19-Dec-17	-8.39%	9	$ 15,269.0	$ 19,999.0	6.54%	Y	Y
26-Dec-17	15.09%	28-Dec-17	-6.89%	3	$ 13,562.0	$ 16,500.0	11.53%	Y	Y
31-Dec-17	11.50%	8-Jan-18	-7.82%	9	$ 12,540.0	$ 17,222.0	7.37%	Y	Y

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THANK YOU

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