

# Institute of Actuaries of India

ACET October 2024

## Mathematics

1. If  $x^{2024} + y^{1947} = 1$ , for real numbers  $x$  and  $y$ , then
- A.  $x$  is a function of  $y$ , and  $y$  is a function of  $x$ .
  - B.  $x$  is a function of  $y$ , but  $y$  is NOT a function of  $x$ .
  - C.  $x$  is NOT a function of  $y$ , but  $y$  is a function of  $x$ .
  - D.  $x$  is NOT a function of  $y$ , and  $y$  is NOT a function of  $x$ .
- 1 mark
2. If  $f(x) = e^x$  and  $g(x) = x^2$ , then  $f(g(x)) < g(f(x))$  when  $x$  lies in:
- A.  $(-\infty, 0)$
  - B.  $(0, 2)$
  - C.  $(2, \infty)$
  - D.  $(-\infty, 0) \cup (2, \infty)$
- 1 mark
3. The minimum value of  $\sin^2 x + \cos^2 x + \tan^2 x + \csc^2 x + \sec^2 x + \cot^2 x$  is:
- A. 7
  - B. 6
  - C. 3
  - D. None of the above
- 3 marks
4. If  $\alpha = \int_{20}^{24} \frac{1}{x} dx$ , then the value of  $e^\alpha$  lies in:
- A.  $(1, 1.2]$
  - B.  $(1.2, 1.5]$
  - C.  $(1.5, 2]$
  - D. None of the above
- 1 mark
5. If  $\alpha$  and  $\beta$  are roots of  $x^2 + ax + b = 0$  (and  $b \neq 0$ ), then the roots of  $bx^2 - ax + 1 = 0$  are:
- A.  $-\alpha$  and  $-\beta$
  - B.  $-1/\alpha$  and  $-1/\beta$
  - C.  $1/\alpha$  and  $1/\beta$
  - D. None of the above
- 1 mark
6. Evaluate  $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\cos(2\theta)}{\cos(\theta) - \sin(\theta)}$ :
- A. 0
  - B. 1
  - C. 2
  - D.  $\sqrt{2}$
- 2 marks
7. A geometric progression  $a_n$  is such that  $a_1 = 1$  and  $a_{2024} = 2$ . If  $a_n = 4$ , then  $n$  equals:
- A. 4047
  - B. 4048

- C. 4049
- D. None of the above

1 mark

8. If the magnitudes of the three vectors  $\vec{a}$ ,  $\vec{b}$ , and  $(\vec{a} + \vec{b})$  are equal, then the angle between  $\vec{a}$  and  $\vec{b}$  is:
- A.  $3\pi/4$
  - B.  $\pi/4$
  - C.  $\pi/3$
  - D.  $2\pi/3$

1 mark

9. The dot product between two vectors  $[2,0,24]$  and  $[20,2,4]$  is divisible by:
- A. 11
  - B. 13
  - C. 17
  - D. 19

1 mark

10. If A is a  $2024 \times 2024$  matrix such that  $|A| \neq 0$ , then  $\frac{|2A|}{|A|}$  equals:
- A. 2
  - B. 2024
  - C.  $2^{2024}$
  - D.  $2024^2$

1 mark

11. For a  $2 \times 2$  matrix M,  $\text{adj}(\text{adj}(M))$  equals:
- A. M
  - B. I (i.e.  $2 \times 2$  identity matrix)
  - C. O (i.e.  $2 \times 2$  zero matrix)
  - D. None of the above

1 mark

12.  $\int_0^1 e^x(x+1)dx =$
- A. 1
  - B. e
  - C. 2e
  - D. None of the above

2 marks

13. If  $e^x = \tan y$ , then the value of  $\frac{dy}{dx}$  at  $x = 0$  equals:
- A. 0
  - B.  $1/2$
  - C. 1
  - D. None of the above

2 marks

14. Given that the sum of the first 23 terms of an AP is 2024, which terms of that AP can be exactly known?
- A. 12<sup>th</sup> term
  - B. 11<sup>th</sup> term

- C. All terms
- D. No terms

1 mark

15. The coefficient of  $x^{20}$  in  $(x^2 - \frac{1}{x})^{24}$  is:

- A.  $\binom{24}{9}$
- B.  $-\binom{24}{9}$
- C.  $\binom{24}{10}$
- D. None of the above

