



**1st Capacity Building Seminar on
Key aspects of Risk Management
in Life Insurance Companies**

**MOODY'S
ANALYTICS**

**Economic Scenario Generator
and Stochastic Modelling**

Jonathan Lau, FIA

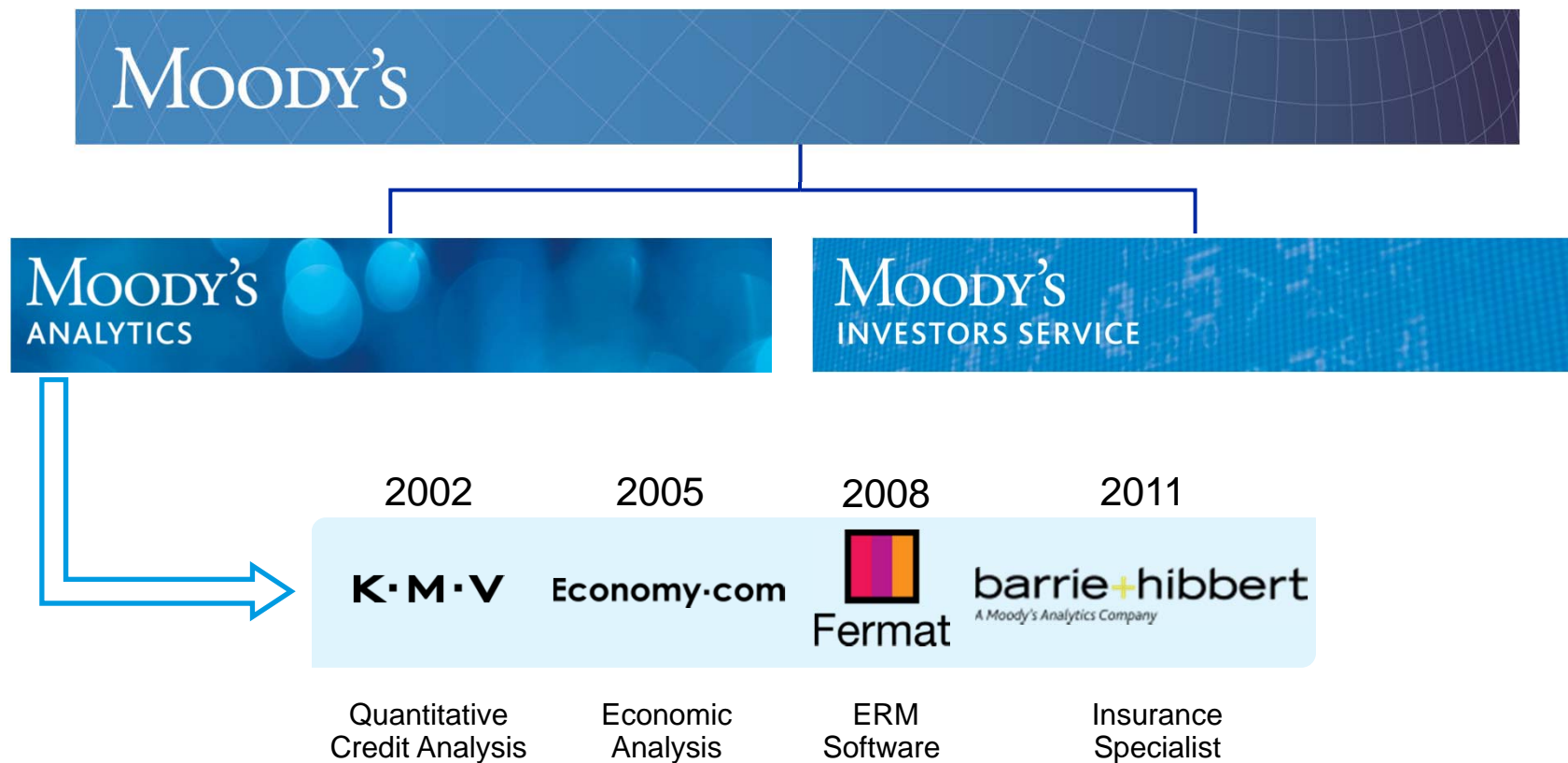
Moody's Analytics

9 August 2014, Mumbai

Stochastic Modelling for Insurance

Economic Scenario Generator

Moody's Analytics Overview – beyond credit ratings



Research-Led Risk Management Solutions for Financial Institutions

Strong & Growing Presence in the Global Insurance Market

- » 200 Insurance Relationships
- » 70% of Insurers in Global Fortune 500 clients
- » Combine B&H & Moody's expertise to extend what we offer to the insurance sector
- » Focus on supporting the Capital modeling & ERM activities of insurers
- » Leveraging both the research expertise and enterprise infrastructure.



Agenda – Stochastic Modelling for Insurance Companies

- » Stochastic Modelling for Insurance and Asset Management
 - ESG (Economic Scenario Generator) Overview
 - Different Uses of ESGs
- » ESG Model Selection and Calibration
- » Stochastic Modelling for Indian Insurers and Key Challenges
- » ESG Models

Objectives

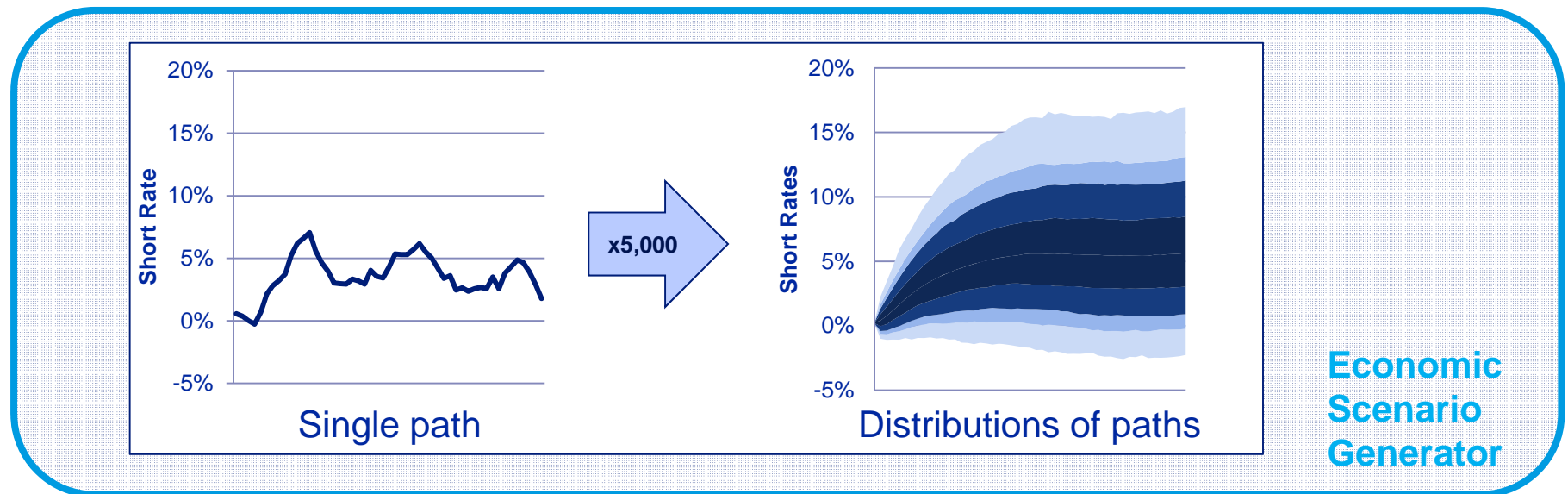
- » Explain the use of ESG by insurance companies
 - **Market Consistent ESG** and **Real World ESG**
- » Explain the approach to validating ESGs for insurance companies
 - Choosing the appropriate asset model
 - ESG is NOT a black-box
 - Validation and documentation
 - The challenges for calibrating models to Indian markets
 - Answering the challenges for Indian Insurers
- » Example of ESG models (Interest Rates, Equity and Credit)

1

Overview – Stochastic Modelling

What are Stochastic Simulations?

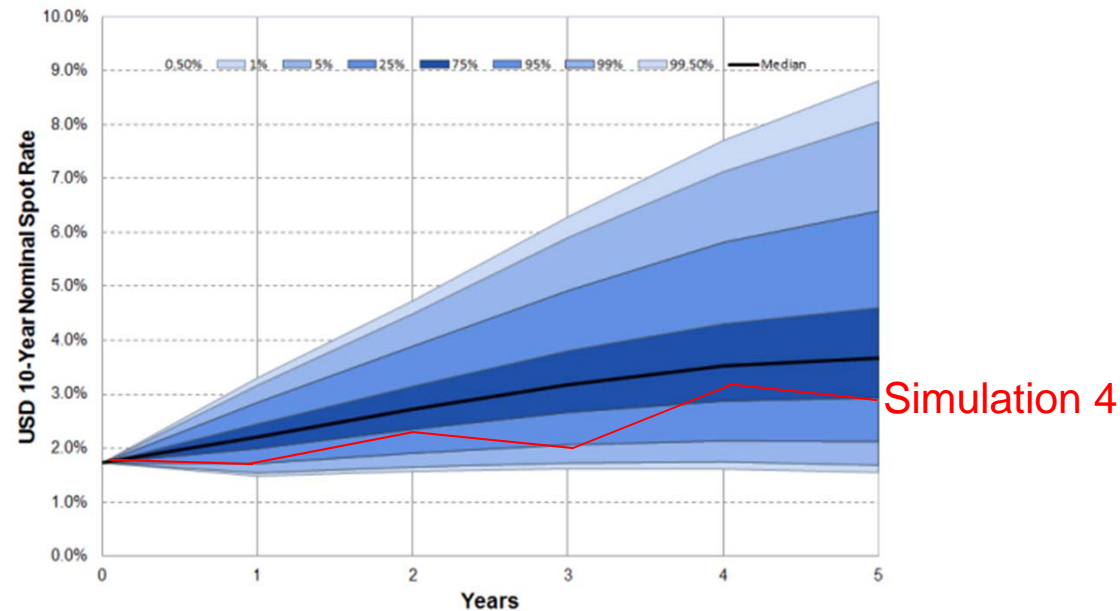
- » Future is **unknown**
- » We may have **expectations** about the future but we are never **certain** about it
- » Simulate **many** future scenarios based on mathematical stochastic models
- » Use scenarios in **Monte Carlo** simulations by ALM systems
- » **Average** of the **Monte Carlo** simulations converge to our expectation



Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods.

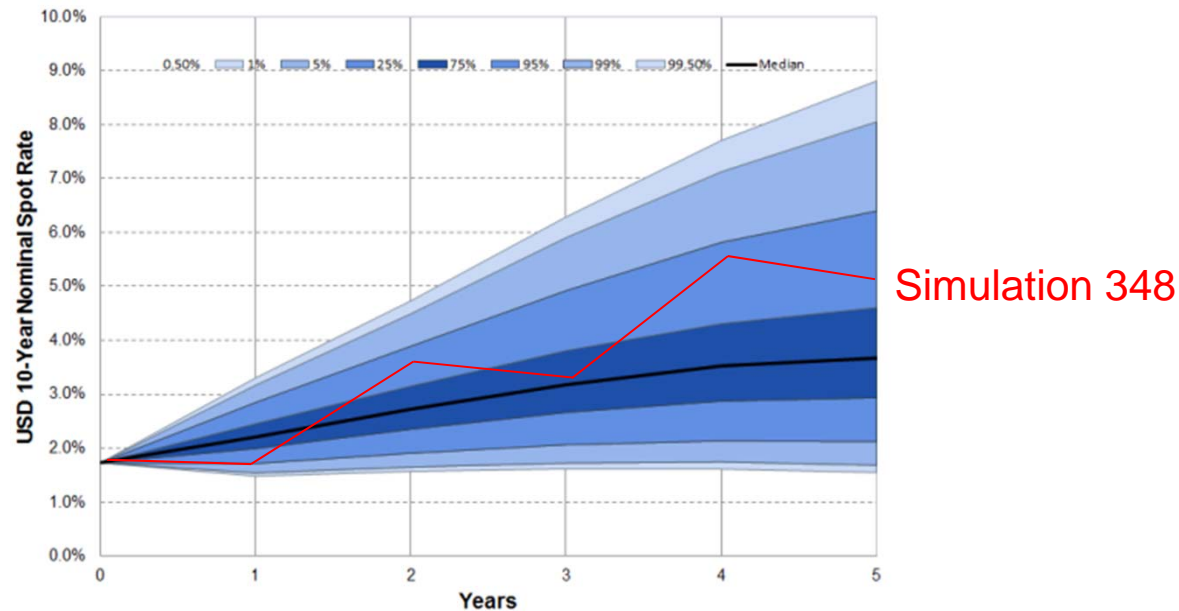
Example: 10-year Spot Rate Projected over 5 years



Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods

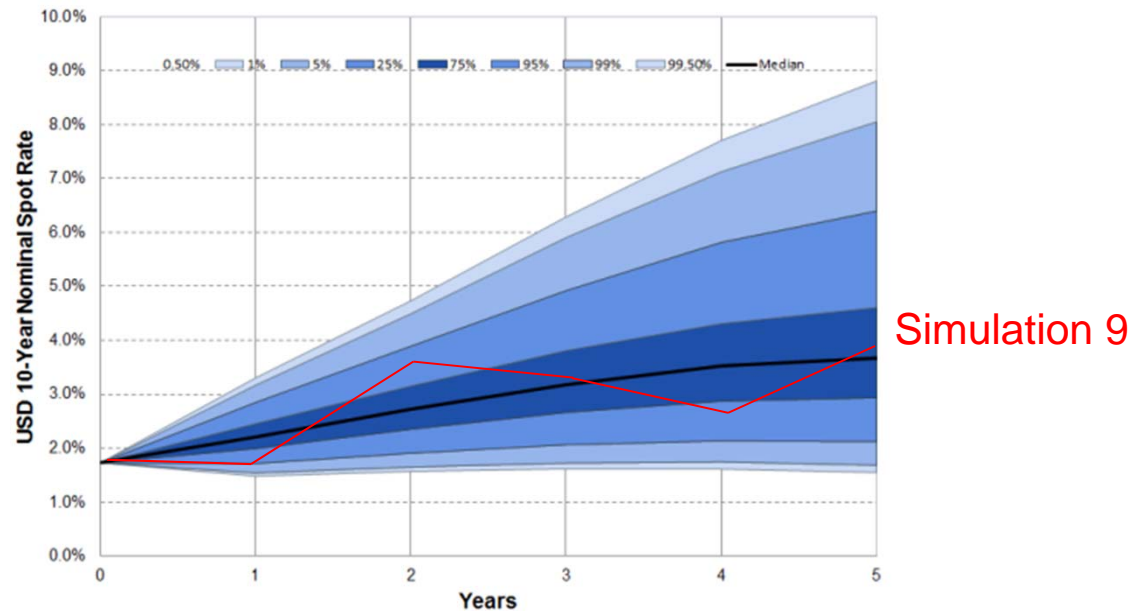
Example: 10-year Spot Rate Projected over 5 years



Stochastic Economic Scenario Generator

The ESG uses Monte Carlo Simulation to generate thousands of simulations of risk factors across multiple time periods

Example: 10-year Spot Rate Projected over 5 years



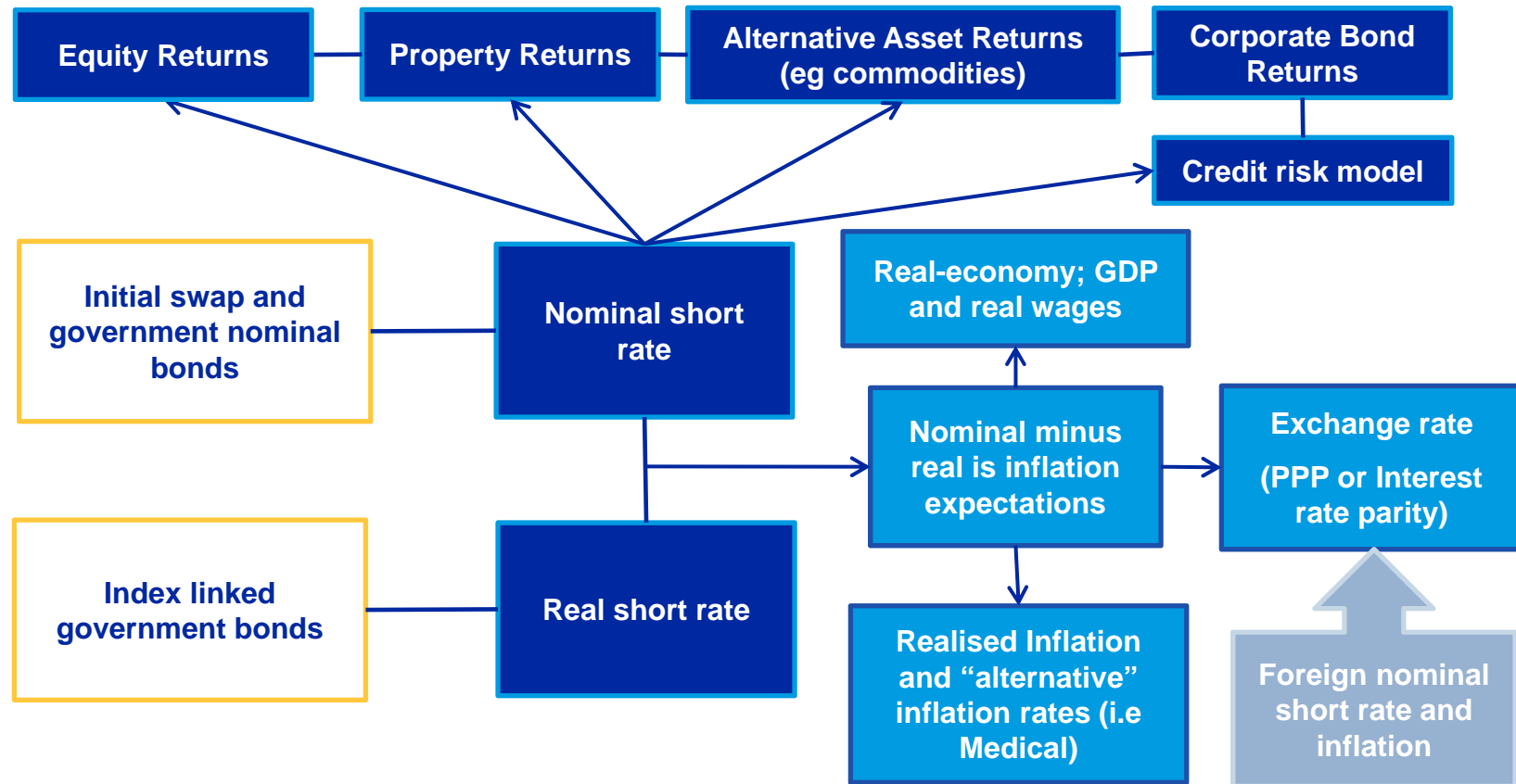
Risk Factors generated by the ESG

» The ESG generates Monte Carlo simulations for the joint behaviour of multiple risk factors :

