Institute of Actuaries of India

ACET January 2026 Solutions

Mathematics

1. If $f(x) = \ln(\sqrt{x^2+9})$, find $\min f(x)$.

$$f(x) = \ln(\sqrt{x^2+9}) = \ln((x^2+9)^{1/2}) = \frac{1}{2}\ln(x^2+9).$$

Since $x^2 \geq 0$, we have $x^2 + 9 \geq 9$. Minimum occurs when $x^2$is minimum i.e. $x = 0$. So:

$$\min f(x) = \frac{1}{2}\ln(9) = \ln 3$$

Answer: B

2. Compare $f(x) = |x^2 - 4x|$and $g(x) = x^2 - 4|x|$.

Consider cases.

- **Case 1:** $x \geq 0$Then $|x| = x$, so $g(x) = x^2 - 4x$. So $f(x) = |x^2 - 4x| = |g(x)| \geq g(x)$. Equality only when $x^2 - 4x \geq 0$(i.e. $x \leq 0$or $x \geq 4$, but here $x \geq 0$, so $x \in [0,0] \cup [4,\infty) \Rightarrow x = 0$or $x \geq 4$).
- **Case 2:** $x < 0$Then $|x| = -x$. $g(x) = x^2 - 4(-x) = x^2 + 4x$. $f(x) = |x^2 - 4x|$.

Check a specific negative: $x = -1$. $f(-1) = |1 + 4| = 5$. $g(-1) = 1 - 4 = -3$. So $f(-1) > g(-1)$.

Try to see if equality or opposite can ever hold. In both sign regions we see $f(x)$is absolute value of something, while $g(x)$is not; numerical checks around 0, 1, 2, -1, -2 show $f(x) \geq g(x)$and sometimes strictly greater. There is no point where $f(x) < g(x)$for all x or equality for all x.

Hence none of A, B, C holds for all real x.

Answer: D

3. Number of real roots of $3x^3 - 2x + 7 = 0$.

Let $f(x) = 3x^3 - 2x + 7$. Cubic with real coefficients always has at least one real root. Check its monotonicity:

$$f'(x) = 9x^2 - 2.$$

Critical points solve $9x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2/9} = \pm\frac{\sqrt{2}}{3}$. So there are two turning points ⇒ possibly 1 or 3 distinct real roots.

Evaluate f at these:

- $f(\frac{\sqrt{2}}{3}) = 3\left(\frac{\sqrt{2}}{3}\right)^3 - 2\left(\frac{\sqrt{2}}{3}\right) + 7 = 3 \cdot \frac{2\sqrt{2}}{27} - \frac{2\sqrt{2}}{3} + 7 = \frac{2\sqrt{2}}{9} - \frac{2\sqrt{2}}{3} + 7 = 2\sqrt{2}\left(\frac{1}{9} - \frac{1}{3}\right) + 7 = 2\sqrt{2}(-\frac{2}{9}) + 7 = -\frac{4\sqrt{2}}{9} + 7 > 0.$
- $f(-\frac{\sqrt{2}}{3}) = 3\left(\ \ -\frac{\sqrt{2}}{3}\right)^3 - 2\left(\ \ -\frac{\sqrt{2}}{3}\right) + 7 = -\frac{2\sqrt{2}}{9} + \frac{2\sqrt{2}}{3} + 7 = 2\sqrt{2}\left(-\frac{1}{9} + \frac{1}{3}\right) + 7 = 2\sqrt{2} \cdot \frac{2}{9} + 7 > 0.$

Both local extremum values are positive, and as $x \to -\infty$, $f(x) \to -\infty$(leading coefficient positive, odd degree), and as $x \to +\infty$, $f(x) \to +\infty$. The graph dips from +∞ to a positive minimum then rises again, so it crosses the x-axis exactly once on the far left.

Thus: **exactly 1 real root**.

Answer: B

4. $\boldsymbol{I = \int_0^{2025} [x]\{x\} dx}$**. Classify** $\boldsymbol{I}$**.**

On any interval $[n, n+1)$ with integer n, $[x] = n$, $\{x\} = x - n$. So:

$$I = \sum_{n=0}^{2024} \int_n^{n+1} n(x-n)\, dx = \sum_{n=0}^{2024} n \int_n^{n+1} (x-n)\, dx.$$

Let $t = x - n$, limits 0 to 1:

$$\int_n^{n+1} (x-n)\, dx = \int_0^1 t\, dt = \frac{1}{2}.$$

So

$$I = \sum_{n=0}^{2024} n \cdot \frac{1}{2} = \frac{1}{2}\sum_{n=0}^{2024} n = \frac{1}{2} \cdot \frac{2024 \cdot 2025}{2} = \frac{2024 \cdot 2025}{4}.$$

Product $2024 \cdot 2025$is divisible by 4? 2024 is divisible by 8 (since 2000+24), so definitely by 4. Therefore numerator is divisible by 4, so $I$is an integer and that integer is divisible by 4.

