ACTUARIAL EXAMINATIONS

3rd IAI Connect

20 Aug 2014

PREPARING FOR ACTUARIAL EXAMS – PROBLEMS

- Work load – limited time to focus on exams
- No ready guidance, coaching, classroom training
- Different background
- Competitive set-up – questions expected to be just more than basic
- Limited time during the exam to think and write and complete

EXAM GUIDES – SOLUTION TO THE PROBLEM

- Active study – e.g. understand each word/expression in a mathematical definition or understand the application of the concept
- Make your own notes while studying
- Maintain regularity
- Browse through as many sample questions and solutions as possible – questions and solutions of previous exam diets
- Focus on a wider scope, instead of just syllabus material – some of the materials have underlying concepts which involve subjects covered in earlier courses
- Study supplementary course materials e.g. relevant references – Options, Futures and Other Derivatives by John C. Hull
- Practice writing under mock exam situation

TESTING CANDIDATES

- **What do examiners expect ?**

  - Know the book work well
  - Understand the concepts well to be able to apply – reasonable and logical attempt always gets you marks!
  - Make reasonable assumptions if you need to attempt a question rather than just leave it un-attempted!
  - Although you won't be tested directly on underlying concepts from earlier courses, you are expected to demonstrate your understanding
  - You are expected to appreciate the specific section where a question has been asked- the attempt should be triggered by that
  - Clear writing – please work on your hand writing!!

USEFUL TIPS

- **Interpretation of use of language in a question**

  - Understand the difference between “list” and “ discuss”

  - Carefully study the application of “alternate” approach – you are expected to deduce the solution using an alternate approach compared to one you have already demonstrated – don’t just repeat!

USEFUL TIPS

- **Interpretation of marks allocations**

  - Try to assign marks yourself to a question – example: you are expected to describe a mathematical expression in simple sentence depending on marks allocated
  - A computation in a sub part of same question might be repeated with different parameters but lower marks – think if you can deduce the computation from a standard equation rather than working it out from first principle again
  - Don’t skip steps: show computations clearly as marks are allocated for all computations

USEFUL TIPS

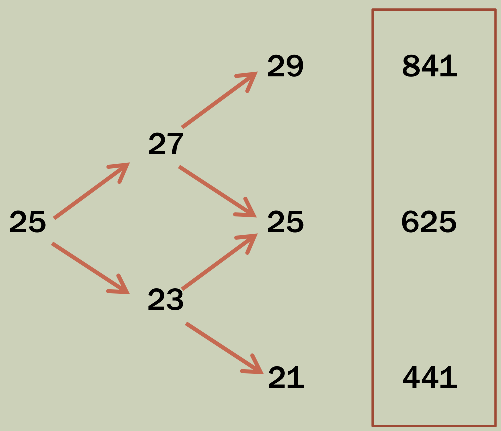
Example: May 2014

A stock price is currently Rs.25. It is known that at the end of every month, the stock price either goes up or goes down by Rs.2. The risk free interest rate is 10% per annum with continuous compounding. Suppose S_T is the stock price at the end of two months.

Define $X_t = S_t^2$. What is the value of a derivative that pays off X_T at the end of 2 month? (5)

USEFUL TIPS

Solution: May 2014



Stock price	25
u	1.08
d	0.92
r	10%
t	0.0833
p	0.5523
d	0.4477
$e^{-2r/12} * [p^2 fuu+2p(1-p)fud+(1-p)^2 fdd]$	643.1981

- ❑ Payoff diagram 2
- ❑ Computation of parameters 1
- ❑ Formula and computation of option value 2

USEFUL TIPS

Example: May 2013

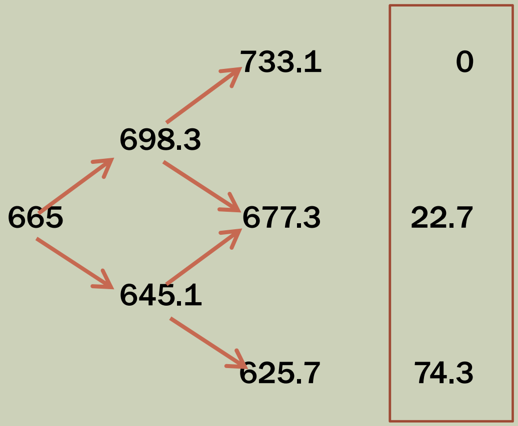
The price of a non-dividend paying share goes up 5% or down 3% over each month. Assume a monthly risk-free rate (continuously compounded) of 1.5%. Using the two-period binomial tree model calculate each of the following:

i. Value of a two month European call with a strike price of Rs. 700 and current share price of Rs. 665 (4)

ii. Value of a two month European put with the same strike price (3)