- Nominal Interest Rates
- Real Interest Rates
- Inflation Indices
- Equity and dividend returns
- Property and rental returns
- Credit Spreads, rating transitions, risky bonds returns
- Alternative asset returns
- Interest rate implied volatility and equity implied volatility
- Exchange rates
- Macroeconomic indicators such as GDP, wage indices
- Non market risk such as mortality and lapse rates

» **Coherent modelling in Real World and Market Consistent environment**

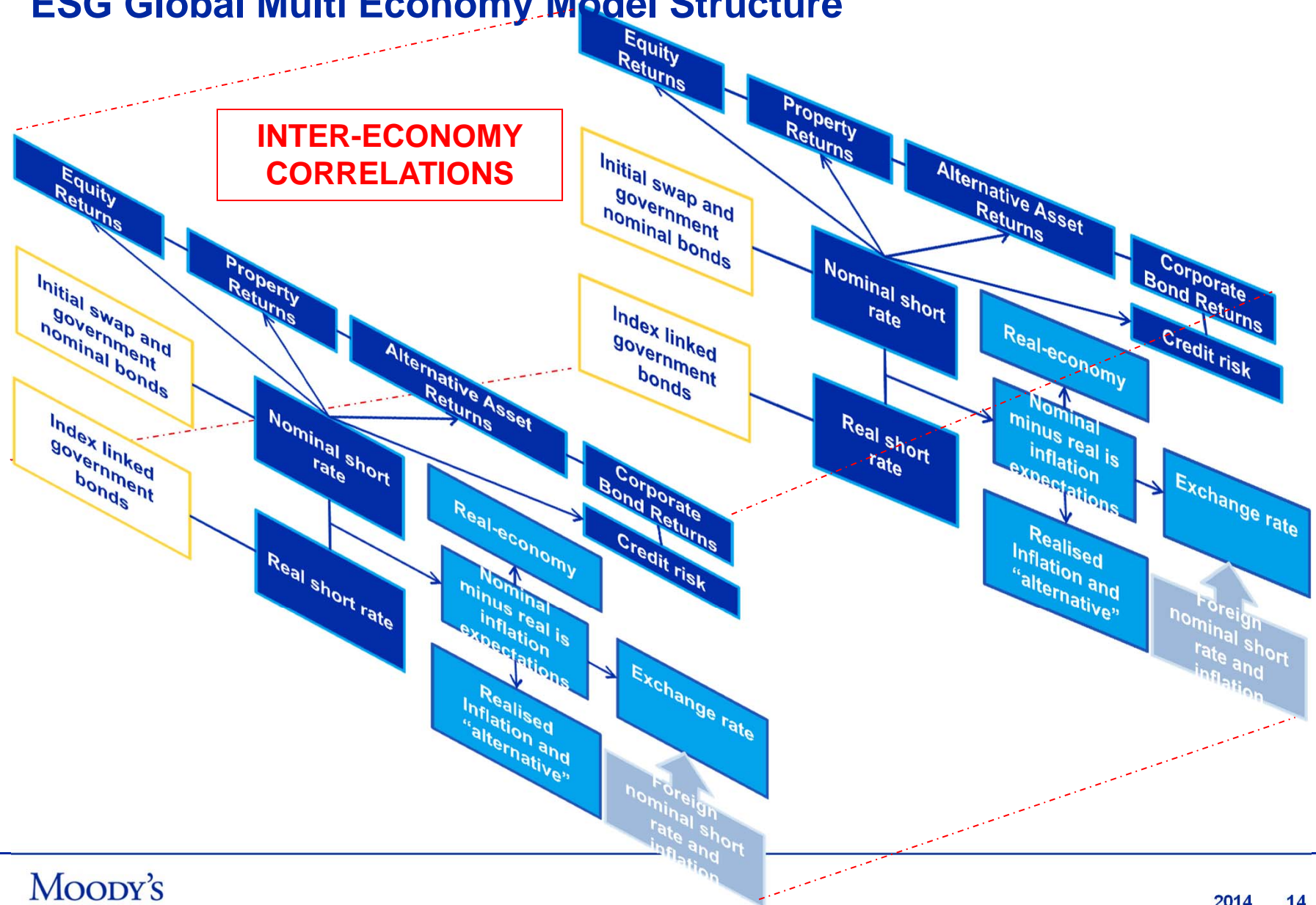
B&H Economy Model Structure



Joint distribution

- » Correlation relationships between shocks driving each model
- » Economically rational structure

ESG Global Multi Economy Model Structure



Use of the ESG in the insurance sector

Calculation of cost of options and guarantees
(EV, Fair Value, Best Estimate Reserves)

Technical Provision (Time Value)

Economic Capital calculation

Internal models, ORSA

ALM, Asset Allocation, Business Planning

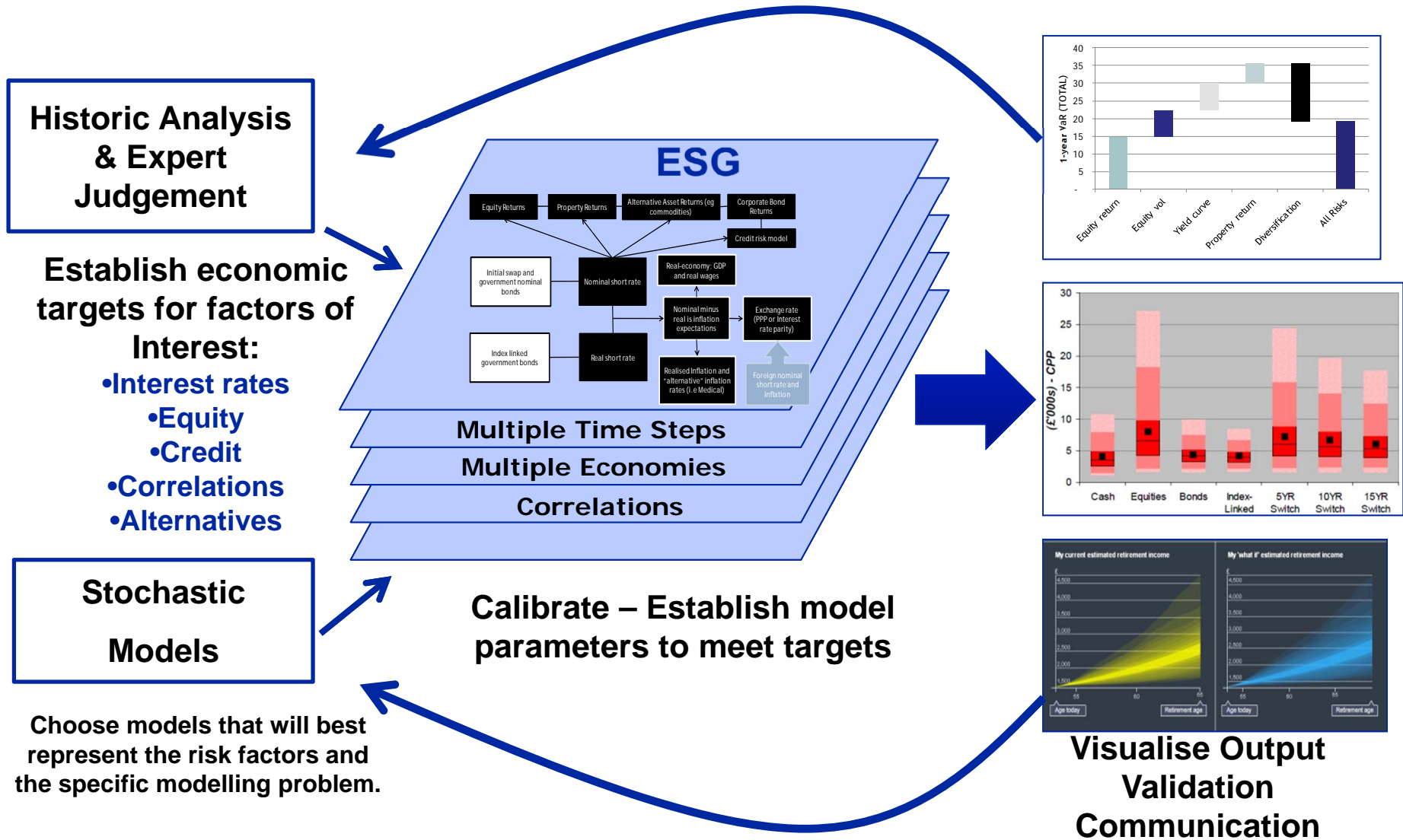
Hedging

Advanced uses of stochastic models

Pricing and product development

Retail advisory

Stochastic Economic Scenario Generator



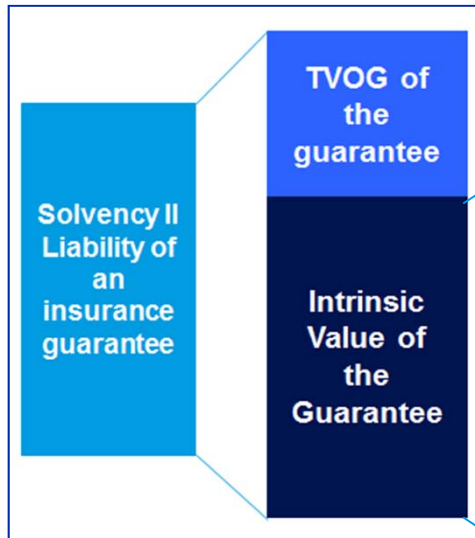
Market Consistent ESG – Example

Market Consistent ESG

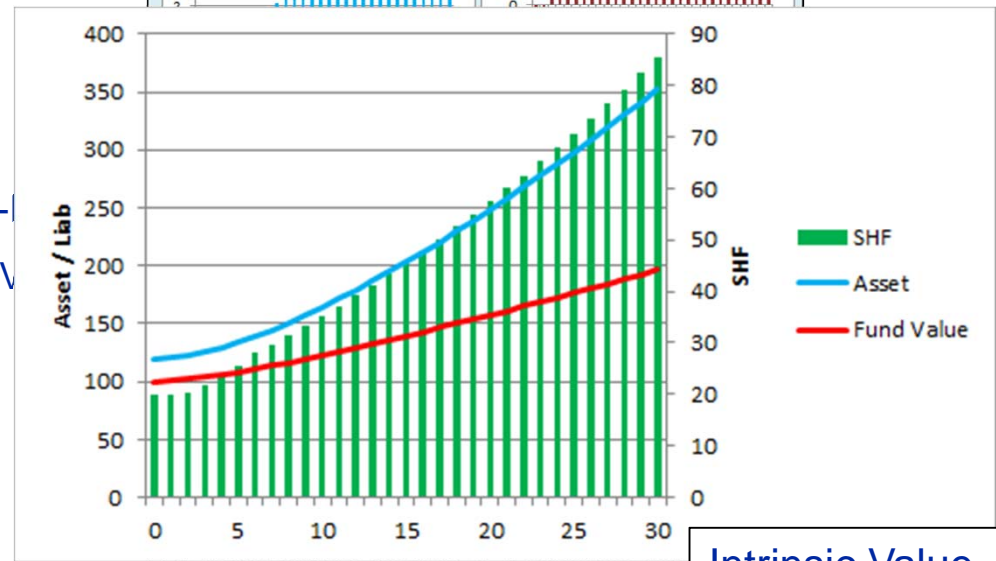
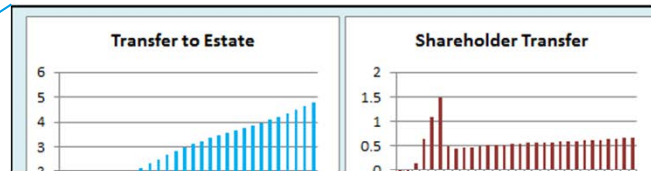
- » Mathematical models used to value complex cashflows
 - Can be asset or liability cashflow
 - No arbitrage theory
- » Model prices replicate market prices
 - Models calibrated to market prices to achieve this
- » Model simulates scenarios that can be used to value cashflows where a market price does not exist

Valuation of Path Dependent Insurance Liability

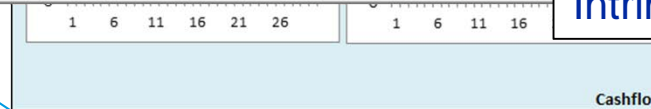
Deterministic Market-Consistent Roll Forward Using Risk-Free Rates



Risk-free Roll-
Deterministic V

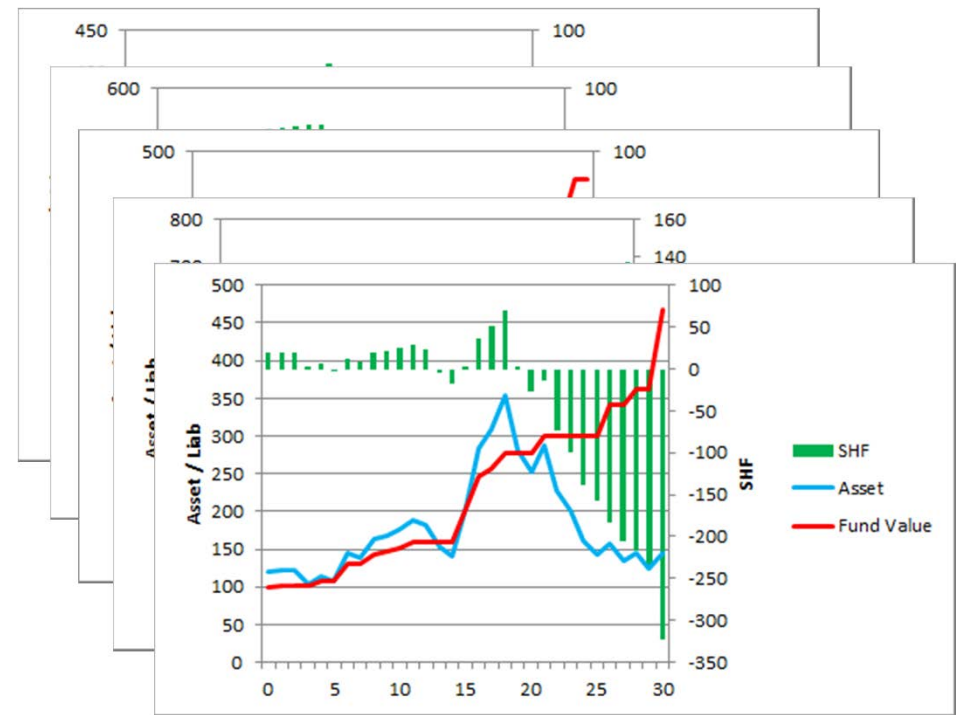
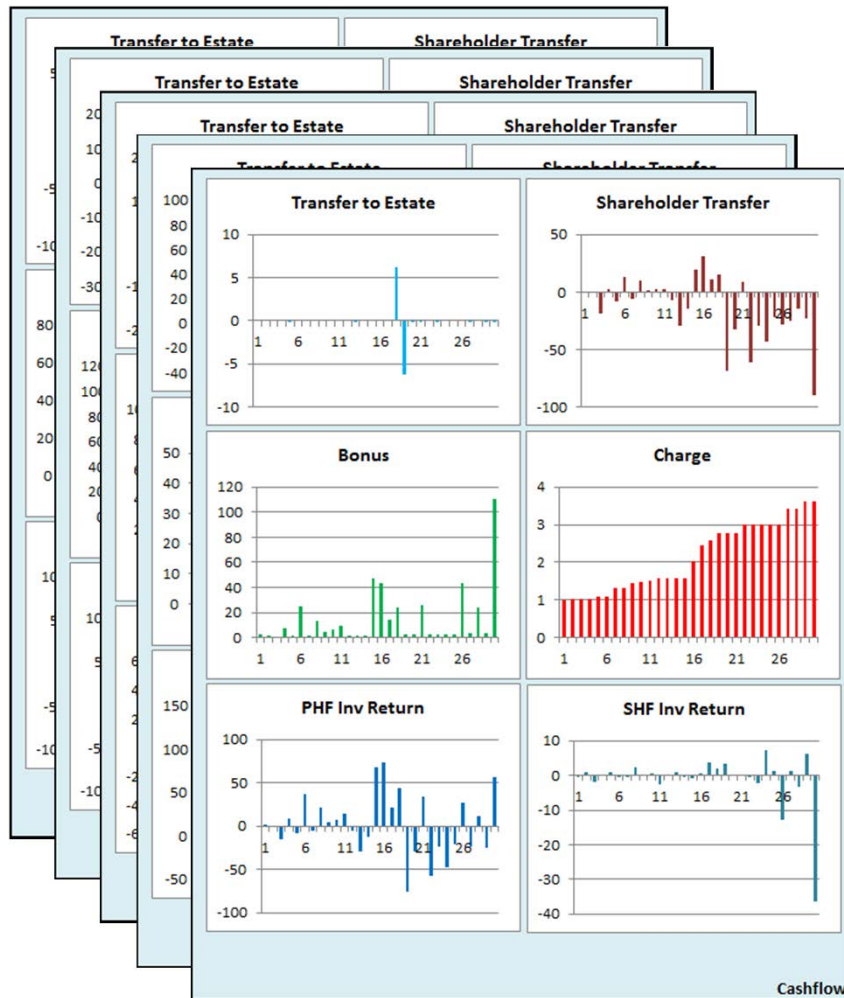