Answer: C

5. Roots of $\boldsymbol{x^2 - (a+b)x + ab = 0}$**when** $\boldsymbol{a, b}$**are roots of** $\boldsymbol{x^2 - 12x + 20 = 0.}$

From original quadratic: Sum of roots $a + b = 12$. Product $ab = 20$.

The new quadratic is exactly:

$$x^2 - (a+b)x + ab = x^2 - 12x + 20$$

which has roots a, b. So the roots are a, b.

Answer: A

6. $\lim_{x\to 0} \frac{\tan(7x) - \tan(3x)}{x}$**.**

Use small-angle approximation or derivative of tan at 0 (which is 1). Near 0, $\tan(kx) \approx kx$. So:

$$\tan(7x) - \tan(3x) \approx 7x - 3x = 4x.$$

Then:

$$\lim_{x\to 0} \frac{\tan(7x) - \tan(3x)}{x} \approx \lim_{x\to 0} \frac{4x}{x} = 4.$$

Formal: this is \((7 - 3)\tan'(0) = 4·1 = 4.

Answer: A

7. GP with $a_5 = 48$, $a_{10} = 3$. Find smallest $k > 10$with integer a_k, then $k + a_k$.

Let first term = a, common ratio = r. $a_5 = ar^4 = 48$. $a_{10} = ar^9 = 3$.

Divide:

$$\frac{ar^9}{ar^4} = r^5 = \frac{3}{48} = \frac{1}{16}.$$

So:

$$r = \left(\frac{1}{16}\right)^{1/5}.$$

Now $ar^4 = 48$, so:

$$a = \frac{48}{r^4}.$$

General term:

$$a_n = ar^{n-1} = \frac{48}{r^4} \cdot r^{n-1} = 48r^{n-5}.$$

We want $a_k$integer for minimal $k > 10$. We know $r^5 = 1/16$, so $r^{n-5} = (r^5)^{(n-5)/5} = (1/16)^{(n-5)/5}$.

For n such that $n-5$ is a multiple of 5, say $n-5=5m$:

$$a_n = 48(1/16)^m = 48/16^m.$$

We already know at $n=10$, $a_{10}=3$(m=1). That's integer. They want $k>10$. Next with $n-5$ multiple of 5 is $n=15$(m=2):

$$a_{15} = 48/16^2 = 48/256 = 3/16$$

not integer. Next $n=20$, m=3:

$$a_{20} = 48/16^3 = 48/4096 = 3/256$$

not integer. In fact for $m \geq 2$ the numerator 48 doesn't contain enough powers of 2 to cancel 16^m. So beyond 10, no further term will be integer.

Small subtlety: the question says "smallest integer greater than 25" in the original style, but here I used k>10. In our version it is "greater than 10". Given none for $n>10$ are integers, the correct mathematical conclusion is: there is **no such k**.

So the correct option is:

Answer: D (None of the above)

8. If $\vec{v} = 6\mathbf{i} - 8\mathbf{j}$, unit vector in direction of $\vec{v}$.

Magnitude:

$$|\vec{v}| = \sqrt{6^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10.$$

Unit vector:

$$\hat{v} = \frac{1}{10}(6\mathbf{i} - 8\mathbf{j}) = 0.6\mathbf{i} - 0.8\mathbf{j} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}.$$

Answer: A

9. Dot and cross product commutativity.

- Dot product: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \Rightarrow$ TRUE.
- Cross product: $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a}) \Rightarrow$ not equal in general, so FALSE.

So X is true, Y is false.

Answer: C

10. If $\det(A) = 5$ for 3×3 matrix A, find $\det(A^{-1})$.

For any invertible matrix, $\det(A^{-1}) = 1/\det(A)$. So $\det(A^{-1}) = 1/5$.

Answer: A

11. For 2×2 matrix M, what is adj$(\text{adj}(M))$?

For 2×2 matrix M with determinant $d \neq 0$:

- adj$(M) = dM^{-1}$.
- Also, adj(M) = matrix of cofactors transposed.

General fact: for an $n \times n$ matrix, adj$(\text{adj}(M)) = (\det\ M)^{n-2} M$ if M is invertible. For $n = 2$:

$$\text{adj}(\text{adj}(M)) = (\det\ M)^0 M = M.$$

So adj$(\text{adj}(M)) = M$.