1 mark

16. The value of  $\sum_{n=2}^{\infty} \frac{2024}{n^2-1}$  is:

- A. Less than 1250
- B. Between 1250 and 1500
- C. Between 1500 and 1750
- D. Above 1750

1 mark

17. If the integral  $\int_0^2 x^3 dx$  is approximated by the Trapezoidal rule with ( $h = 0.5$ ) instead of evaluating the integral exactly, the error (defined as approximated value less true value) equals:

- A. 0.25
- B. 0
- C. -0.25
- D. None of the above

2 marks

18. For solving the equation  $e^x = 20x$  using Newton-Raphson method starting with initial value  $x_0=1$ , the approximate root where we land after two iterations is

- A. 0.053
- B. 4.5
- C. 0
- D. None of the above

3 marks

19. If  $\omega$  denotes a complex cube root of 1 (other than 1 itself), then evaluate  $\omega^{2024} + \omega^{10}$ .

- A. 1
- B. -1
- C.  $\omega$
- D. None of the above

2 marks

20. If  $z = \frac{i-3}{i+2}$ , then  $z^{2024}$  equals:

- A. 1
- B.  $2^{2024}$
- C.  $2^{1012}$
- D. None of the above

2 marks

## Statistics

21. Two regular, unbiased six-faced dice (with faces numbered 1 through 6) are rolled and the higher of the two numbers is recorded. Let this be a random variable  $N$ . The median of  $N$  is:
- A. 3.5
  - B. 4
  - C. 5
  - D. None of the above

2 marks

22. For the set of observations  $\{1, 1, 2, 2, 2, 3, 3, 3, 4, 5, 5, n\}$ , if the median is equal to the unique mode, then  $n$  equals:
- A. 2
  - B. 3
  - C. 2.5
  - D. None of the above

1 mark

23. Seven friends – Sheldon, Leonard, Raj, Howard, Penny, Amy and Bernadette – need to stand in a line for a group photograph. If they order themselves randomly, the probability of Sheldon and Amy standing next to each other is:
- A.  $\frac{2}{7}$
  - B.  $\frac{1}{6}$
  - C.  $\frac{1}{7}$
  - D. None of the above

1 mark

24. The average marks of all boys in a class is 52, while the average marks of all girls in that class is 60. Then the average marks of all students (boys and girls) in that class will be
- A. 56
  - B. Indeterminate, but will surely be between 54 and 58.
  - C. Indeterminate, but will surely be between 52 and 60.
  - D. None of the above

1 mark

25. For two events  $X$  and  $Y$ ,  $P(X) = \frac{2}{7}$ ,  $P(X|Y) = \frac{1}{3}$ ,  $P(Y) = \frac{3}{10}$ , then  $P(Y|X)$  equals:
- A.  $\frac{1}{3}$
  - B.  $\frac{7}{20}$
  - C.  $\frac{1}{10}$
  - D. None of the above

1 mark

26. For a research project, a team comprising one or more actuaries and one or more data scientists needs to be constituted. Given that there are three actuaries  $\{A, B, C\}$  and three data scientists  $\{D, E, F\}$  to choose from, the total number of distinct (i.e. not exactly identical) teams that can be formed are:
- A. 9
  - B. 49
  - C. 64
  - D. None of the above

2 marks

27. If  $X$  follows a uniform distribution on  $[0,1]$ , the interquartile range of  $20X+24$  is:

- A. 10
- B. 20
- C. 24
- D. None of the above

1 mark

28. Let P and Q be two families having two children each. You happen to look in the courtyard of P's house and you see a boy playing. Separately, you know that the elder child in Q is a boy. Assume that a child is equally likely to be a boy or a girl (independent of their sibling). Let p and q be probabilities that both children in families P and Q respectively are boys. Then which of the following is true?

- A.  $p > q$
- B.  $p < q$
- C.  $p = q$
- D. Cannot be determined

2 marks

29. Let N follow a Poisson distribution with mean m, and X follow an Exponential distribution with mean  $\mu$ . Then which of the following is true?