Intrinsic Value = 0



Valuation of Path Dependent Insurance Liability

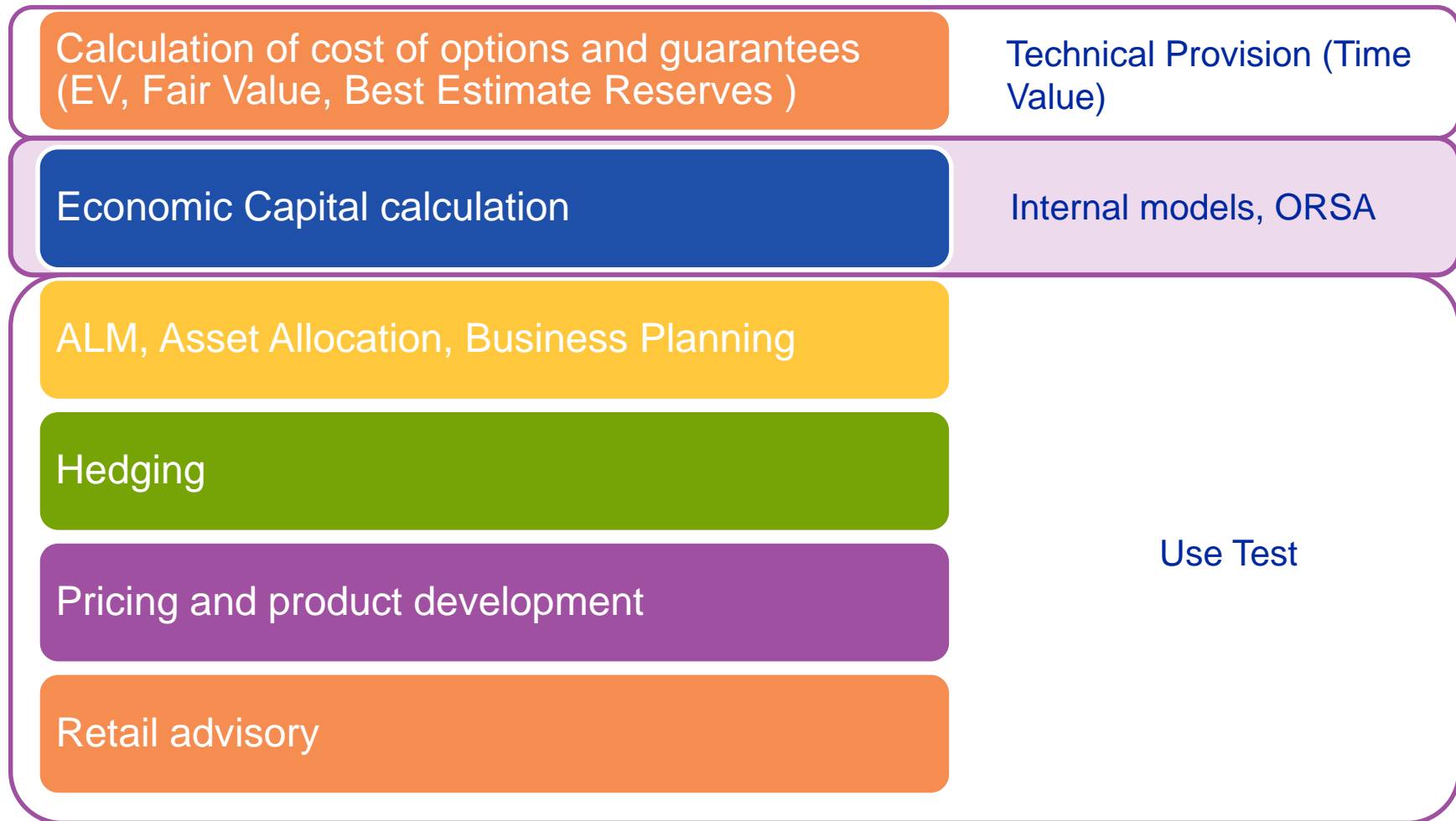
Run ALM Many Times Using Stochastic Market-Consistent Scenarios



- » **Average value** represents stochastic value
- » The **difference** between the stochastic value and the intrinsic value is the **time value**

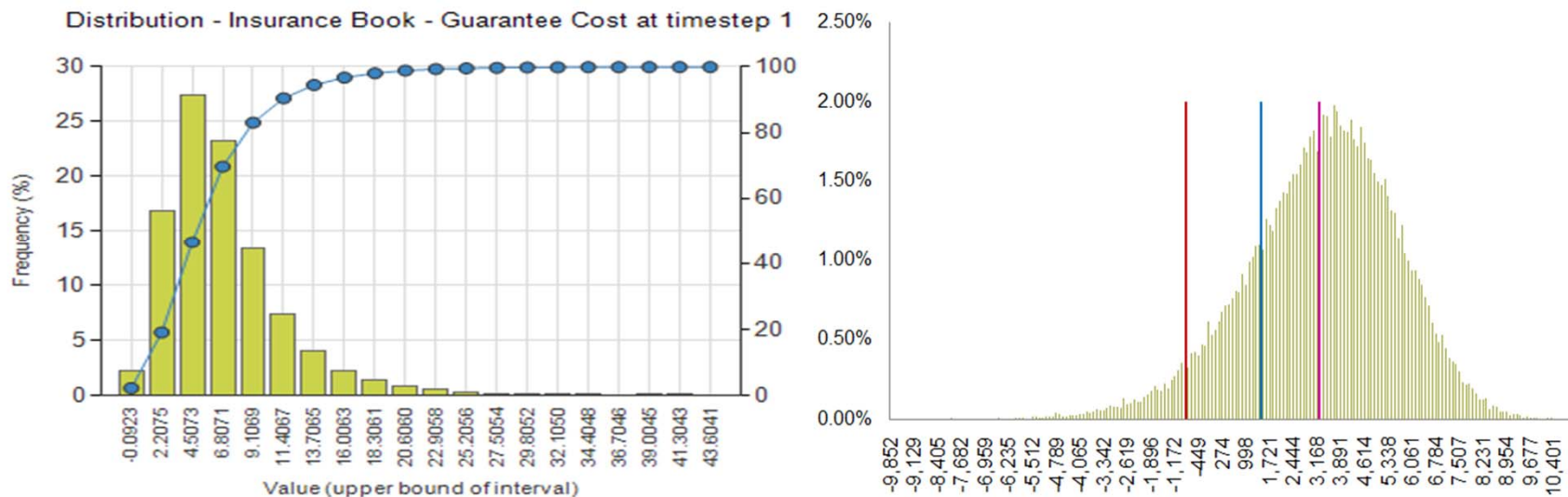
Real World ESG – Example

Use of the ESG in the insurance sector



Example Use – Determine the tail for SCR

- » Real World ESG models are calibrated to realistic distributional targets
- » Probability distribution of risk factors (equity, interest rates, etc) translated into probability distribution of the Net Asset Value



- » Holistic approach captures dependency between risk factors
- » Internal model approach also contains Use Test information such as risk exposure decomposition and reverse stress test material.

Approach (1): Stress and Correlate

Interest Rate	Equity/Property	Credit	Other Markets
Shift	Level	Spread Level	FX
Twist	Volatility	Transitions	
Curvature			
Volatility			

Non ESG Risks		
Catastrophe	Longevity	Lapse
	Mortality	Expense
	Morbidity	

V@R

IR	Eq	Credit	Misc
22,515,826	74,565,212	5,123	0
54,486,312	58,321,588	551,314	
484			
224,182			

V@R

UW - NL	UW - Life	Lapse	Expense
1,561,516	546,546	5,465,412	4,462,132
2,651,684	9,851,951	-5,568	
93,232			
961,323			
88,896			



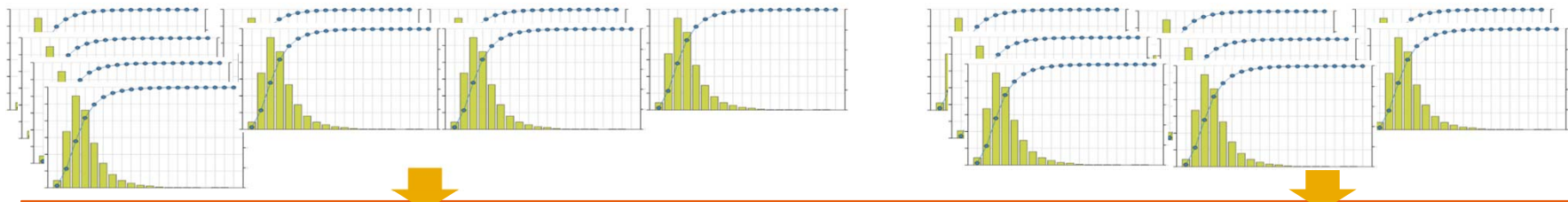
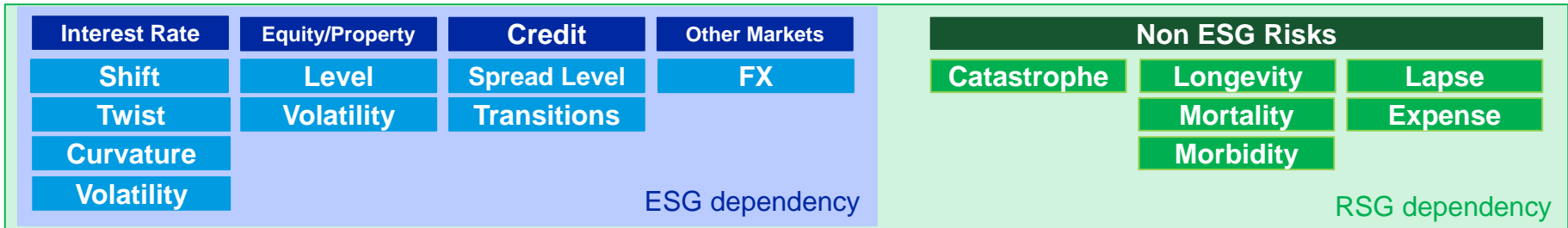
*Capital Aggregation Matrix does not reflect actual correlations between risk factors



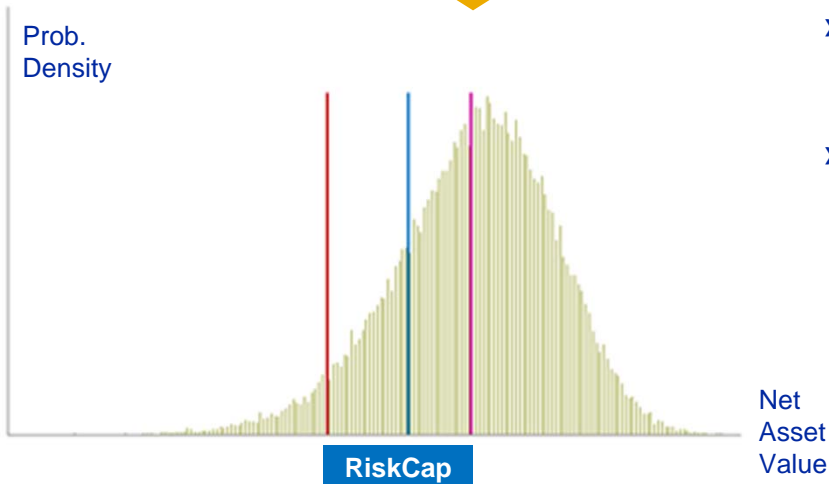
Problems:

- » Does not capture dependency effects that are firm specific
- » Capital aggregation matrix requires subjective input and does not reflect actual correlations between risk factors

Approach (2): Holistic Balance Sheet



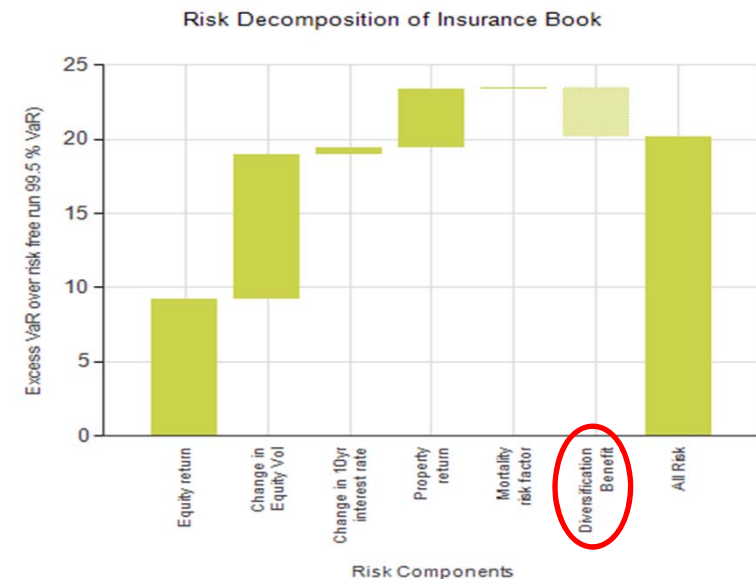
Cashflow engine



- » Risk Capital reflect company specific risk profile
- » Contains useful metrics beyond Stress and Correlate approach
 - Probability of insolvency
 - Upside potential statistics
 - Conditional tail expectation

Solvency Capital / Economic Capital

- » Capital allocation
 - By risk factors
 - By line of businesses/products
- » Capital efficiency through optimising
 - Investment strategy
 - Management action
 - New business strategy
 - M&A strategy
- » Risk framework that are specific to the insurance company
 - Specific to risk profile and cashflow of the company
 - Provide financial confidence internally and externally

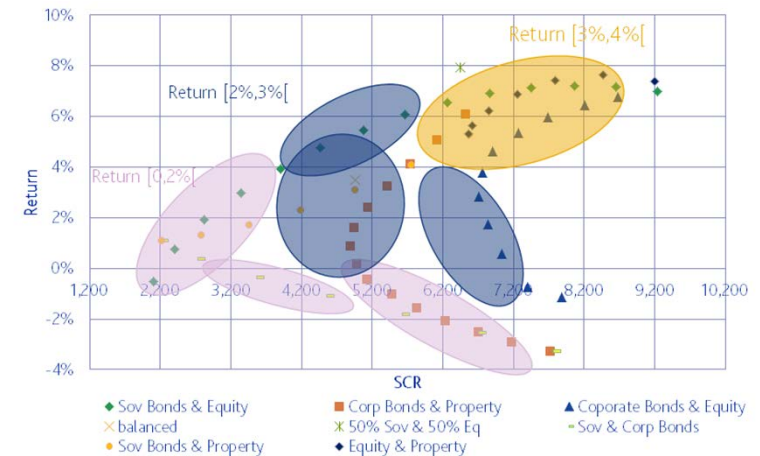


Other uses of Real World ESG

Experience from B&H

Strategic Asset Allocation and Portfolio Optimisation

- » Maximises investment returns
 - Minimises volatility
 - Minimises VaR
 - Minimises risk capital
- » Used by insurance companies (life and non-life), pensions funds and asset managers

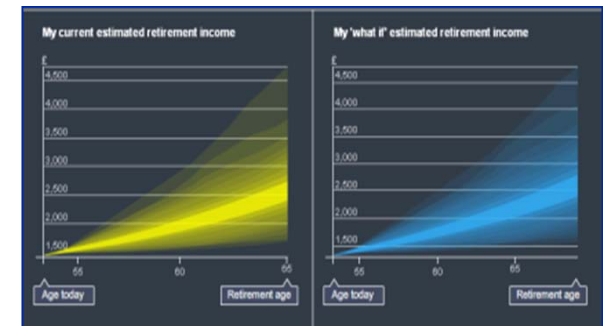


ALM Hedging

- » Matching investment strategies to liability profile

Retail Advisory

- » Spectrum charts instead of simplistic “high-medium-low” numbers
- » Welcomed by regulators and policyholders for increased transparency



Choosing Stochastic Models

Stylised Facts & Data

Goal is to produce **realistic** and **justifiable** projections of financial and macroeconomic variables.

Use all credible historical **data**, market expectations via **options** and expert **judgement**.

Our approach involves 3 main activities:

- 1) Developing and documenting a set of stylized facts and beliefs.
- 2) Use these to select/build/structure, calibrate and validate models.
- 3) Look at real world markets to validate and review the stylized facts and models.