Answer: A

12. Evaluate $\int_0^1 e^{2x}\,(1+x)\,dx$.

Let's split:

$$\int_0^1 e^{2x}\,(1+x)dx = \int_0^1 e^{2x}\,dx + \int_0^1 x\,e^{2x}dx.$$

First:

$$\int_0^1 e^{2x}\,dx = \frac{1}{2}e^{2x}\,|_0^1 = \frac{1}{2}(e^2 - 1).$$

Second: integrate by parts. Let $u = x$, $dv = e^{2x}dx$. Then $du = dx$, $v = \frac{1}{2}e^{2x}$.

$$\int_0^1 x\,e^{2x}dx = \frac{1}{2}xe^{2x}\,|_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx = \frac{1}{2}(1 \cdot e^2 - 0) - \frac{1}{2}\cdot\frac{1}{2}(e^2 - 1) = \frac{1}{2}e^2 - \frac{1}{4}(e^2 - 1)$$

$$= \frac{1}{2}e^2 - \frac{1}{4}e^2 + \frac{1}{4} = \frac{1}{4}e^2 + \frac{1}{4}.$$

Now sum the two:

$$\int_0^1 e^{2x}\,(1+x)dx = \frac{1}{2}(e^2 - 1) + \left(\frac{1}{4}e^2 + \frac{1}{4}\right) = \left(\frac{1}{2} + \frac{1}{4}\right)e^2 + \left(-\frac{1}{2} + \frac{1}{4}\right) = \frac{3}{4}e^2 - \frac{1}{4}.$$

Check options: none of A, B, C match $\frac{3}{4}e^2 - \frac{1}{4}$.

Answer: D

13. If $e^x = \tan\ y$, find $\frac{dy}{dx}$ at $x = 0$.

We have:

$$e^x = \tan\ y.$$

Differentiate both sides w.r.t x:

$$e^x = \sec^2 y \cdot \frac{dy}{dx}.$$

So:

$$\frac{dy}{dx} = \frac{e^x}{\sec^2 y}.$$

At $x = 0$: $e^0 = 1$. Also, $1 = \tan\ y \Rightarrow y = \pi/4$(principal branch). Then $\sec^2(\pi/4) = 2$.

So:

$$\frac{dy}{dx}\Big|_{x=0} = \frac{1}{2}.$$

Answer: B

14. Find the derivative:

$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

1. Critical points: x=0 and x=2.
   But note: factorization shows f'(x)=3x(x-2).

2. Test intervals:

- For x < 0: $f'(x) > 0$ → **increasing**
- For 0 < x < 2: $f'(x) < 0$ → decreasing
- For x > 2: $f'(x) > 0$ → **increasing**

So the function is strictly increasing on $(-\infty, 0)U(2, \infty)$

Answer: C

15. Coefficient of x^{12}in $(x^3 - 2)^8$.

General term:

$$T_k = \binom{8}{k}(x^3)^{8-k}(-2)^k = \binom{8}{k}x^{3(8-k)}(-2)^k.$$

We want exponent of x: $3(8-k) = 12$.

$$3(8-k) = 12 \Rightarrow 8 - k = 4 \Rightarrow k = 4.$$

So coefficient = $\binom{8}{4}(-2)^4$. That matches option A.

Answer: A

16. Value of $\sum_{n=1}^{2024} \frac{1}{n(n+1)}$.

Use partial fractions:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}.$$

So:

$$\sum_{n=1}^{2024} \frac{1}{n(n+1)} = \sum_{n=1}^{2024} \left(\frac{1}{n} - \frac{1}{n+1}\right) = 1 - \frac{1}{2025}.$$

So value is $1 - 1/2025 \approx 0.9995$, less than 1.

Answer: A

17. Trapezoidal rule with $h = 0.25$for $\int_0^1 (x^2 + 1)dx$.

We partition [0,1] into 4 intervals of width h=0.25: points 0,0.25,0.5,0.75,1.

Compute f(x)=x^2+1:

- f(0)=1
- f(0.25)=1+0.0625=1.0625
- f(0.5)=1+0.25=1.25
- f(0.75)=1+0.5625=1.5625
- f(1)=2

Trapezoidal approximation:

$$T = \frac{h}{2}[f(0) + 2(f(0.25) + f(0.5) + f(0.75)) + f(1)].$$

Sum inside:

$$f(0.25) + f(0.5) + f(0.75) = 1.0625 + 1.25 + 1.5625 = 3.875.$$

Then:

$$T = \frac{0.25}{2}[1 + 2(3.875) + 2] = 0.125[1 + 7.75 + 2] = 0.125 \cdot 10.75 = 1.34375.$$

Closest to this is 1.33, and so the correct answer is B

Answer: B

(For reference, exact integral is $\int_0^1 (x^2 + 1)dx = [x^3/3 + x]_0^1 = 1/3 + 1 = 4/3 \approx 1.3333$. Trapezoidal gave 1.34375, slightly above.)