- A. 2N follows a Poisson distribution with mean 2m, and 2X follows an Exponential distribution with mean  $2\mu$ .
- B. 2N does NOT follow a Poisson distribution with mean 2m, and 2X does NOT follow an Exponential distribution with mean  $2\mu$ .
- C. 2N follows a Poisson distribution with mean 2m, but 2X does NOT follow an Exponential distribution with mean  $2\mu$ .
- D. 2N does NOT follow a Poisson distribution with mean 2m, but 2X follows an Exponential distribution with mean  $2\mu$ .

1 mark

30. Suppose B follows a binomial distribution with mean 12 and standard deviation 2, then the maximum value that B can take is:

- A. 18
- B. 20
- C. 24
- D. None of the above

1 mark

31. A discrete random variable X has the following cumulative distribution function:

x	0	1	2	3	4	5
F(X)	0	0.3	0.4	0.6	0.9	1

Find the probability that X is even given that X is prime:

- A.  $2/5$
- B.  $1/5$
- C.  $1/4$
- D. None of the above

1 mark

32. Let X and Y be random variables with the following joint probability distribution:

		X			
		0	1	2	3
Y	0	0.20	0.15	0.1	0.05
	1	p	0.15	0.1	0

Here,  $p$  is an unknown real number. Then the covariance between  $X$  and  $Y$  is:

- A. Less than  $-0.1$
- B. Greater than  $0.1$
- C. Between  $0$  and  $0.1$
- D. Between  $-0.1$  and  $0$

3 marks

33. Based on positively correlated pairs of observations  $(x,y)$ , it is observed that the standard deviation of  $y$  is four times that of  $x$ . Further, the ratio of regression coefficient of  $y$  on  $x$  to that of  $x$  on  $y$  is:

- A.  $4$
- B.  $1/4$
- C.  $1/16$
- D.  $16$

1 mark

34. If  $X$  follows a normal distribution with mean  $0$  and variance  $2024$ , then the correlation coefficient between  $X^{2024}$  and  $X^{1947}$  will be:

- A. positive
- B. zero
- C. negative
- D. indeterminate

1 mark

35. If a square is drawn with the side length determined by the roll of a fair, six-faced die (faces numbered from  $1$  to  $6$ ), the expected value of the area of the square is:

- A.  $49/4$
- B.  $37/2$
- C.  $91/6$
- D.  $25/2$

2 marks

36. The price of a stock (currently valued at  $100$ ) can either increase by  $10\%$  or decrease by  $10\%$  each month. The probability of an increase is  $1.5$  times that of a decrease. Then, the probability that the stock price will be less than  $100$  after two months is:

- A.  $0.16$
- B.  $0.48$
- C.  $0.64$
- D. None of the above

3 marks

37. For a light bulb, the probability that it continues to work after  $t$  days of usage is given by  $e^{-\frac{t}{1000}}$ . Given that the bulb has already lasted  $2024$  days, the probability that it will last for  $1000$  more days equals:

- A.  $1/e$
- B.  $1/e^2$
- C.  $1/e^{3.024}$
- D. None of the above

1 mark

38. If  $A$  and  $B$  are disjoint events, each with a positive probability, then they are:

- A. Necessarily independent
- B. May or may not be independent – it depends on their probability values.

- C. May or may not be independent – it depends on the universal set.
- D. Definitely not independent

1 mark

39. A test contains 100 multiple-choice questions with 5 given options of which exactly one is correct. Each correct answer fetches one mark, and there is no negative marking. If a student randomly attempts all the 100 questions, the mean and the standard deviation of his total marks will be:

