

# 7th Capacity Building Seminar In Health Insurance

## Practical Aspects of Designing Morbidity table



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# Agenda



- What is Morbidity
- Calculation of Exposed to Risk
- Graduation / Smoothing of Incidence Rates
- Practical Considerations
- Example of a Multifactor Analysis of Incidence Rates

# What is Morbidity

- Morbidity is the rate at which disease occurs in a group of people over a given period of time
- In actuarial parlance, it is represented as  $i(x)$  which represents the probability of sickness or injury for an individual aged  $x$  between time period  $t$  and  $t+1$ .
- A morbidity table provides morbidity rates that may vary by multiple factors / variates

# Morbidity rates may vary by..



- Age
- Gender
- Duration
- Underwriting – Short Form / Long Form; Med / Non-Med
- Geographic location
- Smoker Status
- Product type
- Disease type
- Distribution channel
- Occupation
- Activity of Daily Living or Daily Working
- Waiting period / Survival period
- Rider / Standalone
- Claim triggers (ADL / ADW) etc

# Morbidity tables for various products

- Fixed (Defined) Benefit products
  - Critical Illness
  - Hospital Cash Benefit / Surgical Cash Benefit
  - Total and Permanent Disability due to accident and/ or sickness
  - Income Protection
  - Long-Term Care
- Indemnity Products
  - In patient hospitalization
  - Out patient / day care

# Today's Exercise...

Is only limited to calculation of crude and smoothed incidence rates / morbidity rates for inpatient hospitalization products

We intend to cover the following aspects:

- Calculation of Exposed to Risk (exposure)
- Calculation of Crude incidence rates
- Smoothing of rates
- Different approaches for graduation
- Univariate / Bivariate analysis of incidence rates
- Multifactor example to unwind the effect of more than one variate

# **Calculation of Exposed To Risk & Derivation of crude and smoothed morbidity rates**

# Graduation techniques



Some of the graduation techniques

1. Whittaker – Henderson
2. Cubic Spline
3. Heligman Pollard



# Whittaker – Henderson

The Whittaker -Henderson method attempts to graduate the crude rates by obtaining a balance between the adherence to data and the smoothness of the rates.

The graduated rates are obtained by minimizing  $Q$  below:

$$Q = \sum_{j=0}^N w_j (q_{x+j} - \hat{q}_{x+j})^2 + \sum_{j=0}^{N-3} K_j (\Delta^3 q_{x+j})^2$$

Where  $w_j = \frac{N \cdot E_x}{\sum E_x}$  are the weights that assign higher weights to ages with higher exposure

And  $K_j$  are smoothing coefficients

$\hat{q}_x$  – Crude Rate

$q_x$  – Graduated Rate

# Whittaker – Henderson

Ensures that the graduated rates are as close to crude rates as possible

$$Q = \sum_{j=0}^N w_j (q_{x+j} - \hat{q}_{x+j})^2 + \sum_{j=0}^{N-3} K_j (\Delta^3 q_{x+j})^2$$

Ensures that the graduated rates are as smooth as possible

Goodness of Fit

Smoothness

This method attempts to balance both these opposing objectives.

# Cubic Spline

The Cubic Spline method fits a piecewise curve to the crude rates. It fits a smooth curve between each of the knots, which are predetermined age ranges selected to optimize the graduation process.

The graduated rates are obtained by minimizing  $Q$  below:

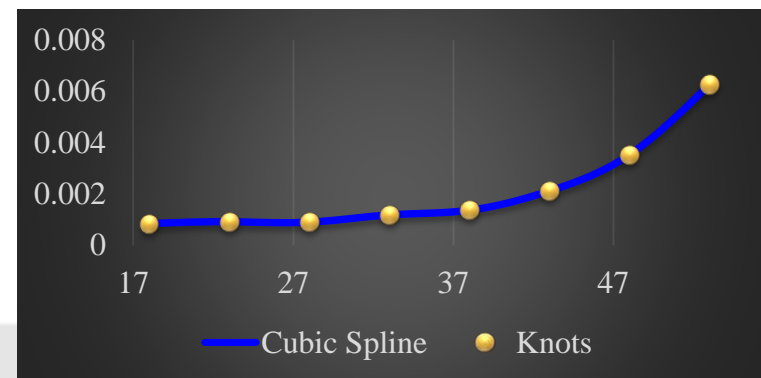
$$Q = \sum_{j=0}^{N-1} w_{x+j} (q_{x+j} - \hat{q}_{x+j})^2$$

Where  $q_x = a_0 + a_1x + a_2x^2 + a_3x^3 + \sum_{i=1}^n b_i G_i(x)$  is the cubic equation