18. Newton–Raphson for $e^x = 5x$, starting $x_0 = 1$. Find x_1.

Let $f(x) = e^x - 5x$. Then:

$$f'(x) = e^x - 5.$$

Newton step:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

At $x_0 = 1$:

$$f(1) = e - 5 \approx 2.718 - 5 = -2.282.$$

$$f'(1) = e - 5 \approx -2.282.$$

So:

$$x_1 = 1 - \frac{-2.282}{-2.282} = 1 - 1 = 0.$$

So $x_1 = 0$, which is not among A, B, C; thus "None of the above".

Answer: D

19. If ω is cube root of unity (≠1), compute $\omega^{2025} + \omega^{2027}$.

We know $\omega^3 = 1$, and $\{1, \omega, \omega^2\}$with $1 + \omega + \omega^2 = 0$.

Reduce exponents mod 3:

$$2025 \div 3 \Rightarrow \text{remainder } 0 \Rightarrow \omega^{2025} = \omega^0 = 1.$$

$$2027 \div 3 \Rightarrow \text{remainder } 2 \Rightarrow \omega^{2027} = \omega^2.$$

So expression = $1 + \omega^2$. But $1 + \omega + \omega^2 = 0 \Rightarrow 1 + \omega^2 = -\omega$.

So sum = $-\omega$. This is not equal to $0, 1, \omega, \omega^2$ for non-real ω (−ω is not in the set $\{1, \omega, \omega^2\}$). So none of the above listed options are correct. Hence None of the above.

Answer: D

20. If $z = (3 - 4i)/(1 + 2i)$, compute | z |.

First compute z:

Multiply numerator and denominator by conjugate of denominator $1 - 2i$:

$$z = \frac{(3 - 4i)(1 - 2i)}{(1 + 2i)(1 - 2i)}.$$

Denominator:

$$(1 + 2i)(1 - 2i) = 1 + 4 = 5.$$

Numerator:

$$(3 - 4i)(1 - 2i) = 3(1) + 3(-2i) - 4i(1) + (-4i)(-2i)$$

$$= 3 - 6i - 4i + 8i^2 = 3 - 10i + 8(-1) = 3 - 10i - 8 = -5 - 10i.$$

So:

$$z = \frac{-5 - 10i}{5} = -1 - 2i.$$

Magnitude:

$$|\, z \,| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{1 + 4} = \sqrt{5}.$$

Answer: C

Statistics

21. Spinner with probabilities proportional to j+1

Outcomes {0,1,2}. Weights:

- w_0 = 0+1 = 1
- w_1 = 1+1 = 2
- w_2 = 2+1 = 3

Total = 1+2+3 = 6.

So probabilities:

- P(N=0) = 1/6
- P(N=1) = 2/6 = 1/3
- P(N=2) = 3/6 = 1/2

CDF:

- P(N ≤ 0) = 1/6
- P(N ≤ 1) = 1/6 + 1/3 = 3/6 = 1/2
- P(N ≤ 2) = 1

Median = smallest m with P(N≤m) ≥ 1/2 ⇒ m = 1.

Answer: B (1)
Question status: Correct, unique answer.

22. 5 heights and 3-student means

Given:

$$h_1 \le h_2 \le h_3 \le h_4 \le h_5, \quad \frac{h_1+h_2+h_3+h_4+h_5}{5} = 170 \Rightarrow h_1+\dots+h_5 = 850.$$

Smallest 3-mean:

$$\frac{h_1+h_2+h_3}{3} = 162 \Rightarrow h_1+h_2+h_3 = 486.$$

Largest 3-mean:

$$\frac{h_3+h_4+h_5}{3} = 176 \Rightarrow h_3+h_4+h_5 = 528.$$

Add:

$$(h_1+h_2+h_3)+(h_3+h_4+h_5) = 486+528 = 1014.$$

LHS = $(h_1+h_2+2h_3+h_4+h_5)$.
Subtract total 850:

$$(h_1+h_2+2h_3+h_4+h_5) - (h_1+h_2+h_3+h_4+h_5) = 1014 - 850 \Rightarrow h_3 = 164.$$

Answer: A (164 cm)
Question status: Correct, unique answer and in options.

23. Committee with at least 2 economists, more engineers

Total engineers: 8, economists: 5. Committee size = 6. Let (E,C) be numbers of engineers, economists.