These are all ongoing activities:

- » Frequent calibration
- » Regular Real World Target updates and methodology reviews

Weighting Schemes & Data

Calibration is an art

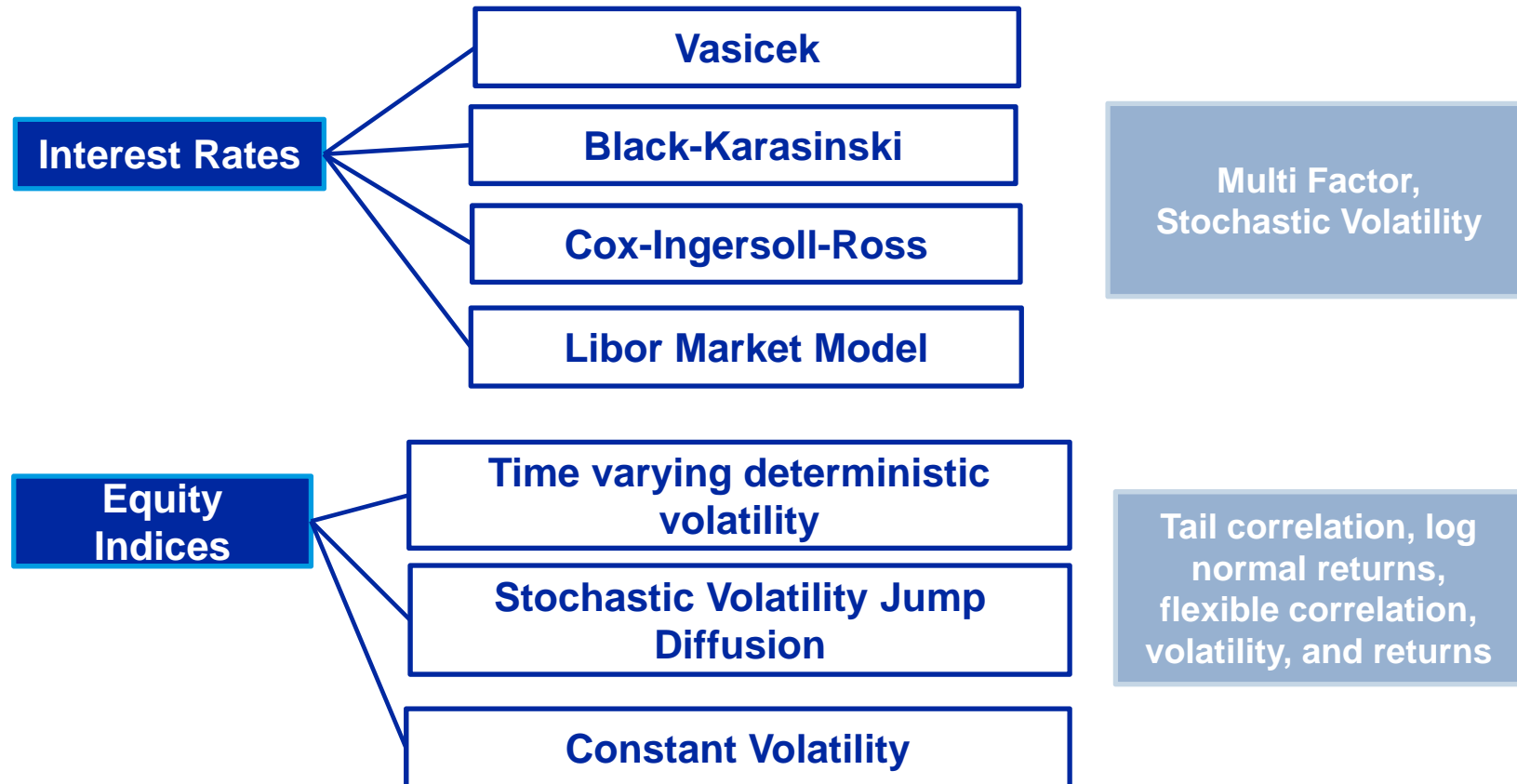
- » Subjectivity in: data sources, data policies, weighting, judgement

Goal is to produce **realistic** and **justifiable** projections of financial and macroeconomic variables.

Use all credible data available:

- » Combine with market data of expectations: e.g. option implied volatility, consensus data
- » Filter and clean data: liquidity of instruments, depth of market
- » Exponentially-weighted moving average ensures more weight is placed on recent observations
- » Consistency across asset classes

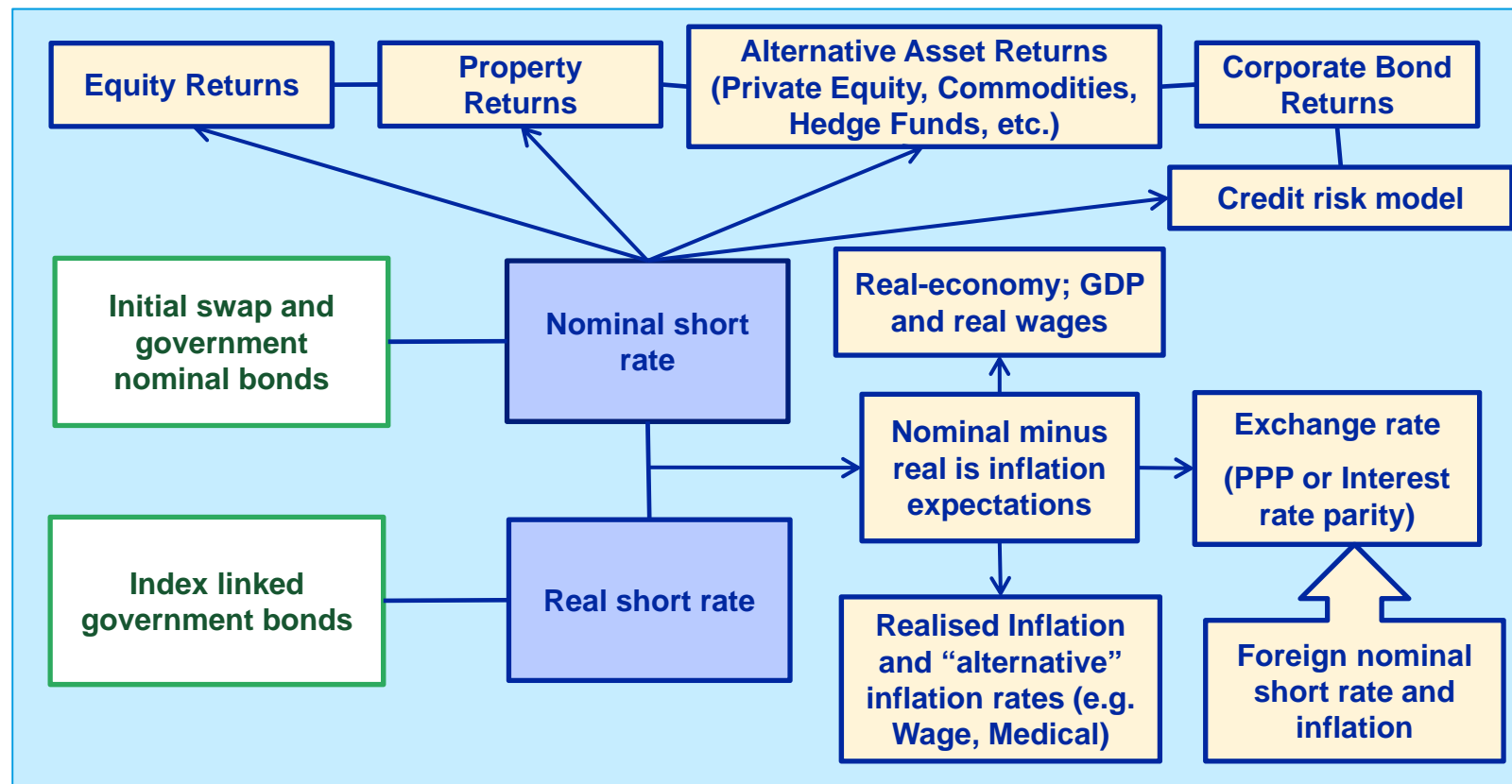
Models & Calibrations



And others for credit, inflation, exchange rates, MBS, derivatives etc.

All models documented in academic literature and MA research papers

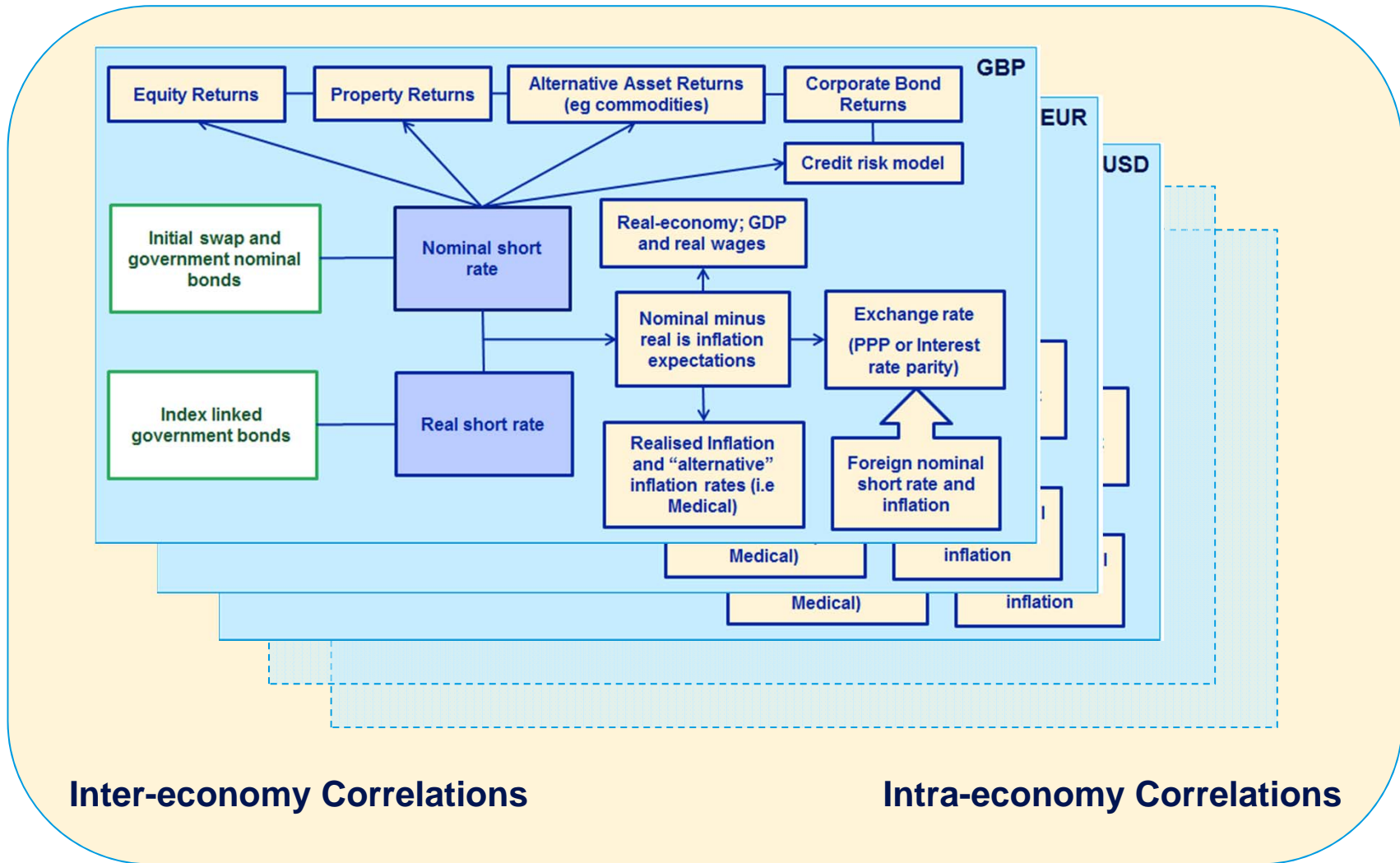
B&H Economic Scenario Generator (ESG)



Mathematical stochastic models simulates returns of financial assets

Correlation ensures plausible economic relationship between asset classes and economies

Correlations and Dynamic Behaviours



Communicating Stochastic Models

Knowledge transfer

- » MA/B&H ESG is NOT a black box.
 - Transparency is a core value to the B&H services
- » Knowledge transfer is provided through
 - ESG trainings
 - Bespoke trainings/workshops
 - Detailed model documentations
 - Calibration reports (economic analysis + validation reports)
 - ESG Users group meetings (current topics and presentation of new models)
 - Access to online research library
 - Access to technical support

Knowledge Database

MOODY'S ANALYTICS | Risk Management Solutions
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Barrie & Hibbert Knowledge Base

Through our work with clients on specific issues, we frequently uncover revealing insights about some of the major challenges facing whole sectors.

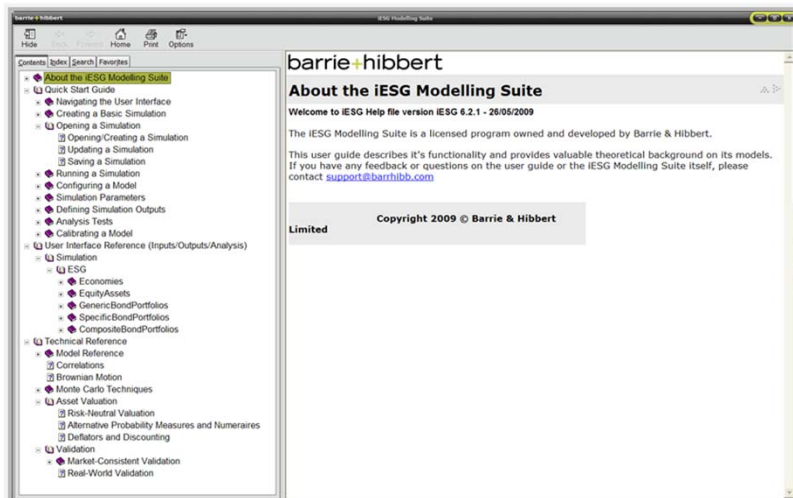
Wherever possible we share our research with our clients and more widely to stimulate debate and discussion. This type of dialogue is not just about providing thought leadership, it's also an important way to refine our research and models while continually enhancing the value of our work for all our clients.

We store all our research in the Barrie & Hibbert Knowledge Base. Anyone can view the Knowledge Base and its document

- » Models methodologies, Economic research,
- » Calibration documentation and Technical Advisory Panel

Documentation

Help menu in ESG



Technical documentation

2.2 Calibration method

Using the fact that spread is an increasing function of π value (with all other parameters fixed), we can write the cumulative distribution function of spread in terms of the cumulative distribution function of π as follows:

$$\Pr[S < s] = \Pr[S(\pi) < s] = \Pr[\pi < \pi^{-1}(s)] = F(\pi^{-1}(s))$$

where F is the cumulative distribution function of the non-central chi-squared distribution and $\pi^{-1}(s)$ is the π value corresponding to the spread level s , obtained by numerical inversion of the relationships above (spread \rightarrow default probability $\rightarrow \pi$).

Similarly, any desired percentile of the spread distribution can be found by computing the corresponding percentile of the π distribution and converting that π value to a spread.

Since our calibration targets are expressed in terms of *moments* of the spread distribution (i.e., average and standard deviation), we use the following expression for the p^{th} moment:

$$E[S^p] = \int_0^{\infty} p s^{p-1} \Pr[S > s] ds$$

Calibration report

1 Real-World Interest Rate Calibrations

1.1 Nominal Rates - Extended 2-factor Black-Karasinski - Time-Varying Term Premium

Exhibit 1.1.1: Parameters in Extended 2FBK

Calibration Information		
Model:	Extended 2-factor Black-Karasinski	
Economy:	USD	
Calibrated To:	Govt. Nominal Bond Yields	
Term premium:	Time-varying	
Calibration Parameters		
Parameter	30 Sep 2009	30 Jun 2009
κ_1	0.3000	-
κ_2	0.0750	-
κ_3	0.3000	-
κ_4	0.2500	-

Commentary
Parameters in the extended 2FBK (exhibit 1.1.1) model are calibrated to historical volatility of the short rate and 10 year spot yields. These are updated once per year (Q2). The initial instantaneous nominal forward curve (exhibit 1.1.3 shows the initial curve and associated spot curve) is a direct input in the extended 2FBK model. The curve is derived by fitting regression splines to the market bond yields (exhibit 1.1.2) and extrapolating to a limiting forward rate. You can read more in Antonio and Roseburgh (KB article 2008-1145) and Sorensen (KB article 2008). See Li, Liu and Sorensen for latest targets (Knowledge Base article 2009-1538). The extrapolation targets are updated in Q3.

The target path for the short rate (exhibit 1.1.4) is economically sensible in that: a) We do not make frequent changes to our long term interest rate expectations, b) interest rates are mean reverting and c) short term forward rates have informational content about future interest rates while long term forward rates do not. Read more in Liu and Sorensen (KB article 2009-1521). This target path is updated each calibration round. We would only expect to see larger changes between year 1 and 10 from one quarter to another.

We set the market price of risk associated with the two factors equal to one other. We calibrate this to a trade-off between a) fitting the target path for the short rate (exhibit 1.1.4) and b) ensuring that risk premia on government bonds are smooth. You can read more in Cheyne and Liu (KB article 2009-1525).

With the model calibration and the arbitrage free interest rate model we can obtain a projection of the nominal short rate, 10 year spot yield, yield curves and excess return on government bonds and implied distributions. Such summary statistics can be found in any remaining exhibits. Short rates are continuously compounded to align with our targets. Yields and excess returns are annually compounded.

Exhibit 1.1.4: Economic Target and ESG Valuation Path

Exhibit 1.1.2: Market Bond Yield vs Spline Model Fit

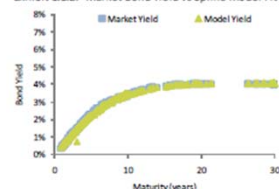


Exhibit 1.1.3: Implied Spot and Forward Curve

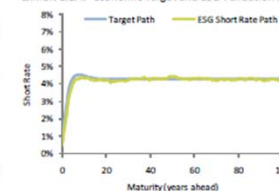
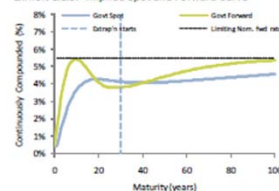


Exhibit 1.1.5: Central Distribution, Short Rate

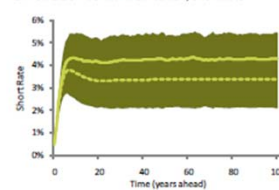
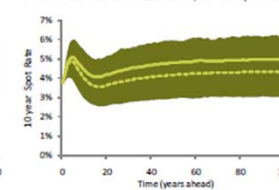


Exhibit 1.1.6: Central Distribution, 10 Year Spot Rate



The ESG proposition of B&H

» Software

- Professional software, Intuitive User interface
- Compatible with many operating systems and ALM solutions
- Includes an API
- Grid computing

» ESG modelling

- Joint stochastic modelling of multiple assets, multiple economies, multiple use
- Bond portfolios and composite portfolios
- MBS and derivatives (FRNs, swaps, swaptions, options...).