- A. 20, 4
- B. 20,  $\sqrt{20}$
- C. 20, 20
- D. None of the above

2 marks

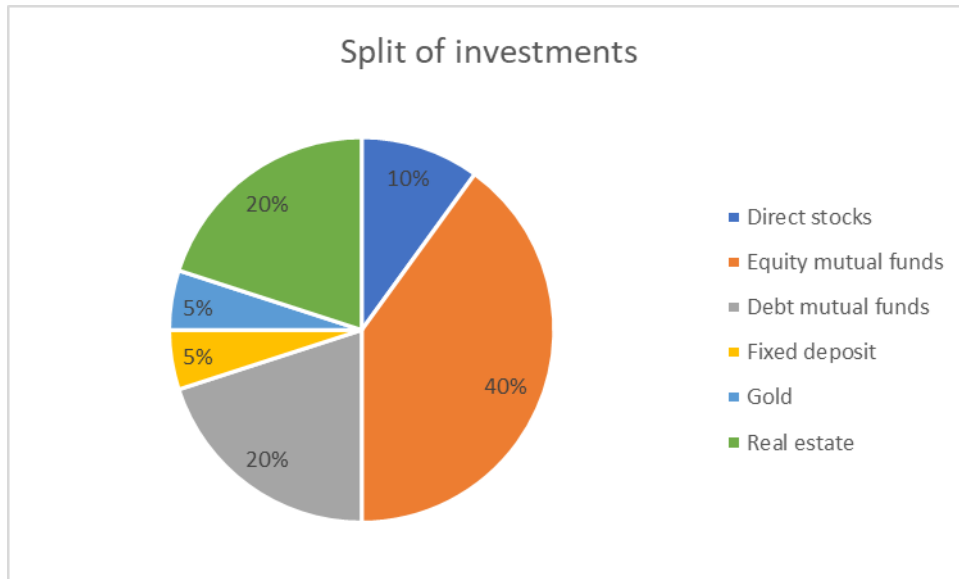
40. In a meeting of 20 people – 10 men and 10 women, every individual shakes hands with participants of the opposite gender, and hugs participants of the same gender. Then the total number of handshakes and hugs, respectively, is:

- A. 200, 180
- B. 100, 90
- C. 180, 200
- D. 90, 100

2 marks

## Data Interpretation

The following pie chart shows the allocation of investments of an investor named Mr. Paisawala, whose total mutual fund investments amount to Rs. 1.2 crores. Questions 41 to 43 are based on the information.



41. The total size of Mr. Paisawala's fixed income investments (i.e. debt mutual funds and fixed deposits) is:
- A. Rs. 50 lakh
  - B. Rs. 30 lakh
  - C. Rs. 40 lakh
  - D. None of the above
- 1 mark
42. If Mr. Paisawala wants to limit the total mutual fund exposure to 50% of total investments by transferring some of the equity mutual fund investment to direct stocks, the direct stocks component will become
- A. Bigger than real estate component
  - B. Equal to real estate component
  - C. Smaller than real estate component
  - D. Insufficient information
- 1 mark
43. If, one year from now, the equity investments (direct stocks and equity mutual funds), the real estate and gold appreciate by 15%, 12.5% and 10% respectively, and the fixed income investments remain unchanged, the total investment value after one year will become:
- A. Rs. 2.42 crore
  - B. Rs. 1.326 crore
  - C. Rs. 2.21 crore
  - D. None of the above

2 marks

The table below contains the band-wise details of number of returns filed, and the total and average salary incomes in a particular assessment year. Questions 44 to 47 are based on this information.



## 1.2 All Taxpayers - Range of Salary Income (AY 2018-19)

Range (in INR)	No. of Returns	Sum of Salary Income (In Crore INR)	Average Salary Income (In Lakh INR)
< 0	-	-	-
= 0	2,96,80,223	-	-
>0 and <=1,50,000	19,81,749	18,095	0.91
>1,50,000 and <= 2,00,000	11,59,340	20,611	1.78
>2,00,000 and <= 2,50,000	16,01,553	36,740	2.29
>2,50,000 and <= 3,50,000	37,65,023	1,13,845	3.02
>3,50,000 and <= 4,00,000	20,08,938	75,298	3.75
>4,00,000 and <= 4,50,000	20,36,705	86,662	4.26
>4,50,000 and <= 5,00,000	20,58,243	97,768	4.75
>5,00,000 and <= 5,50,000	17,93,339	94,029	5.24
>5,50,000 and <= 9,50,000	81,55,335	5,80,567	7.12
>9,50,000 and <= 10,00,000	4,60,298	44,848	9.74
>10,00,000 and <=15,00,000	22,37,558	2,67,135	11.94
>15,00,000 and <= 20,00,000	7,19,882	1,23,831	17.20
>20,00,000 and <= 25,00,000	3,80,802	84,736	22.25
>25,00,000 and <= 50,00,000	5,04,258	1,68,129	33.34
>50,00,000 and <= 1,00,00,000	1,21,084	81,171	67.04
>1,00,00,000 and <=5,00,00,000	46,279	80,380	173.69
>5,00,00,000 and <=10,00,00,000	2,039	13,669	670.38
>10,00,00,000 and <=25,00,00,000	660	9,610	1,456.21
>25,00,00,000 and <=50,00,00,000	106	3,505	3,306.93
>50,00,00,000 and <=100,00,00,000	35	2,276	6,503.36
>100,00,00,000 and <=500,00,00,000	9	1,154	12,831.77
>500,00,00,000	-	-	-
<b>Total</b>	<b>5,87,13,458</b>	<b>20,04,069</b>	