USEFUL TIPS

Solution (ii): May 2013



Stock price	665
u	1.05
d	0.97
r	1.5%
t	1
p	0.5639
d	0.4361
$e^{-2r} * [p^2 fuu+2p(1-p)fud+(1-p)^2 fdd]$	24.5467

- ❑ Payoff diagram 2
- ❑ Formula and computation of option value 1

USEFUL TIPS

Example: May 2014

1. Define martingales in discrete time (2)

- A discrete time stochastic process $X_0, X_1, X_2 \ldots$ is said to be a martingale if **$E[|X_n|] < \infty$** for all n, **and $E[X_n | X_0, X_1, X_2 \ldots, X_m] = X_m$ for all $m < n$.** (1)
- In other words, the current value X_m of a martingale is the optimum estimator of its future value (1)

2. State Ito's lemma. (2)

- Ito's lemma states that for an **Ito drift-diffusion process X_t, satisfying stochastic differential equation $dX_t = \mu dt + \sigma dW_t$** where **$W_t$ is a Wiener process** and **any twice differentiable scalar function f(t,x) of two real variables t and x**, one has

$$df(t, X_t) = \left(\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2}\right)dt + \sigma \frac{\partial f}{\partial x} dW_t$$

USEFUL TIPS

Example: May 2013 – Application of knowledge on statistics

Let B_t , t>= 0 be Standard Brownian motion with B_0 = 0
Show that exp($23B_t$ – 529t/2) is also a martingale. (4)

Example: May 2014 – Application of knowledge on statistics

A stock price follows geometric Brownian motion with an expected return of 16% and a volatility of 35%. The current price is Rs. 254.

What is the probability that a European call option on the stock with an exercise price of Rs. 258 and a maturity date in six months will be exercised?

What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised? (3+1)

USEFUL TIPS

Solution Hint

<u>Application of moment generating function can make life easy</u>

$B_t = B_t - B_s + B_s$ and $B_t - B_s \sim N(0, (t - s))$ and $E(\exp(t'z)) = \exp(t'^2\sigma^2/2)$ where $z \sim N(0, \sigma^2)$

<u>Statistical Distributions</u>

Stock price S_t ~ geometric Brownian motion i.e. $S_t = S_0 e^{(\mu - \frac{\sigma^2}{2})t + \sigma W_t}$

$\Rightarrow$ $Ln(S_t/S_0)$ ~ Normal $(\mu - \frac{\sigma^2}{2}, \sigma^2 t)$

$\Rightarrow$

$$\frac{Ln(S_t / S_0) - (\mu - \frac{\sigma^2}{2})}{\sigma\sqrt{t}} \sim \text{Normal } (0,1)$$

Use $P[S_T > K] = P[S_T/S_0 > K/S_0] = P[Ln(S_T/S_0) > Ln(K/S_0)]$

USEFUL TIPS

Example: May 2014

Suppose X_1, X_2, ...are i.i.d with mean μ. Show that $Z_n = \sum_{i=1}^{n}(X_i - \mu)$ is a martingale (2)

Tentative Approach: Martingale is a process with no drift. Hence E(expression) = 0 which implies it is a martingale???!!

A mean zero process is not always a martingale!

Need to prove $E(Z_n \mid Z_m) = Z_m$

$$E[\sum_{i=1}^{n}(X_i - \mu) \mid X_m, m < n] = E[\sum_{i=1}^{m}(X_i - \mu) \mid X_m] + E[\sum_{i=m+1}^{n}(X_i - \mu) \mid X_m] = E[\sum_{i=1}^{m}(X_i - \mu) \mid X_m, m < n] + 0 = Z_m$$

USEFUL TIPS

- **Time management**

  - Do not spend “extra” energy in just one question because you happen to know it – it won’t get you “extra” marks
  - Do not unnecessarily spend time doing the computations over and over again if those have not been explicitly asked– use smart techniques
  - Read question carefully before answering – try to evaluate links in sub parts of a question

QUESTIONS ?

THANK YOU