» Calibration Services

- Standard calibrations for a variety of economies and variety of assets
- Bespoke calibrations services
- Access to calibrations tools
- Economic research
- Automation platform

» Support, maintenance, training

- Support
- Training
- Documentation
- Maintenance services

Key Challenges for Indian Insurers

Challenges in Indian Capital Markets

Mathematical assets need to be calibrated to market data (bond yields, equity prices, etc)

- » Lack of good quality **data**
 - Data coverage is not consistent
 - Market data does not have long enough history
 - Lack of liquidity in certain parts of asset market
 - Affects frequency of data
 - Bid-Offer spread/transaction costs mask the underlying market values
- » High **volatility** challenges the stability of results

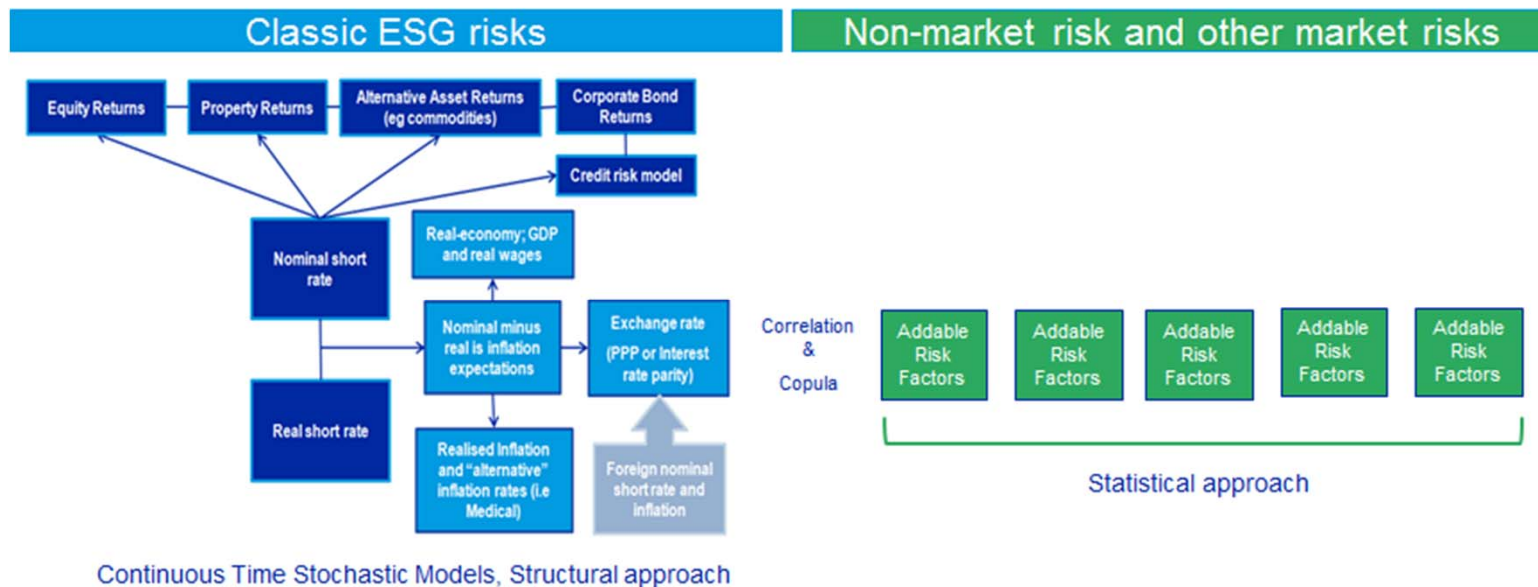
Answering the challenge:

- » Consistent choice of index across all economies for consistent and comparable data
- » Adjust weighting scheme to reflect the shorter data history
- » Set global targets to make economic sense of the stochastic scenarios instead of blindly calibrating to poor quality data. B&H provides model calibrations to 28+ economies.

Beyond Market Risks

Insurance capital should also cover non-market risks/insurance risks

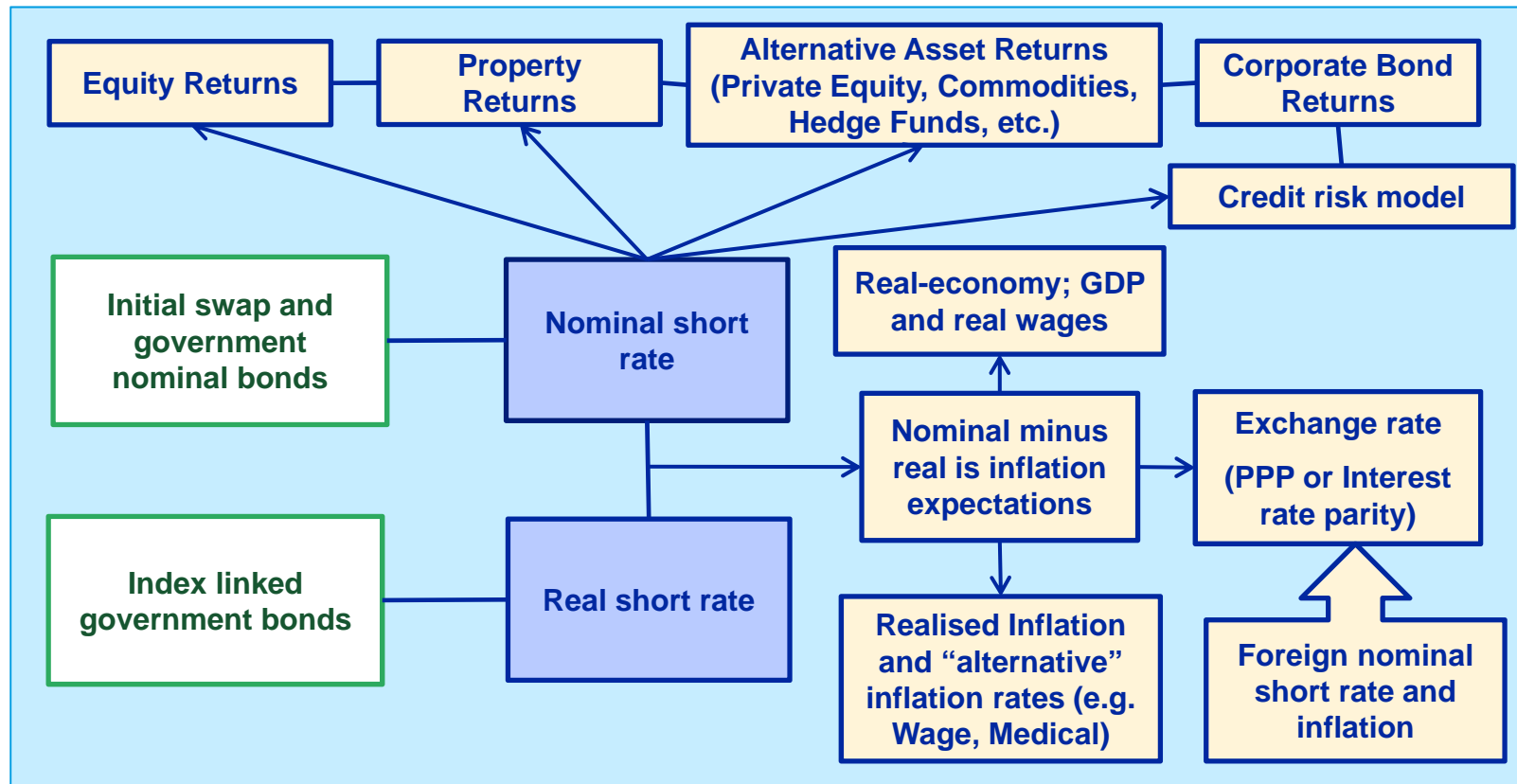
- » Non-market risks often only affects the Liability side of the balance sheet
- » Quite often insurance companies model non-market risks and market risks independently
 - But need to bear in mind potential dependencies. E.g. equity risks and lapse risks



The Models

Economic Scenario Generator

B&H Economic Scenario Generator (ESG)



Mathematical stochastic models simulates returns of financial assets

Correlation ensures plausible economic relationship between asset classes and economies

A

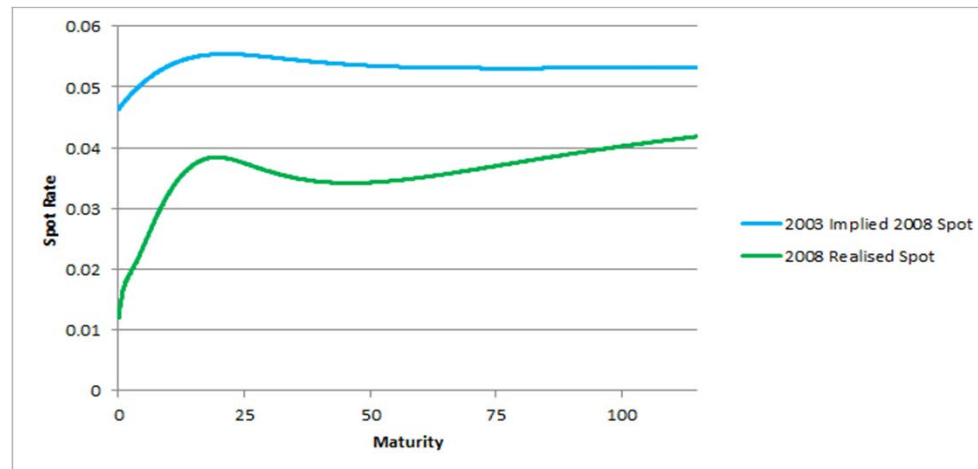
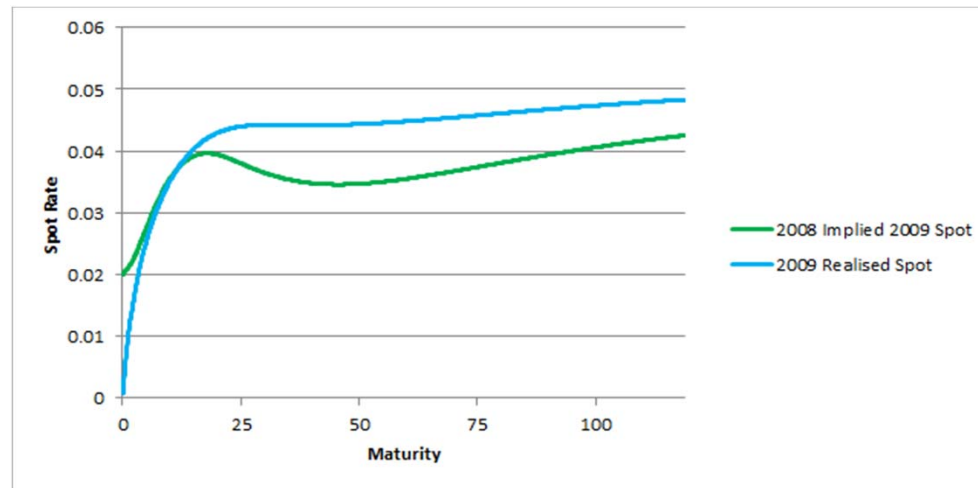
Nominal Rates

Extended 2Factor Black Karasinski (2FBK)

- » Log-Normal model
- » Simulate the short rate
- » Model dynamics:
 - Mean-Reverting processes

$$\begin{aligned}d \ln r(t) &= \alpha_1 (\ln m(t) - \ln r(t)) dt + \sigma_1 dZ_1 \\d \ln m(t) &= \alpha_2 (\mu'(t) - \ln m(t)) dt + \sigma_2 dZ_2\end{aligned}$$

Example Market Yield Curve vs Realised Yield Curve

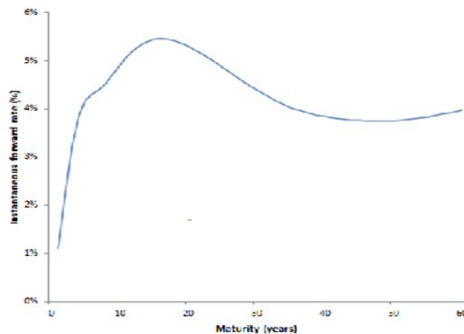


Additional Parameter: Market Price of Risk

- » In the risk-neutral world the expected return on all assets (e.g. bonds) is the risk-free rate.
- » In the reality investors demand a premium for holding bonds (e.g. Interest rate risk)
- » The *Market Price of Risk* (γ) adjusts the Brownian motions

$$\begin{aligned}dZ_1 &= dW_1 + \gamma_1(t)dt \\dZ_2 &= dW_2 + \gamma_2(t)dt\end{aligned}$$

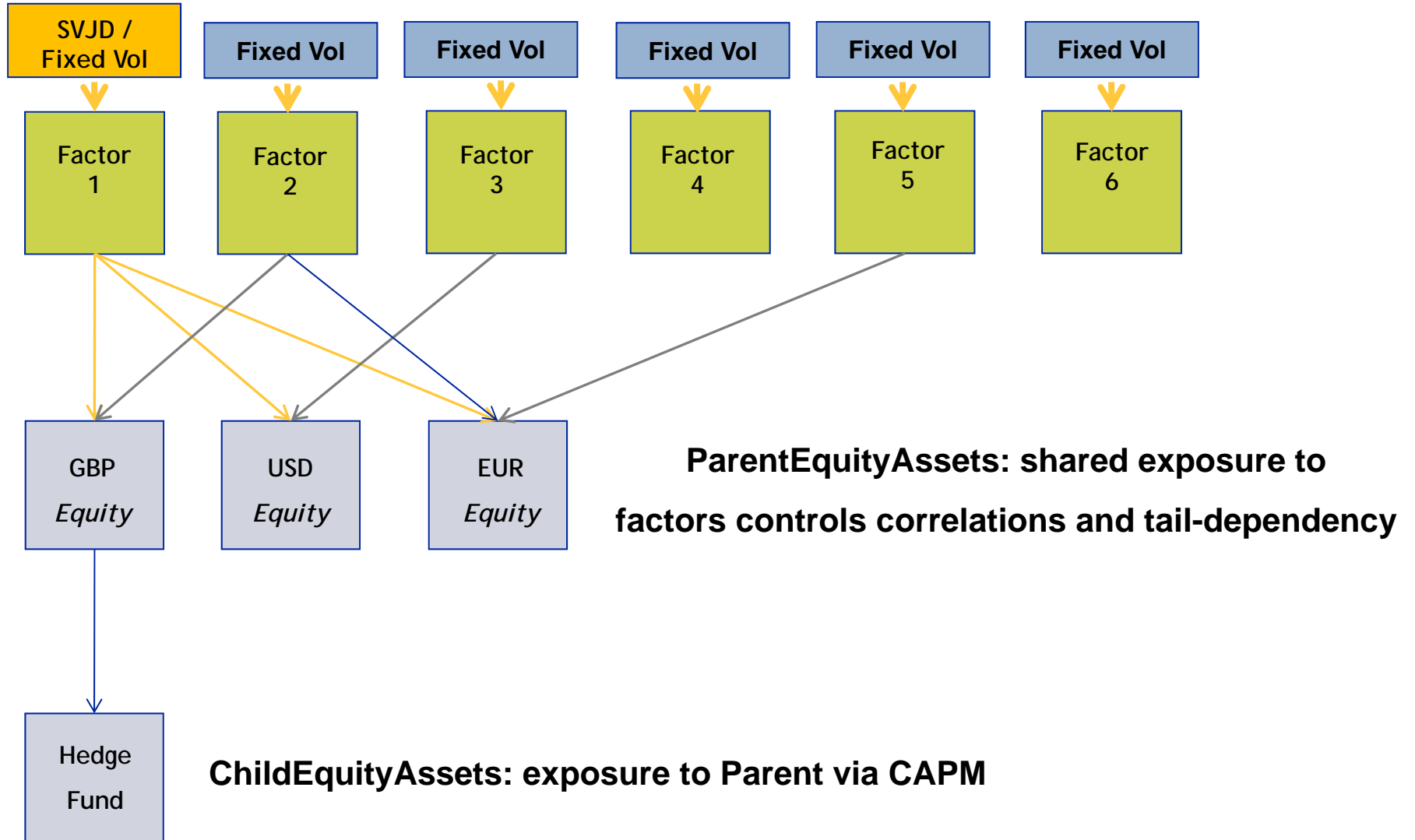
- » γ value set to adjust interest rate paths
 - We set target short rate paths and calibrate γ