Conditions:

- $E + C = 6$
- $C \ge 2$
- $E > C$

Check possibilities:

- $C=2 \Rightarrow E=4 \Rightarrow E>C$, OK.
- $C=3 \Rightarrow E=3 \Rightarrow E>C$ fails (equal).
- $C=4 \Rightarrow E=2 \Rightarrow E>C$ fails.

So only (E,C) = (4,2).

Number of committees:

$$\binom{8}{4}\binom{5}{2} = 70 \cdot 10 = 700.$$

Answer: D (700)
Question status: Correct, unique.

24. Two batches, combined mean

Batch A: n_1=40, mean=65.
Batch B: n_2=n, mean=75, all marks equal (so variance 0). Combined mean=70.

Mean equation:

[\frac{40\cdot 65 + n\cdot 75}{40+n} = 70.]

Compute:

(40·65 = 2600).

[2600 + 75n = 70(40+n) = 2800 + 70n \Rightarrow 75n - 70n = 2800 - 2600 \Rightarrow 5n = 200 \Rightarrow n=40.]

Answer: D (40)
Question status: Now consistent (variance not used), unique answer.

25. P(B|A)

Given (P(A)=0.3), (P(B)=0.5), (P(A\mid B) = 0.4).

First:

[P(A\cap B) = P(A\mid B)P(B) = 0.4·0.5 = 0.2.]

Then:

[P(B\mid A) = \frac{P(A\cap B)}{P(A)} = \frac{0.2}{0.3} = \frac{2}{3}.]

Answer: C (2/3)
Question status: Correct.

26. Die then coin tosses, P(X=0)

Die outcome n (1–6) with probability 1/6. Then coin tossed n times.

Conditional probability:

[P(X=0 \mid n) = (1/2)^n.]

Unconditional:

[P(X=0) = \sum_{n=1}^6 \frac{1}{6} (1/2)^n = \frac{1}{6} \sum_{n=1}^6 (1/2)^n.]

Geometric sum:

$$\sum_{n=1}^6 (1/2)^n = \frac{1}{2}\cdot\frac{1-(1/2)^6}{1-1/2} = 1 - 1/64 = 63/64.$$

So:

$$P(X=0) = \frac{1}{6}\cdot\frac{63}{64} = \frac{63}{384} = \frac{21}{128}.$$

Answer: A (21/128)
Question status: Correct, options adjusted to include the exact value.

27. IQR of Y

$X \sim \text{Uniform}(0,2)$. For a Uniform(a,b), quartiles:

- $Q1 = a + 0.25(b-a)$
- $Q3 = a + 0.75(b-a)$.

Here $a=0, b=2 \Rightarrow Q1=0.5$, $Q3=1.5 \Rightarrow IQR(X)=1$.

$Y = 5X - 3$, so for linear transform $Y = aX + b$, $IQR(Y) = |a|\cdot IQR(X) = 5\cdot 1 = 5$.

Answer: B (5)
Question status: Correct.

28. Exponential, conditional survival

Mean=4 ⇒ rate $\lambda=1/4$. Survival:

$$P(T>t) = e^{-t/4}.$$

We want:

$$P(T > 3+5 \mid T > 3) = P(T>8 \mid T>3).$$

Memorylessness ⇒ equals $P(T>5) = e^{-5/4}$.

Answer: A ($e^{-5/4}$)
Question status: Correct.

29. Variance under linear transform

$X \sim N(10,9)$. So $Var(X)=9$. $Y=2X-5$.

$Var(Y) = 2^2 \cdot 9 = 36$.

Answer: C (36)
Question status: Correct.

30. Symmetric Binomial

Binomial(n,p) is symmetric iff p=1/2 (for any n).

Answer: B (p = 1/2)
Question status: Correct.

31. Poisson parameter

Poisson(λ), condition: (P(X=3) = 2 P(X=4)).

Using pmf:

[\frac{e^{-\lambda}\lambda^3}{3!} = 2 \cdot \frac{e^{-\lambda}\lambda^4}{4!}.]

Cancel common factors:

[\frac{1}{6} = 2 \cdot \frac{\lambda}{24} = \frac{\lambda}{12} \Rightarrow \lambda = 2.]

Answer: A (2)
Question status: Correct.

32. Exponential, given ratio of tails

Exponential with mean θ ⇒ rate = 1/θ. Survival: (P(X>t)=e^{-t/\theta}).

Given:

[\frac{P(X>2)}{P(X>5)} = e.]

Compute:

[\frac{e^{-2/\theta}}{e^{-5/\theta}} = e^{3/\theta} = e \Rightarrow 3/\theta = 1 \Rightarrow \theta = 3.]

Variance = θ^2 = 9.

Answer: D (9)
Question status: Correct.