and  $G_i(x) = (x - x_i)^3$  for  $x \geq x_i$   
 $= 0$  for  $x < x_i$

$\hat{q}_x$  – Crude Rate

$q_x$  – Graduated Rate



# Heligman Pollard

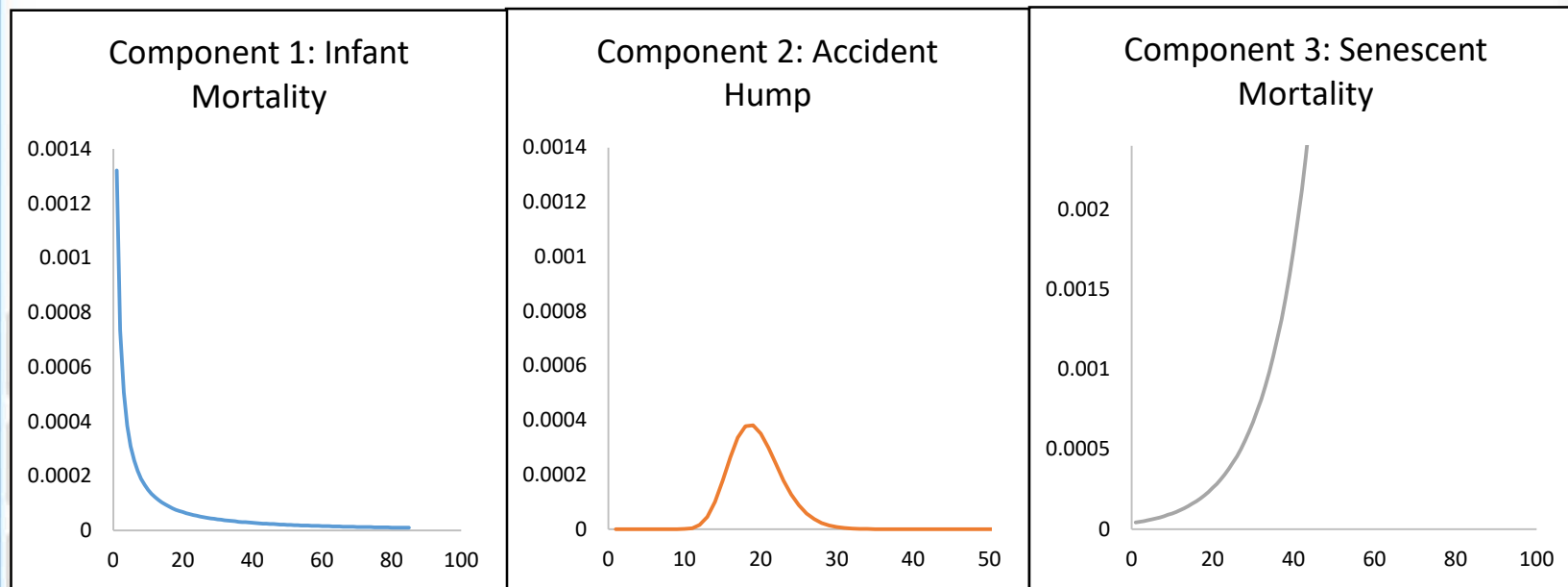
The mortality law suggested by Heligman and Pollard is:

$$\frac{q_x}{p_x} = A(x+B)^C + De^{-E(\ln x - \ln F)^2} + GH^x$$

Where  $q_x$  is the probability of dying within 1 year for a person aged  $x$  exactly and

$$p_x = 1 - q_x$$

Each component represents a distinct component of mortality:

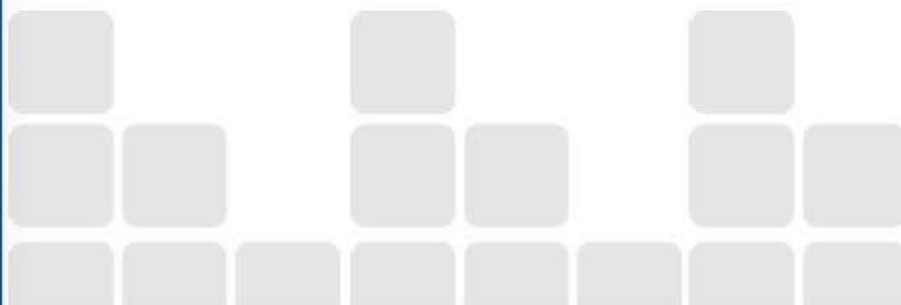


# Test of Graduated rates



Test	Purpose	Working	Criteria
Standardized Deviations Test	Testing Overall Goodness of Fit	Checks for normality of Standardized deviations	$z_x = \frac{(Actual - Expected)}{sqrt(Expected)}$ We expect $z_x$ 's to follow Standard Normal Distribution
Chi-Squared Test	Testing Overall Goodness of Fit	Calculates sum of squares of differences between expected and actual deaths	We expect $\sum z_x^2$ to follow $\chi^2$ distribution (Degrees of freedom depend on the method of graduation)
Sign Test	Detecting Overall Bias	Calculates number of positive deviations of the graduated rate from the crude rates	$P = \text{Count of Positive } z_x \text{ values}$ We expect P to follow Binomial distribution with parameters (n,0.5) where n is the number of observations
Grouping of Signs Test	Detecting Runs and Clumps	Calculates groups of positive deviations throughout the graduation	$G = \text{Groups of Positive } z_x$ We expect neither too many nor too few groups
Cumulative Deviations Test	Testing for Over graduation	Calculates overall deviation	Test statistic = $\frac{\sum(Actual - Expected)^2}{\sum Expected}$
Serial Correlation Test	Testing for Over graduation	Calculates correlation between successive standardized deviations	Calculate correlation between successive $z_x$ values. It is expected to follow $N(0, 1/m)$
Third Differences Test	Test for smoothness	Calculates the third order difference of the graduated rates	Find third difference ( $\Delta^3 q_x$ ) of the graduated rates. They are expected to be small and to move gradually.

# Univariate / Bivariate Analysis of Morbidity Rates



First principle approach to unwind  
the effects of more than one variate

Thank you !!