B

Equity

B&H Equity Six-Factor Model

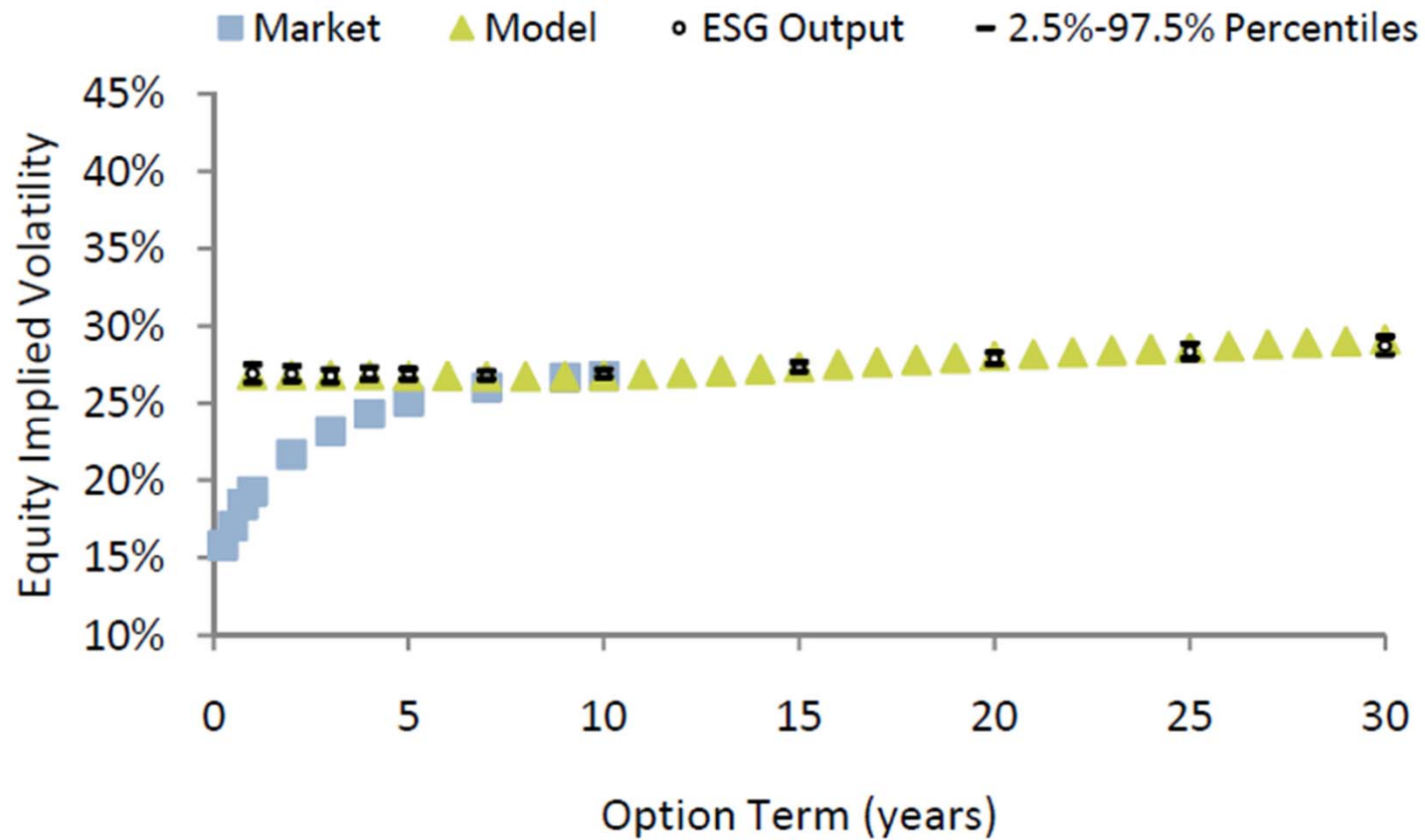


B&H Equity Model Choices

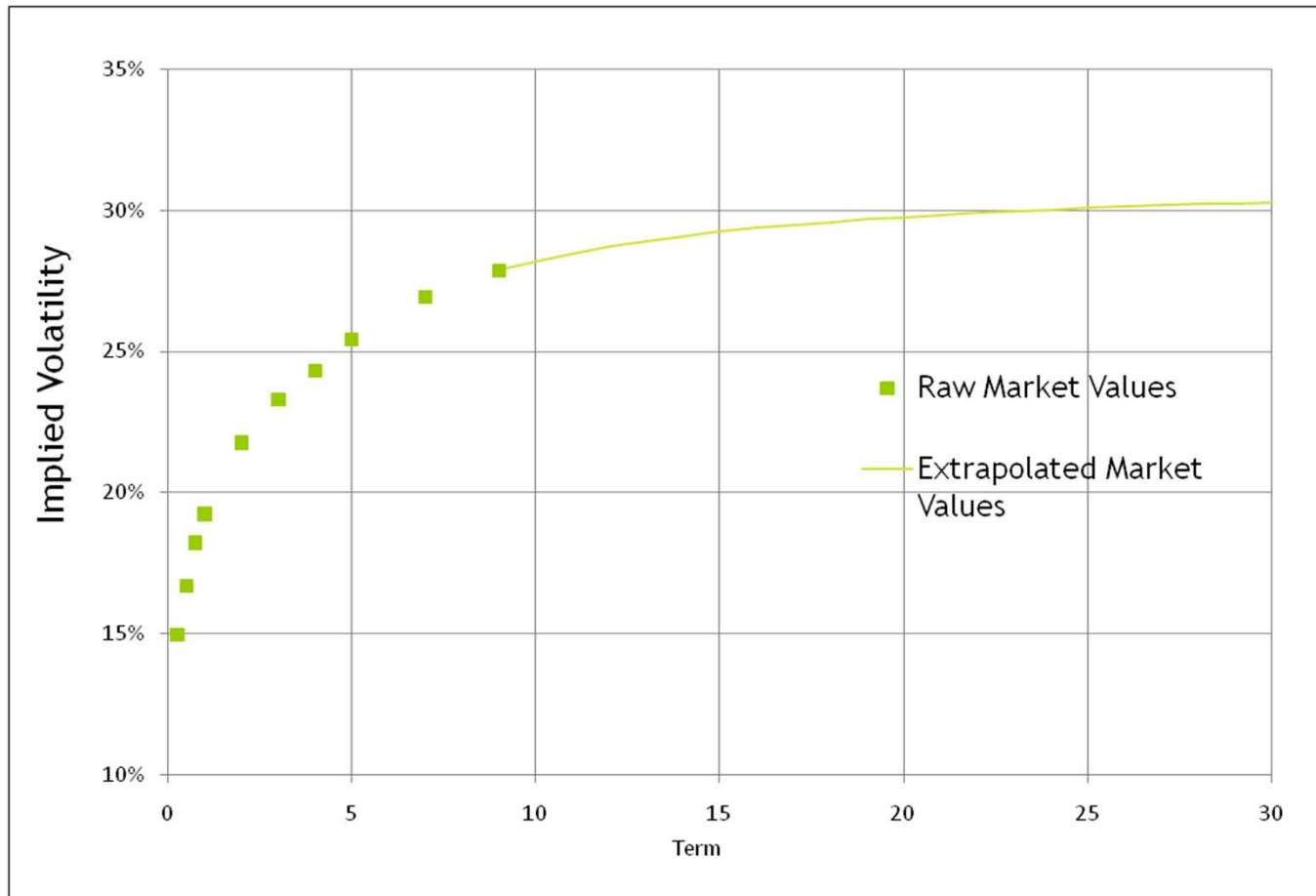
Excess Return Model

- » Constant Volatility
 - Black-and-Scholes type Geometric Brownian Motion
- » Time Varying Deterministic Volatility
- » Stochastic Volatility Jump Diffusion

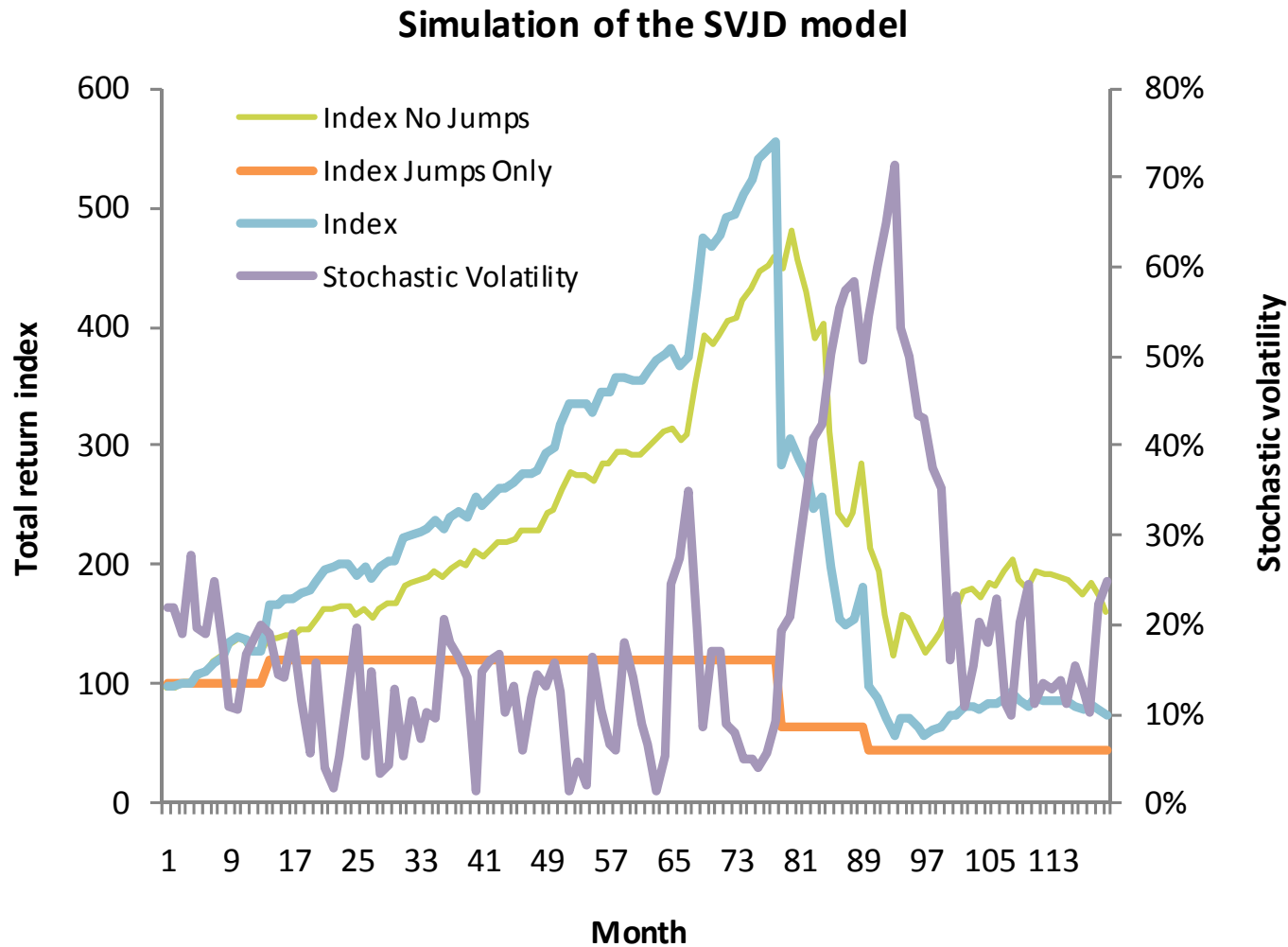
Constant Volatility



Time Varying Deterministic Volatility



B&H Equity - Stochastic Volatility Jump Diffusion



Lognormal Model: Dynamics

- The process for equity (excess return) index:

$$dS = \mu S dt + \sigma S dZ$$

- From Ito's lemma

$$d \ln S = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dZ$$

- So that changes in the index follow

$$\ln \left(\frac{S(t + \Delta t)}{S(t)} \right) = \left(\mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} Z$$

Time-Varying Deterministic Volatility (TVDV) model

- Volatility varies (deterministically) with time

$$d\ln S = \left(\mu - \frac{\sigma(t)^2}{2} \right) dt + \sigma(t) dZ$$

- Fit market data term structure
- Implied Volatility:

$$\sigma_{imp}(T) = \sqrt{\frac{1}{T} \int_0^T \sigma(t)^2 dt}$$

Equity – Stochastic Volatility Jump Diffusion

Stochastic Volatility Jump Diffusion

- » Stochastic volatility part, Heston model (red)
- » Jump diffusion part, Merton model (blue)

$$\frac{dS_t}{S_t} = (\mu - \lambda\bar{\mu})dt + \sqrt{v_t}dW_t^{(1)} + (\eta_t - 1)dN_t$$

$$dv_t = \alpha(\theta - v_t)dt + \xi\sqrt{v_t}dW_t^{(2)}$$

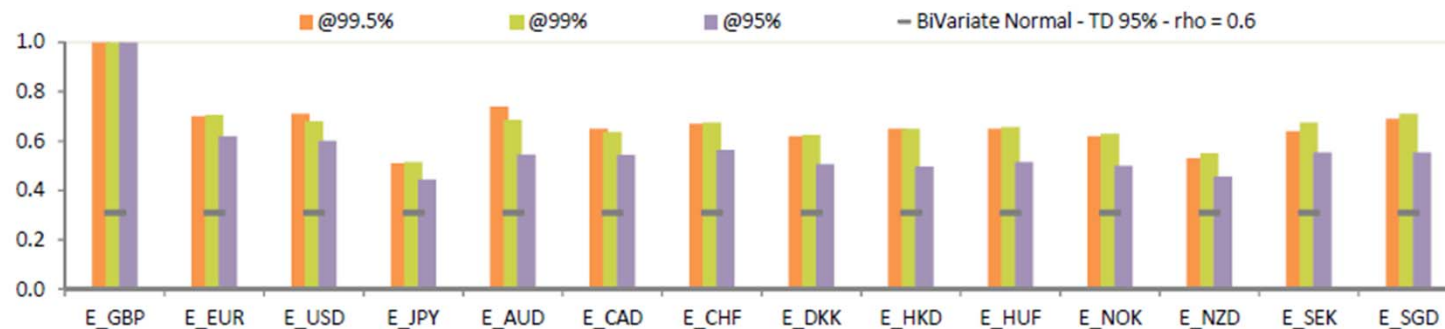
$$\text{corr}(W_t^{(1)}, W_t^{(2)}) = \rho$$

$$dN_t \sim \{0, 1\}$$

$$\eta_t \sim e^{N(\bar{\mu}, \sigma^2)}$$

Tail Dependence: Definition

- » Factor exposure implies tail dependency
- » What happened to other indices given on the conditional another index is in its tail?
- » Tail dependence is not targeted (limited amount of data) but is measured



Example: Equity tail-dependency – GBP Equity against other developing countries

Properties and Alternatives

Equity type assets:

- Properties
- Infrastructure
- Commodities (Generic, Energy, Precious Metal, etc)
- Private Equity
- Hedge Fund

Setting Targets - Equity

MA B&H Real-World Equity Volatility targets

- » **Short Term:** 30-day at-the-money option implied volatilities
 - Adjusted by a scalar of 0.98
 - Scalar determined through regression on long term historical data
- » **Long Term:** Exponentially weighted moving average of up to 120 years of historical data
 - Average age of data for developed markets = 25 years
 - Average age of data for developing markets = 12.5 years
- » **Medium to Long Term:** Produce “volatility term structure” to bridge short and long term
 - Volatility decay by regressing 21-day ahead volatilities against realised volatilities
 - Negative correlation between volatility and returns
 - “Volatility of volatility”

C

Credit

Setting Targets - Credit

MA B&H Real-World Credit targets

The Credit model is made up of a number of elements:

- » Transition Matrix
- » Credit Spread Level
- » Credit Spread Distribution
- » Default Recovery Assumption
- » Correlation and Tail Dependency with Equity Asset

B&H ESG simulate stochastically:

- Spreads
- Transitions/Defaults
- Recovery upon Default

Annual Transition Matrix

Ratings at end of period

Ratings at start of period	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	94.04%	5.69%	0.23%	0.01%	0.01%	0.01%	0.01%	0.01%
AA	2.13%	89.53%	7.28%	0.36%	0.30%	0.23%	0.03%	0.14%
A	1.63%	3.44%	89.52%	4.40%	0.39%	0.39%	0.03%	0.20%
BBB	1.54%	1.56%	5.20%	87.95%	2.06%	0.63%	0.63%	0.43%
BB	0.07%	0.56%	1.28%	6.68%	82.26%	6.70%	0.70%	1.75%
B	0.02%	0.05%	1.28%	1.58%	6.26%	80.56%	5.71%	4.53%
CCC	0.02%	0.04%	0.95%	1.46%	2.66%	8.74%	73.01%	13.13%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Default as absorbing state

- » Markovian: $p_2(A \rightarrow D) = p_1(A \rightarrow D)p_1(D \rightarrow D) + p_1(A \rightarrow A)p_1(A \rightarrow D)$
- » Multiply matrices at different periods to calculate default probabilities
- » Two type of transition matrices: real world (RW) vs risk neutral (RN)

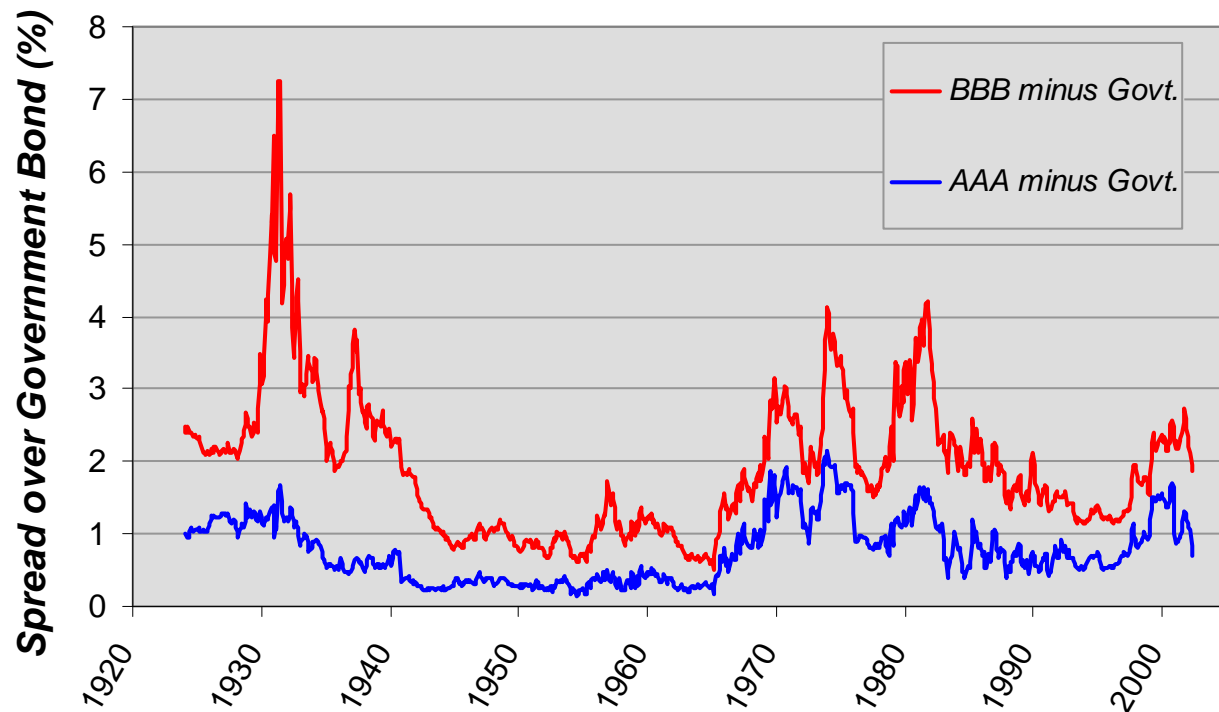
Spreads

Generally, corporate bonds are cheaper (i.e. offer a higher yield) than an equivalent government bond