33. P(X is odd)

From CDF we extract pmf:

- (P(X=0) = 0.2)
- (P(X=1) = 0.5 - 0.2 = 0.3)
- (P(X=2) = 0.8 - 0.5 = 0.3)
- (P(X=3) = 0.95 - 0.8 = 0.15)
- (P(X=4) = 1 - 0.95 = 0.05).

Check sum = 1.

Odd values: 1 and 3.
Probability = 0.3 + 0.15 = 0.45.

Answer: B (0.45)
Question status: Correct.

34. |mean – median| for same X

Mean:

[E[X] = 0·0.2 + 1·0.3 + 2·0.3 + 3·0.15 + 4·0.05 = 0.3 + 0.6 + 0.45 + 0.2 = 1.55.]

Median: smallest x with F(x) ≥ 0.5.
F(0)=0.2, F(1)=0.5 ⇒ median = 1.

Difference:

[|m - M| = |1.55 - 1| = 0.55,]

which lies between 0.5 and 1.

Answer: C (Between 0.5 and 1)
Question status: Correct.

35. Mode of stem-and-leaf data

Data list:

- 21, 24
- 30, 33, 33, 38
- 40, 42, 45, 45, 47
- 50, 51, 53, 54, 54

Frequencies:

- 33 → 2
- 45 → 2
- 54 → 2
  Others → 1.

So there are **three modes**: 33, 45, 54. That means "more than one mode".

Answer: C (The dataset has more than one mode.)
Question status: Now correctly asks about nature of mode, not specific value.

36. Regression line and correlation feasibility

Regression of X on Y:

$$4X - 3Y + 10 = 0 \Rightarrow X = \frac{3}{4}Y - \frac{10}{4}.$$

Slope ($b_{X|Y} = 3/4$). For regressions:

$$b_{X|Y} = \rho \cdot \frac{s_X}{s_Y}.$$

Given ($s_X=2$), ($s_Y=4$):

$$\frac{3}{4} = \rho \cdot \frac{2}{4} = \rho \cdot \frac{1}{2} \Rightarrow \rho = \frac{3}{2},$$

which violates $|\rho| \le 1$.

So the only consistent statement is that the information itself cannot all be correct.

Answer: C (($|\rho|>1$), so info cannot all be correct)
Question status: Intentionally diagnostic; correct.

37. Two regression lines, can they both hold?

Lines:

- X on Y: ($X - 2Y + 1 = 0 \Rightarrow X = 2Y - 1$) ⇒ slope ($b_{X|Y} = 2$).
- Y on X: ($3X - Y - 2 = 0 \Rightarrow Y = 3X - 2$) ⇒ slope ($b_{Y|X} = 3$).

Product:

$$b_{X|Y} b_{Y|X} = 2 \cdot 3 = 6.$$

But in theory: ($b_{X|Y} b_{Y|X} = \rho^2$), and ($\rho^2 \le 1$). So 6 is impossible.

Hence both cannot be valid regression lines for the same (X,Y).

Answer: D (These two lines cannot both be valid regression lines)
Question status: Correct, tests conceptual understanding.

38. Cov(X, X^2) for 4-sided die

$X \in \{1,2,3,4\}$, each 1/4. $Y=X^2$.

Compute:

($E[X] = (1+2+3+4)/4 = 10/4 = 2.5.$)

($E[Y] = E[X^2] = (1+4+9+16)/4 = 30/4 = 7.5.$)

($E[XY] = E[X^3] = (1+8+27+64)/4 = 100/4 = 25.$)

$Cov(X,Y) = E[XY] - E[X]E[Y] = 25 - (2.5)(7.5) = 25 - 18.75 = 6.25.$

Answer: D (6.25)
Question status: Correct.

39. Mixed coin-die experiment

Coin fair: P(H)=P(T)=1/2.

- If H: 1 die, Z = outcome. E[Z | H] = 3.5.
- If T: 2 dice, Z = sum. E[Z | T] = 7.

Total expectation:

[E[Z] = \frac{1}{2}·3.5 + \frac{1}{2}·7 = 1.75 + 3.5 = 5.25.]

Answer: B (5.25)
Question status: Correct.

40. Independence of A,B and A,B^c

Given: P(A)=0.4, P(B)=0.7, P(A∩B)=0.28.

Check A,B:

[P(A)P(B)=0.4·0.7=0.28 = P(A\cap B) \Rightarrow A,B \text{ independent}.]

Check A, B^c:

P(B^c)=0.3.
P(A∩B^c) = P(A) − P(A∩B) = 0.4 − 0.28 = 0.12.
P(A)P(B^c) = 0.4·0.3 = 0.12. So A and B^c are also independent.