44. The number of returns with a salary income exceeding than 1 crore as a proportion of the returns with a positive salary income is (approximately):
- 0.08%
  - 0.17%
  - 5.5%
  - None of the above

1 mark

45. The total income of individuals with a salary income exceeding than 1 crore as a proportion of the overall total income of all individuals is (approximately):
- Less than 3%
  - Between 3% to 5%
  - Between 5% to 7%
  - More than 7%

1 mark

46. The average salary of those who have an income exceeding 25 crore is:
- 46.2 crore
  - 226.4 crore
  - 65.0 crore
  - None of the above

2 marks

47. Across all the returns filed, the median salary income lies in which band:
- 4.5 lakh to 5 lakh
  - 5 lakh to 5.5 lakh
  - 4 lakh to 4.5 lakh

D. None of the above

1 mark

A company manufactures cars at three different manufacturing units – A, B and C.

In 2021, the total cars manufactured were 1200. The number of cars manufactured at B were  $\frac{1}{3}$ rd of those manufactured at C in 2016. The average number of cars manufactured at A and B in 2021 was 300.

In 2022, the sum of the number of cars manufactured at A and C were 1200. The ratio of the number of cars manufactured at C in 2021 to that manufactured at C in 2022 is 4:3. The number of cars manufactured at B in 2022 were equal to the number cars manufactured at B in 2023.

In 2023, the sum of the number of cars manufactured at A and B is equal to the total cars manufactured at C. The total number of cars manufactured in 2023 were 1400. The number of cars manufactured at A were twice of the those manufactured at B in 2021.

Questions 48 to 51 are dependent on this information.

48. The contribution of unit C as a proportion of the total production for that year is least in:

- A. 2021
- B. 2022
- C. 2023
- D. Cannot be determined

1 mark

49. The total number of cars manufactured in 2022 is:

- A. 1200
- B. 1400
- C. 1600
- D. None of the above

2 marks

50. The total number of cars manufactured at unit B across all years is:

- A. 800
- B. 900
- C. 1000
- D. None of the above

1 mark

51. The maximum number of cars manufactured at one unit in one year was:

- A. At unit C in 2021
- B. At unit A in 2022
- C. At unit C in 2023
- D. None of the above

2 marks

# English

52. An 'aviary' is:
- A. A place where they dispose of old airplanes
  - B. An aviation centre or museum
  - C. A greedy person
  - D. A place where birds are kept
- 1 mark
53. Which of the following is incorrect?
- A. An umbrella
  - B. An unit
  - C. An urchin
  - D. An umpire
- 1 mark
54. A student who sacrifices night sleep and studies hard for an exam can be said to be burning the midnight \_\_\_\_.
- A. Fire
  - B. Oil
  - C. Sleep
  - D. Snake
- 1 mark
55. Poet : Poem :: Lexicographer : ?
- A. Laws
  - B. Pen
  - C. Dictionary
  - D. Graphs and charts
- 1 mark
56. "The judges disposed off the bail application after hearing both sides for a day." Identify the erroneous portion in this sentence.
- A. "The judges disposed off"
  - B. "the bail application"
  - C. "after hearing both sides for a day"
  - D. None of the above
- 1 mark
57. "If I \_\_\_\_ a musician, I \_\_\_\_\_ have concerts all over India." Fill in the blanks
- A. was, will
  - B. were, will
  - C. was, would
  - D. were, would
- 1 mark
58. Which of the words is NOT similar in meaning to the others?
- A. Ecstatic
  - B. Elated
  - C. Delighted
  - D. Astonished
- 1 mark

59. "Jack and Jill went up the hill to fetch a \_\_\_\_ of water." Fill in the blanks.