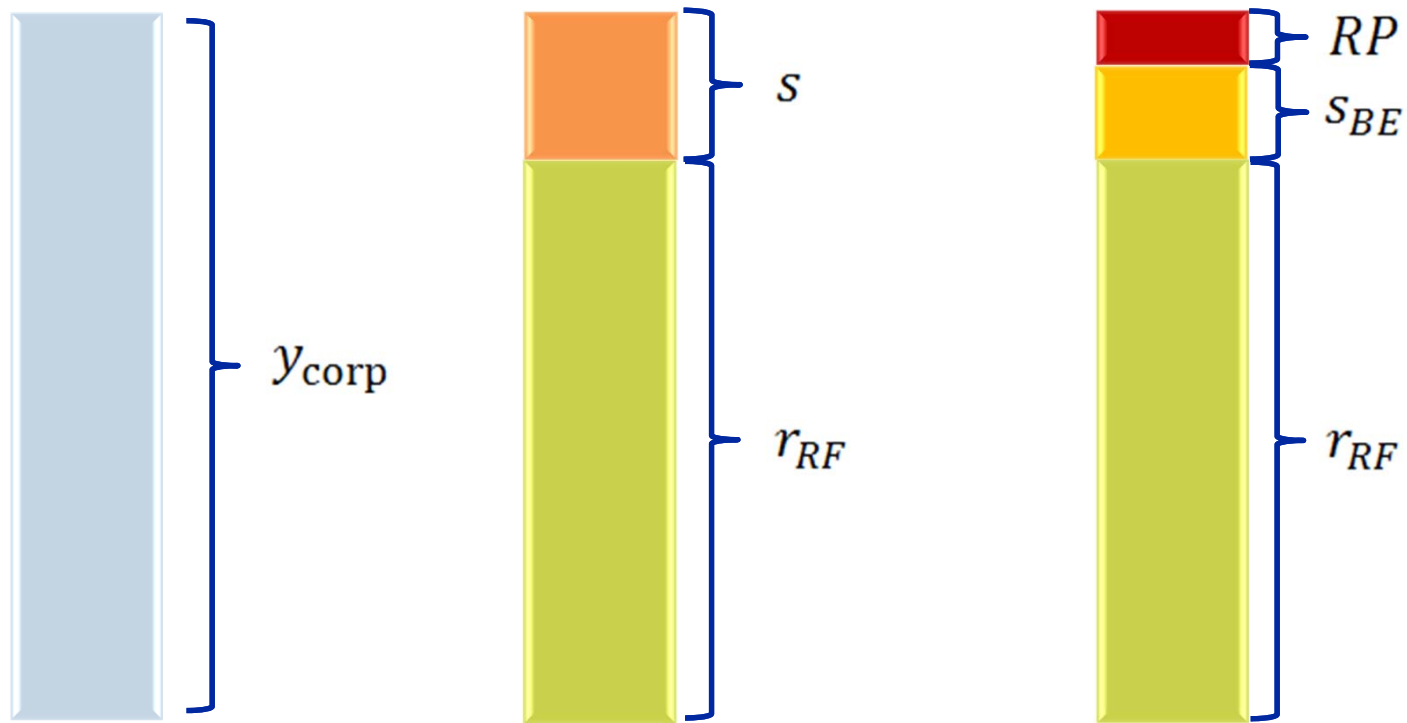
The *spread* is the difference in yield between a corporate bond and a government bond:

$$y_{corp} = r_f + s$$

Spreads vary with time (our model must allow q to vary with time)



Break-even spreads



The spread s accounts for the risk-neutral probabilities of default. The break-even spread s_{BE} accounts for the real-world probabilities of default.

Pricing Credit Risk

RW vs RN pricing of credit risky ZCB

Real World pricing:

- » probability p of bond defaulting before maturity
- » recovery rate R
- » expected cashflow at maturity = $R \times p + (1 - p)$

- » Price = Expected present-value of cashflow

$$P_{corp}(T) = e^{-dT} [R \times p + (1 - p)]$$

where $d = r_f + RiskPremium$

- » Requires a *RiskPremium* to compensate for uncertain return

Pricing Credit Risk

Risk Neutral pricing of credit risky ZCB

Risk Neutral pricing:

- » Risk Neutral probability q of bond defaulting before maturity
- » recovery rate R
- » expected cashflow at maturity = $R \times q + (1 - q)$

- » Price = Expected present-value of cashflow

$$P_{corp}(T) = e^{-r_f T} [R \times q + (1 - q)]$$

where r_f is the risk-free rate

- » Does not require any risk premium to compensate for uncertain return. Earns risk-free rate on average.

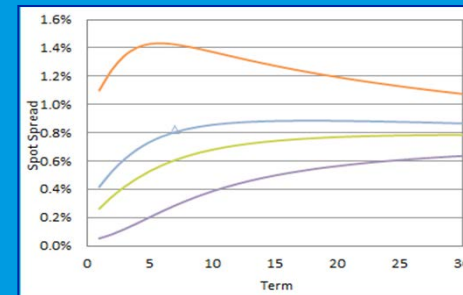
NOTES:

- » Real World and Risk Neutral agree on today's price.
- » Positive *Risk Premium* implies $q > p$, i.e. RN is more pessimistic.
- » *RN pricing means default probabilities need to be unrealistically pessimistic*

Transition and Spreads

	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	94.04%	5.69%	0.23%	0.01%	0.01%	0.01%	0.01%	0.01%
AA	2.13%	89.53%	7.28%	0.36%	0.30%	0.23%	0.03%	0.14%
A	1.63%	3.44%	89.52%	4.40%	0.39%	0.39%	0.03%	0.20%
BBB	1.54%	1.56%	5.20%	87.95%	2.06%	0.63%	0.63%	0.43%
BB	0.07%	0.56%	1.28%	6.68%	82.26%	6.70%	0.70%	1.75%
B	0.02%	0.05%	1.28%	1.58%	6.26%	80.56%	5.71%	4.53%
CCC	0.02%	0.04%	0.95%	1.46%	2.66%	8.74%	73.01%	13.13%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Transitions



Spreads

How to simulate both in the same credit model?

Risk Neutral Transition Matrix

The π process

- » RN matrix is used to calculate credit-risky bond prices and spreads
- » RN transition matrix is calculated by scaling the RW generator matrix Λ_P by the credit stochastic driver π_t :

$$\Lambda_Q = \pi_t \Lambda_P$$

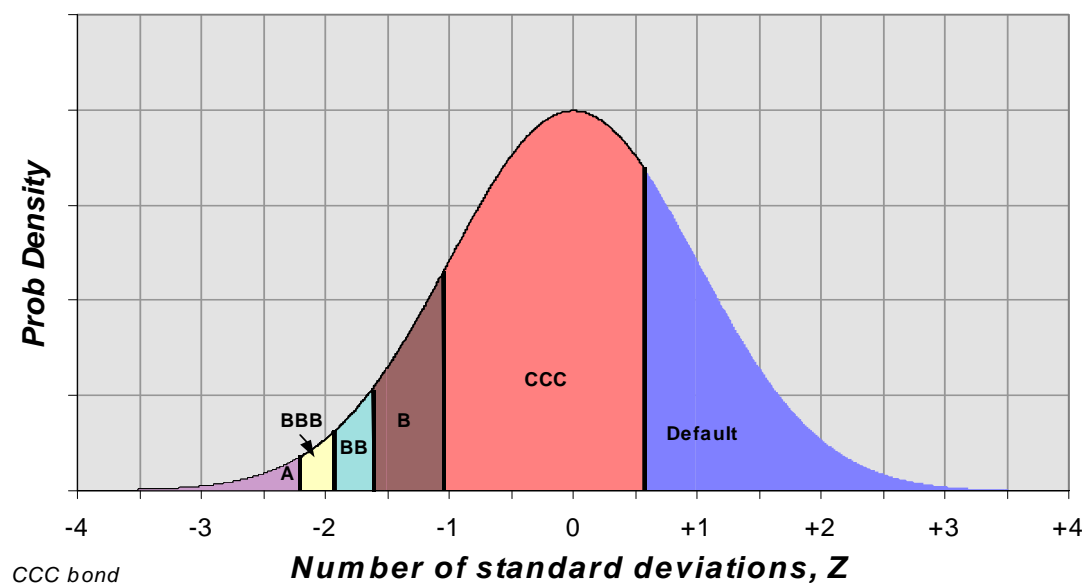
- » π_t is calibrated to market spreads and follows a CIR process:

$$d\pi_t = \alpha(\mu - \pi_t)dt + \sigma\sqrt{\pi_t}dZ_t$$

- $\pi_t > 0 \Rightarrow$ positive spreads
- Model displays mean reversion and has an analytical solution

Simulation of Credit Transitions

- » Credit transitions are generated by slicing up the standard normal distribution according to the transition probabilities
- » The issuer's rating at the end of a period will depend on the value of a generated standard normal variable Z_i
- » For a CCC-rated bond, we have:



Intra-Sector Correlation

- » We must incorporate correlation between different issuers' credit experience
- » This can be done by generating a correlated Z_i :

$$Z_i = \sqrt{\rho}Z_M + \sqrt{1 - \rho}Z_{S_i}$$

where

Z_M is the associated equity shock

Z_{S_i} is the issuer specific shock

ρ is the correlation between Z_i of different issuers (Intra-sector Correlation)

Stochastic Spread in Real-World

Introduce scaling factors:

$$\Lambda_Q^{BASE} = K \Lambda_P^{BASE}$$

where K is a diagonal matrix

- » one scaling factor for each credit rating

	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	94.04%	5.69%	0.23%	0.01%	0.01%	0.01%	0.01%	0.01%
AA	2.13%	89.53%	7.28%	0.36%	0.30%	0.23%	0.03%	0.14%
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CCC	0.02%	0.04%	0.95%	1.46%	2.66%	8.74%	73.01%	13.13%
Default	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Λ_P^{BASE}

Build a more general credit model from Base generator matrices:

$$\Lambda_P = (A_P + B_P \pi_t) \Lambda_P^{BASE}$$

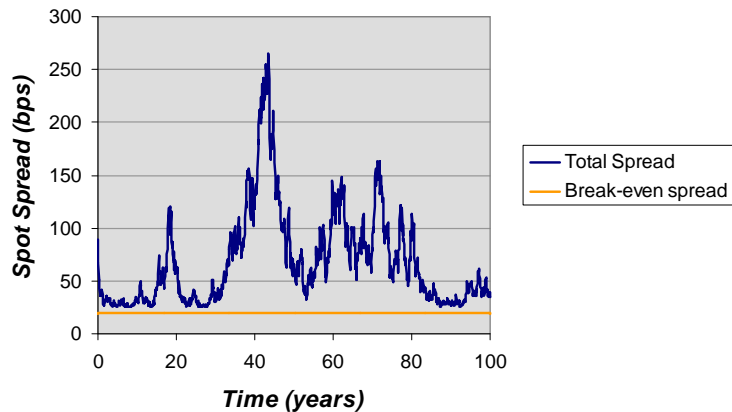
$$\Lambda_Q = (A_Q + B_Q \pi_t) \Lambda_P^{BASE}$$

- » Real-World transition matrix can vary stochastically.
- » Stochastic process drives variation in Credit Risk Premium and Default Probability
- » More freedom when calibrating: better fit to market data.

Split between default and spread

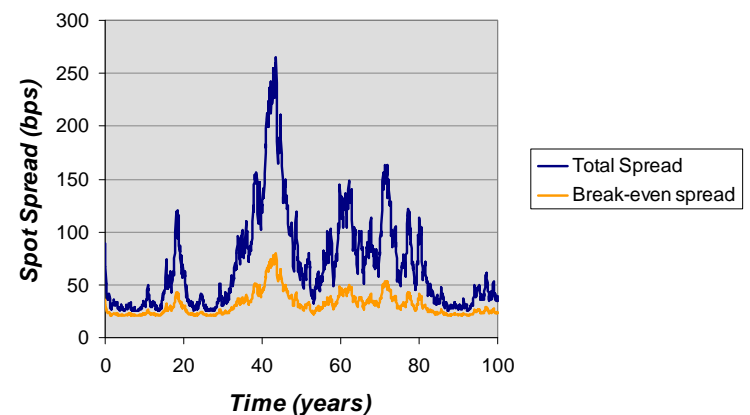
Real-world transitions: $A = 1; B = 0$

Credit spread changes purely a result of changes in credit risk-premium



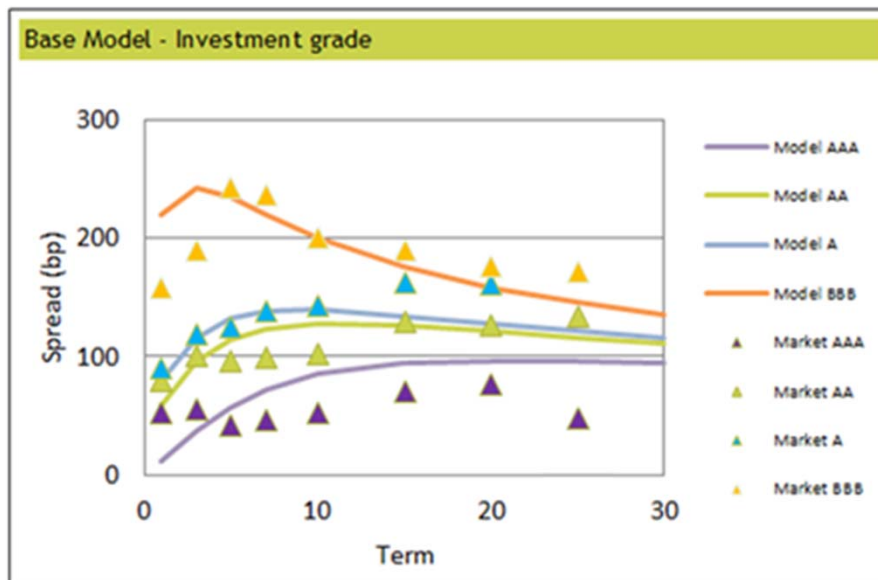
Real-world transitions: $A = 0.75; B = 0.25$

~25% of credit spread changes are due to changes in RW expected default losses



Example market spreads

- » Calibrate to a credit spread (e.g. A7)



D

Inflation

Setting Targets - Inflation

MA B&H Real-World Inflation Calibration

» Index-Linked Bonds

» Inflation Targets

- Global Targets
- Specific Targets
 - By Country
 - By Sector

Inflation Modelling: Affected by Nominal Rates and Real Rates

B&H Inflation models

- » Derived inflation (Basic model)
 - Difference between Nominal Rates and Real Rates
- » InflationPlus
 - Adding unexpected inflation
 - Additional uncertainty
- » Inflation Wedge
 - Specific inflation, e.g. Medical, Wage
 - Evolve around base inflation (CPI/RPI)

B&H Inflation models (Mathematics)

» Derived inflation (Basic model)

- *inflation $\pi = \text{nominal rate} - \text{real rate}$*

» InflationPlus

- $\pi_{realised}(t) = \pi_{expected}(t) + MUI + \sigma_{inflation}Z_{inflation}(t)$
- *MUI = mean unexpected inflation*
- $\pi_{expected}(t) = r_{nominal}(t - 1) - r_{real}(t - 1)$

» Inflation Wedge

- $\pi_{specific} = x_t + \pi_{core}$
- $dx_t = \alpha(\mu_x - x_t)dt + \sigma_{specific}dW_t$

E

FX, Correlations and Tail Dependency

Real World FX Modelling

Goal:

- » Capture economically coherent outcomes for the purposes of projection.