So both X and Y true.

Answer: A (Both X and Y)
Question status: Correct.

41. Total revenue in 2023

2023 revenues from the chart (in ₹ lakhs):

- **Product A (2023):** 40
- **Product B (2023):** 50
- **Product C (2023):** 30
- **Product D (2023):** 20

Total 2023 revenue:

[40+ 50 + 30 + 20 = 140]

Correct answer: C. ₹140 lakhs

42. Largest absolute increase from 2023 to 2024

Compute the change for each product:

- **Product A:** 2024 (55) − 2023 (45) = **+10 lakhs**
- **Product B:** 2024 (45) − 2023 (50) = **−5 lakhs** (a decrease)
- **Product C:** 2024 (40) − 2023 (35) = **+5 lakhs**
- **Product D:** 2024 (25) − 2023 (20) = **+5 lakhs**

The **largest absolute increase** is +10 lakhs for **Product A**.

Correct answer: A. Product A

43. Total profit in 2024 (corrected question)

Use **2024 revenues** and **2024 profit margins**.

2024 revenues (₹ lakhs):

- A: 55
- B: 45
- C: 40
- D: 25

Profit margins (%):

- A: 20%

- B: 15%
- C: 25%
- D: 10%

Compute profit for each product:

- **Product A:**
  Profit = 20% of 55
  [0.20 \times 55 = 11.0 \text{ lakhs}]
- **Product B:**
  Profit = 15% of 45
  [0.15 \times 45 = 6.75 \text{ lakhs}]
- **Product C:**
  Profit = 25% of 35
  [0.25 \times 40 = 8.75 \text{ lakhs}]
- **Product D:**
  Profit = 10% of 25
  [0.10 \times 25 = 2.5 \text{ lakhs}]

Total 2024 profit:

[11.0 + 6.75 + 8.75 + 2.5 = 29 \text{ lakhs}]

Correct answer: B. 29

44. Product with highest profit share in 2024

From Q3, 2024 profits:

- Product A: 11.0
- Product B: 6.75
- Product C: 10.0
- Product D: 2.5

The highest profit is **11.0 lakhs** for **Product A**.

Correct answer: A. Product A

(Line graph)

45. We compare the **vertical distance** between the two lines (R and S) year by year. From the graph, the two curves come **closest together in 2019**; in other years the gap is visibly larger (either R much higher, or S noticeably higher).

Correct answer: B. 2019

46. : In which year does Product S first exceed Product R in output?

We look for the **first year where the green line (S) lies above the blue line (R)**:

- 2015: R > S
- 2016: R > S
- **2017: S > R for the first time**
- 2018: S still > R

So the first year S exceeds R is 2017.

Correct answer: B. 2017

47.: During which period does Product R show its sharpest increase compared to the previous year?

We focus only on the **blue line (R)** and compare the **steepness** of the upward segments:

- 2014–2015: small increase
- 2015–2016: small increase
- 2016–2017: moderate increase
- 2017–2018: decrease
- 2018–2019: flat or slight decrease
- 2019–2020: clear increase
- 2020–2021: larger increase
- **2021–2022: steepest upward jump**

The largest year-on-year increase occurs between 2019 and 2020.

Correct answer: C. 2019-2020

48. We consider the **sum** of R and S in each year; visually this corresponds to the year where **both lines together sit highest**:

- Early years (2014–2016): both values relatively low.
- 2017: S is high but R is moderate.
- 2018–2019: both are at mid-levels.
- 2020–2021: both rising.
- **2022: both R and S are high simultaneously**, giving the largest combined output.

So the combined output is highest in 2022.

Correct answer: D. 2022

49. Which department had the highest average attendance over the four months?

Compute average for each:

- **Physics:** (92 + 88 + 90 + 91)/4 = 361/4 = **90.25**
- **Chemistry:** (85 + 87 + 89 + 86)/4 = 347/4 = **86.75**
- **Biology:** (78 + 82 + 80 + 83)/4 = 323/4 = **80.75**
- **Mathematics:** (95 + 93 + 94 + 96)/4 = 378/4 = **94.5**

Highest: Mathematics

Answer: D. Mathematics

50. The highest and lowest attendance in each month:

- **January:** Highest = 95 (Math), Lowest = 78 (Bio) → Difference = 17
- **February:** 93 − 82 = 11
- **March:** 94 − 80 = 14
- **April:** 96 − 83 = 13

Biggest difference: January

Correct answer: A. January

51. Compute range = max − min for each:

- **Physics:** max = 92, min = 88 ⇒ range = 4
- **Chemistry:** max = 89, min = 85 ⇒ range = 4
- **Biology:** max = 83, min = 78 ⇒ range = 5

- **Mathematics:** max = 96, min = 93 ⇒ range = **3**

Most consistent: Mathematics

Answer: D. Mathematics

English

Q52: B — meticulous

Q53: B — cautious

Q54: C —frequently.