- A. pale
- B. pail
- C. peal
- D. pell

1 mark

60. Which of these is NOT a valid inference from the passage below?

*"The aftermath of the COVID-19 pandemic and ongoing Russia-Ukraine war has exposed cracks in societies that are being further strained by episodic upheaval. Yet the global system has thus far proved surprisingly resilient. A widely anticipated recession failed to materialize last year, and financial turbulence was quickly subdued, but the outlook remains uncertain. Political strife and violent conflicts, from Niger and Sudan to Gaza and Israel, have captured the attention and apprehension of populations worldwide in some instances while attracting little focus in others. These developments have not yet led to wider regional conflicts – nor have they created globally destabilizing consequences such as those seen at the initial outbreak of the war in Ukraine or the COVID-19 pandemic – but their long-term outlook could bring further shocks."*

- A. We could expect a lot of turbulence in the times to come, more than what has been experienced in the recent past.
- B. An economic recession has been on the cards for some time but hasn't quite arrived.
- C. Violent political events across the world have consistently garnered a lot of attention.
- D. COVID-19 caused instability across the world.

3 marks

61. The following sentences are jumbled up. Choose the correct sequence from the options below.

- I. Importantly, its clear warm waters and white sand beaches, along with its proximity to the USA, make the Bahamas a prime tourist destination.
  - II. Bahamas is an archipelagic state of the Lucayan Archipelago and consists of more than 700 islands, of which 30 are inhabited.
  - III. It is not surprising then that a large part, i.e., about 44 percent, of the income generated comes from tourism and that this sector also accounts for more than half of the country's workforce.
  - IV. These islands, albeit only 14,000 square kilometers in land area, stretch over a total area of 260,000 square kilometers of sea.
- A. II, IV, III, I
  - B. II, I, III, IV
  - C. II, I, IV, III
  - D. II, IV, I, III

2 marks

62. Consider the following set of sentences:

- I. I except the award on behalf of my team.
- II. The barber cut my hairs yesterday morning.
- III. I need to by heart this poem for the English exam tomorrow.

Of these, the sentences that are erroneous are:

- A. I and II
- B. I and III
- C. II and III
- D. I, II and III

2 marks

# Logical Reasoning

63. If, in 2023, your birthday fell on a Sunday in 2023, you will celebrate your birthday in 2024 on a:

- A. Sunday
- B. Monday
- C. Tuesday
- D. Can't be sure

1 mark

64. Between noon and midnight (i.e. in 12 hours) and not counting those two instances, how many times will the minute hand exactly overlap with the hour hand on a clock?

- A. 10
- B. 11
- C. 12
- D. 13

1 mark

65. Five sides of a  $5 \times 5 \times 5$  cube are painted red, and the remaining side is painted blue. The cube is then divided into 125 unit cubes (i.e. of dimension  $1 \times 1 \times 1$ ). How many of the smaller cubes will have at least two sides painted red?

- A. 20
- B. 32
- C. 40
- D. None of the above

2 marks

66. If Y is X's mother's father's son, then X is Y's:

- A. Father / mother
- B. Brother / sister
- C. Nephew / niece
- D. Son / daughter

1 mark

67. Five cards numbered 1 to 5 are placed in a sequence. No two prime numbered cards are next to each other. The numbers on two cards, that surround the card numbered 1, add up to 8. Which of the following statements is necessarily true?

- A. Cards numbered 2 and 3 have exactly one card in between.
- B. Card numbered 1 is in the second position in the sequence.
- C. The product of numbers on any two consecutive cards is never more than 15.
- D. Cards numbered 2 and 4 are next to each other.

2 marks

68. If all alphas are betas, and some betas are gammas, then which of the following is necessarily true?

- A. If x isn't gamma, it is also not alpha.
- B. If x isn't beta, it is also not alpha.
- C. If x isn't gamma, it is also not beta.
- D. If x isn't alpha, it is also not beta.

1 mark

69. In a group of 40 students, 20 are male, and 24 have opted for Sanskrit. Then the minimum number of boys who have opted for Sanskrit is:

- A. 0
- B. 4
- C. 20
- D. None of the above

1 mark

70. Which of the following sequences is inconsistent with the others?

- A. Mumbai, Maharashtra, India
- B. Adelaide, South Australia, Australia
- C. United States of America, California, Los Angeles
- D. Toronto, Ontario, Canada

1 mark

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