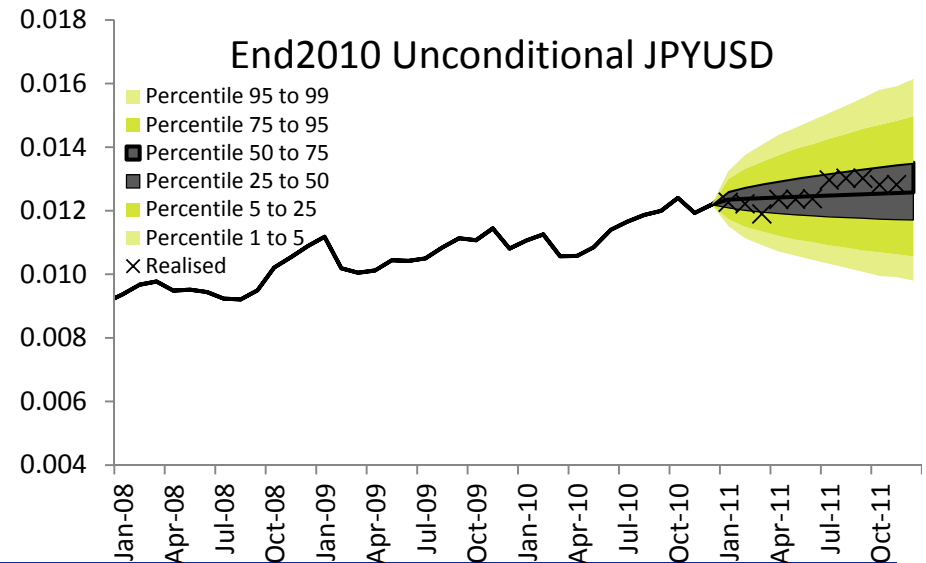
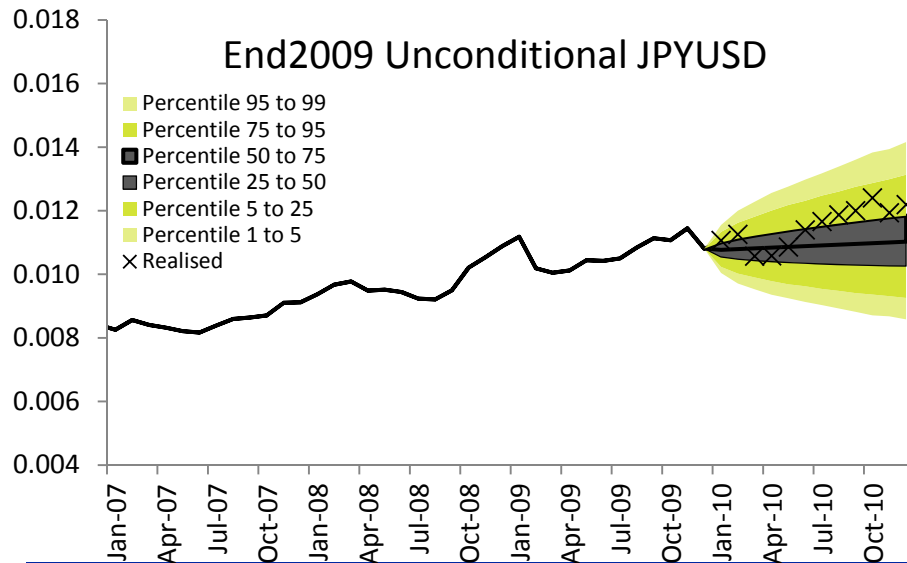
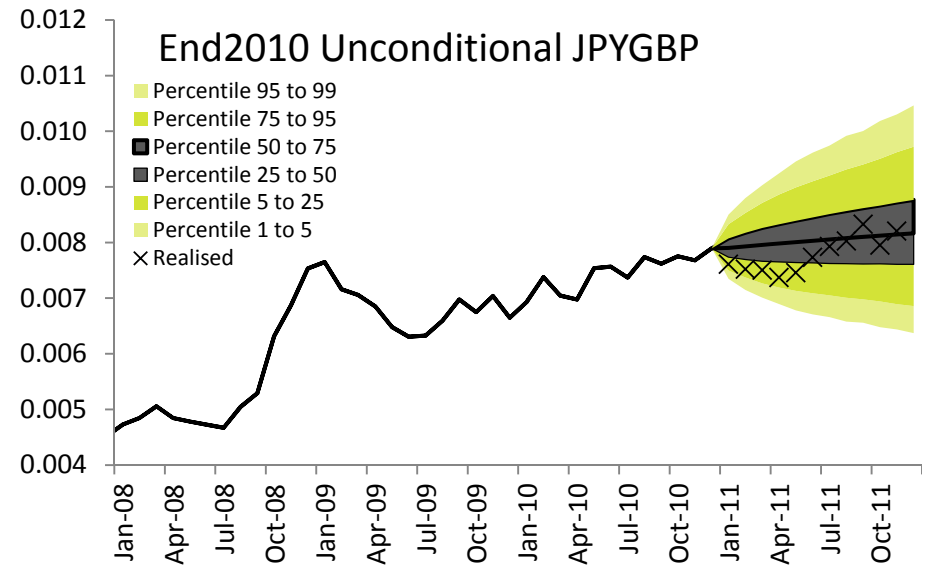
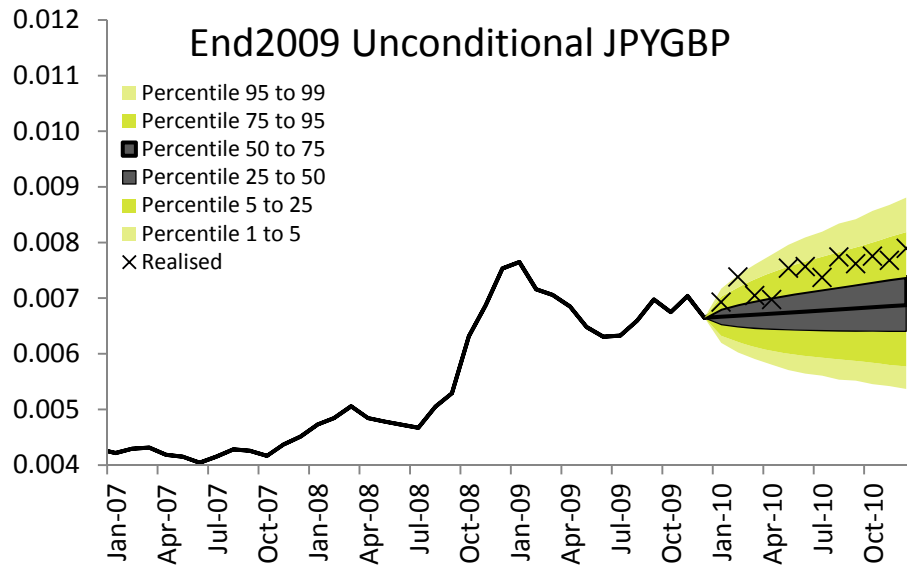
Desirable features:

- » Respect Purchasing Power Parity in the long run
- » Use interest rate differentials in the short term

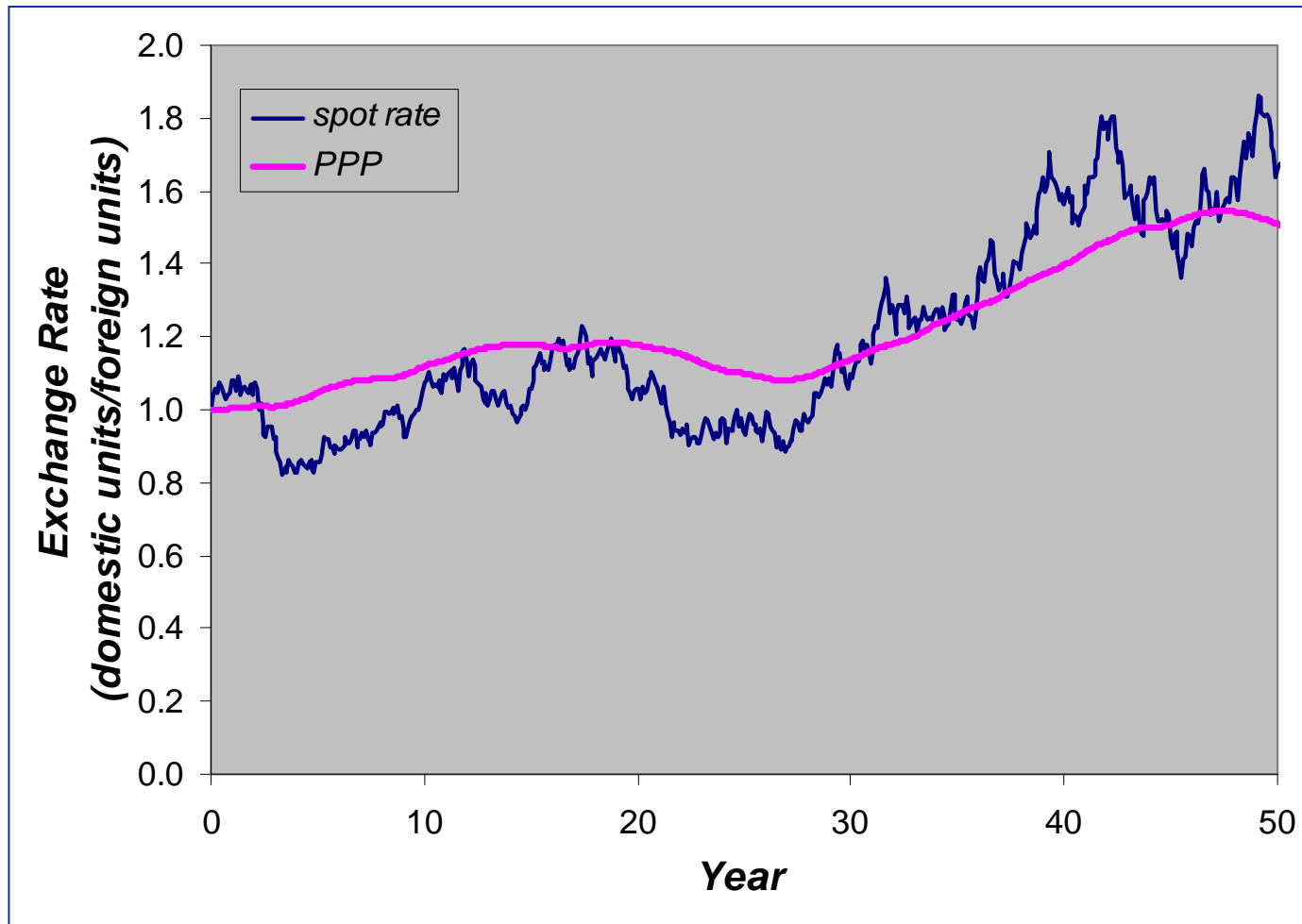
B&H RW Implementation

- » Model **real** exchange rate
- » As a mean reverting process
- » Subject to random shocks in the short term but is pulled towards some mean level over the long term

JPY FX Unconditional Backtests



Example PPP Model Simulation



Real World Correlation targets

Correlation targets are unconditional and global

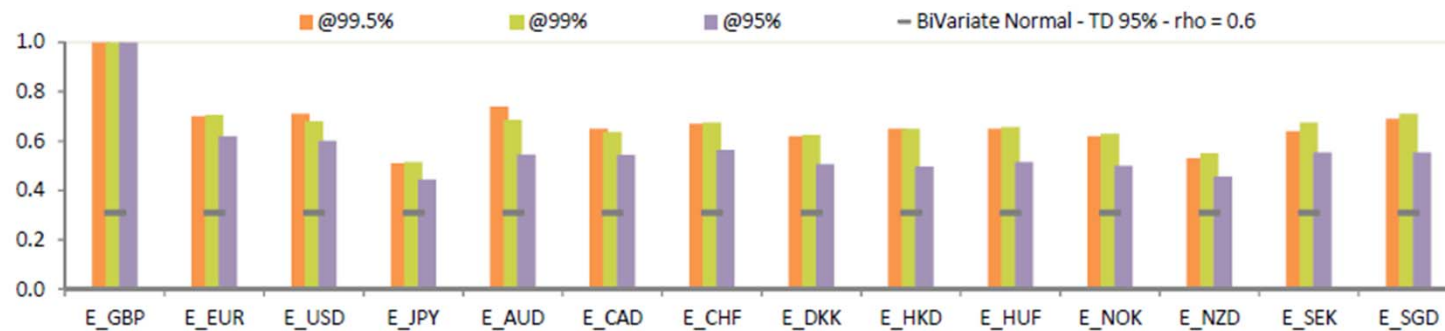
- » Currently investigating rationale for economy-specific targets
- » Weak evidence to suggest statistically credible alternatives.

EndJune2012

	Target	ESG
Equity XS Return Vs. 10Y Govt. Bond XS Return	0.13	0.14
Changes in Log Short Rate Vs. Equity XS Return	-0.09	-0.08
Equity XS Return Vs. 10Y IL Govt. Bond XS Return	0.10	0.06
Changes in Real Short Rate Vs. Equity XS Return	-0.07	-0.01
Equity XS Return Vs. Property XS Return	0.35	0.34
Property XS Return Vs. Changes in Log Short Rate	-0.10	-0.07
Property XS Return Vs. Changes in Real Short Rate	-0.10	-0.07
Property XS Return Vs. 10Y Govt. Bond XS Return	0.10	0.08
Property XS Return Vs. 10Y IL Govt. Bond XS Return	0.10	0.10

Tail Dependency

- » Tail dependency in bond defaults
- » Tail dependency in equity markets
- » Tail dependency is not targeted (limited amount of data) but is measured



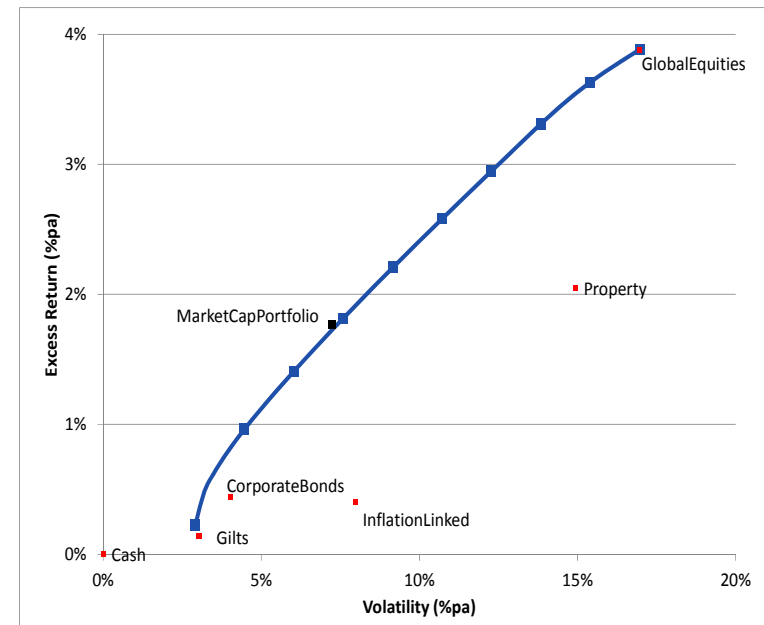
Example: Equity tail-dependency

F

Asset Allocation, Views and Biases

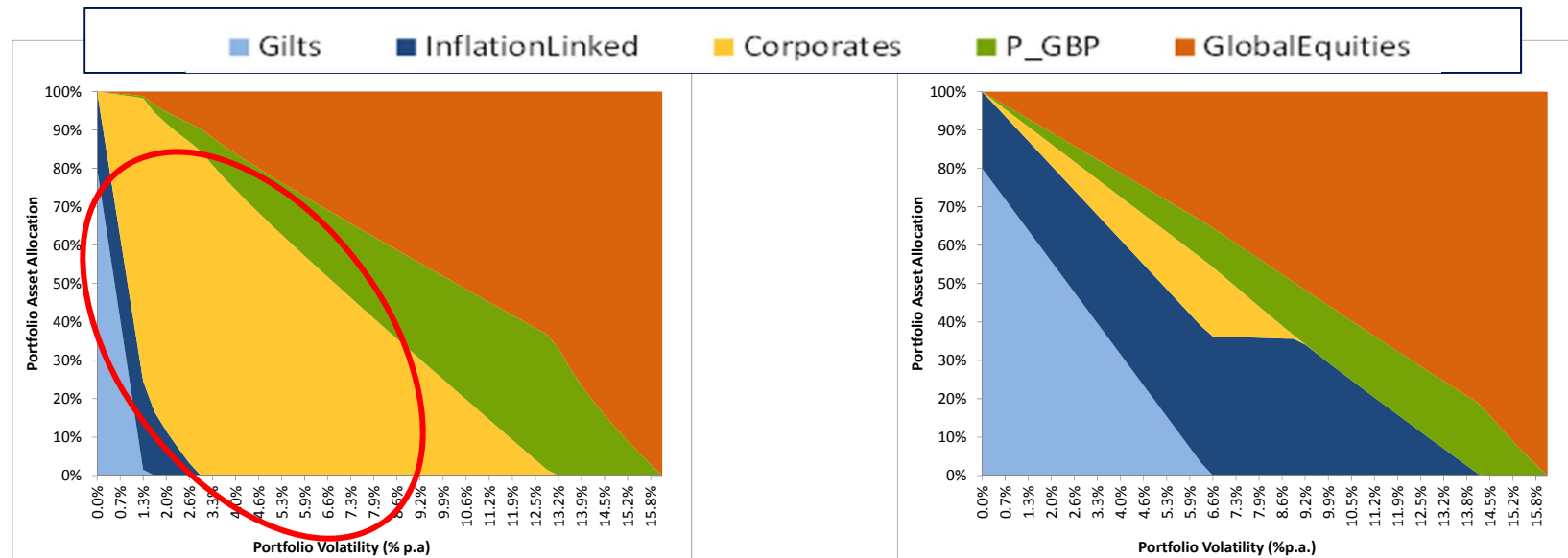
Real-World Scenarios and Asset Allocation

- » Another typical use of Real-World scenarios is for **Strategic Asset Allocation**
- » Investment portfolio optimised using techniques such as Mean-Variance Optimisation
 - Maximise Return
 - Given a Volatility target
- » Real-World scenarios as input
 - To analyse risk-return performances
 - Given different asset-mixes
- » But Real-World embeds views such that
 - Asset allocation biased towards certain assets
 - Because of views taken on risk-premiums being different to market Average



Dynamic Equilibrium Calibration

- » Assumes market-equilibrium in the long-term based on global capitalisation
- » Possible application of Black-Litterman weighting to blend between views and equilibrium.




Bias towards corporate bonds because of views on credit risk-premia

Dynamic Equilibrium scenarios based on global capitalisation

Dynamic Equilibrium Calibration

- » Black-Litterman weighting between views and equilibrium.
- » Shift between importance of best-estimate views (subjective) and market equilibrium (based on global capitalisation)





Examples & Question and Answer

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