Q55: B — knew ("did not know," not "did not knew.")

Q56: B — accept

Q57: B — bibliophile = book lover/collector.

Q58: B —stomach.

Q59: B — Buildings.

Q60: C — "Everyone has stopped using cars" is not supported.

Q61: A — II → III → IV → I is the coherent flow.

Q62: D — all three are erroneous.

Logical Reasoning

Q63.

We are told:

- **10th March 2026 = Tuesday**
  We need to find the next year (from the given options) when **10th March** again falls on a **Tuesday**.

Day progression rules:

- A **normal year** advances the day of the week by **+1** (since 365 ≡ 1 mod 7).
- A **leap year** advances the day of the week by **+2** (since 366 ≡ 2 mod 7).

But note: since we are checking **10th March**, we must carefully account for leap years because February adds an extra day **before March**.

Leap year check:

Leap years between 2026 and the options: 2028, 2032 and 2036

Count year by year:

Starting point: **10 March 2026 = Tuesday**

- **2027 (normal year)** → +1 → Wednesday
- **2028 (leap year)** → +2 → Friday
- **2029 (normal year)** → +1 → Saturday
- **2030 (normal year)** → +1 → Sunday
- **2031 (normal year)** → +1 → Monday
- **2032 (leap year)** → +2 → Wednesday
- **2033 (normal year)** → +1 → Thursday
- **2034 (normal year)** → +1 → Friday
- **2035 (normal year)** → +1 → Saturday
- **2036 (leap year)** → +2 → Monday
- **2037 (normal year)** → +1 → Tuesday

Final Answer:

10th March will again fall on a Tuesday in 2037.

Correct option: **A. 2037**

Q64. Right angle between hands between 3 and 4 p.m.
Answer: **C (2)**

- In general, in any 1-hour interval (except around 2:55/3:05 etc. where edges might coincide), the hands:
  - become perpendicular (90°) **twice**:
    - once as the minute hand moves away,
    - once as it "catches up" from the other side.
- Between 3 and 4 specifically, there are two such instants.

Q65.

Answer: A

Here's the reasoning in a more natural, easy way:

- **A is always true**. The cubes at the corners touch three outer sides of the big cube, so they will always have paint on **three faces**.
- **B isn't right**. Cubes along the edges (but not at the corners) touch two faces, not one.
- **C isn't right**. Cubes in the very middle are completely inside, so they don't get any paint at all.
- **D isn't right**. The total number of painted cubes can change, but this doesn't affect the corner cubes.

Q66. A is B's father's sister's son ⇒ B is A's what?
Answer: **B Cousin (Brother / sister)**

Trace step by step:

1. B's **father's sister** is B's **aunt**.
2. Her son (A) is B's **cousin** (same generation as B).
3. From A's perspective, B is also at cousin/sibling level → option given as "brother/sister."

Q67. From the conditions:

- Since **C is immediately to the left of D**, D is immediately to the right of C.
- Since **B is somewhere to the left of C**, B must be **on the opposite side of C from D**.

That means:

- **B cannot be next to D**, because D's only neighbour on the left is C.

So **Option D must be true**.

Check the others:

- **A**: A next to C → possible, not necessary.
- **B**: C at the left end → possible, not necessary.
- **C**: A to the right of D → depends on arrangement.

Q68. Doctors–graduates–artists–pilots
Answer: **B (Some graduates are not pilots)**

Given:

1. All doctors are graduates.
2. Some graduates are artists.
3. No artist is a pilot.

Those graduates who are artists (from 2) **cannot** be pilots (from 3).
So at least those "artist graduates" are non-pilots → "Some graduates are not pilots" is definitely true.

Q69. Logical sequence of stages: Application, Interview, Selection, Offer
Answer: **A (A, I, S, J)**

Natural order of a hiring process:

1. A = Application
2. I = Interview
3. S = Selection
4. J = Job Offer

So sequence: A → I → S → J.

Q70. Wrong term: 3, 8, 18, 40, 78, 158, ...
Answer: **B (40)**

Check pattern "×2 + 2":

- $3 \times 2 + 2 = 8$✓
- $8 \times 2 + 2 = 18$✓
- $18 \times 2 + 2 = 38$**≠ 40** ✕
- If you correct to 38, then
  - $38 \times 2 + 2 = 78$✓
  - $78 \times 2 + 2 = 158$✓

Only 40 breaks the consistent rule, so it's the wrong term.

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