Institute of Actuaries of India

ACET December 2023 Indicative Solutions

Mathematics

1. C. $|2x+23| = |x+11| \Longrightarrow 2x+23 = x+11$ or $2x+23 = -(x+11)$.
$\Longrightarrow x = 11-23$ or $3x = -11-23$.
$\Longrightarrow x = -12$ or $x = -\frac{34}{3}$.

2. C. Given that $f(x) = 4(x-3)$ and $g(x) = x^2 - 7x + 12 = (x-3)(x-4)$. It follows that $f(0) = g(0) = 0$. Therefore, $\frac{g(x)}{f(x)}$ is undefined when $x = 3$.

3. A. Obviously, choices B, C and D are not true.

Let $A \cup B = A \cap B$. To prove $A = B$, let $x \in A \Longrightarrow x \in A \cup B$
$\Longrightarrow x \in A \cap B \Longrightarrow x \in A$ and $B \Longrightarrow x \in B$. Hence, $A \subset B$.

Similarly it can be proved that $B \subset A$. Hence, $A = B$.

4. B. Given that $f(x) = 2x^2 + 5x - 24$ and $g(x) = 3x + 2$ (x real),
$f(x) + g(x) = 2x^2 + 8x - 22$.
The discriminant $b^2 - 4ac = 64 - 4(2)(-22) = 240 > 0$. Hence, the roots are real and distinct.

5. A.
$$\begin{aligned}
&\left(\sqrt{3}+1\right)^4 - \left(\sqrt{3}-1\right)^4 \\
&= \left[\left(\sqrt{3}\right)^4 + \binom{4}{3}\left(\sqrt{3}\right)^3 + \binom{4}{2}\left(\sqrt{3}\right)^2 + \binom{4}{1}\left(\sqrt{3}\right)^1 + 1\right] \\
&- \left[\left(\sqrt{3}\right)^4 - \binom{4}{3}\left(\sqrt{3}\right)^3 + \binom{4}{2}\left(\sqrt{3}\right)^2 - \binom{4}{1}\left(\sqrt{3}\right)^1 + 1\right] \\
&= 2\left[\binom{4}{3}\left(\sqrt{3}\right)^3 + + \binom{4}{1}\left(\sqrt{3}\right)^1\right] = 2\left[4\left(\sqrt{3}\right)^3 + +4\sqrt{3}\right] = 32\sqrt{3}.
\end{aligned}$$

6. C. $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right) = \tan^{-1}\left(\frac{21+4}{28-3}\right)$.

$= \tan^{-1}(1) = \frac{\pi}{4}$.

7. D. $\sum_{j=1}^{n}(3j-1)^2 = \sum_{j=1}^{n}(9j^2 - 6j + 1) = 9\sum_{j=1}^{n} j^2 - 6\sum_{j=1}^{n} j + \sum_{j=1}^{n} 1 =$
$9\left(\frac{n(n+1)(2n+1)}{6}\right) - 6\left(\frac{n(n+1)}{2}\right) + n = \frac{n}{2}[6n^2 + 3n - 1]$.

8. A.
$$\frac{(1+2i)(2-i)}{(3-2i)(2+3i)} = \frac{2-i+4i-2i^2}{6+9i-4i-6i^2} = \frac{2+3i+2}{6+5i+6} = \frac{3i+4}{5i+12} = \frac{3i+4}{5i+12} \times \frac{5i-12}{5i-12}$$
$$= \frac{15i^2 - 36i + 20i - 48}{(5i)^2 - (12)^2} = \frac{-15-16i-48}{-25-144} = \frac{63+16i}{169}.$$
Hence the conjugate of $\frac{(1+2i)(2-i)}{(3-2i)(2+3i)}$ is $\frac{63}{169} - \frac{16i}{169}$.

9. B.
$$f(x)=\begin{cases}7 & \text{if } x\le 3,\\ mx+n & \text{if } 3<x<12,\\ 18 & \text{if } x\ge 12.\end{cases}$$

In order that the function is continuous at $x=3$, $f(3-)=f(3+)$, that is, $7=3m+n$. Similarly, for $x=7, f(7-)=f(7+)$. This gives $12m+n=18$.
The solution of these two equations is $m=\frac{11}{9}$ and $n=\frac{10}{3}$.

10. D. Given that $x^3+y^3=\cos x+y$, then $\frac{d}{dx}(x^3+y^3)=\frac{d}{dx}(\cos x+y)$,
that is, $3x^2+3y^2\frac{dy}{dx}=-\sin x+\frac{dy}{dx}\Rightarrow 3x^2+\sin x=\frac{dy}{dx}(1-3y^2)$.
Hence $\frac{dy}{dx}=\frac{3x^2+\sin x}{1-3y^2}$.

11. C. $f(x)=-2x^3-9x^2-12x+1, f'(x)=-6x^2-18x-12$.
$f'(x)=0 \Rightarrow x^2+3x+2=0\Rightarrow (x\mp 1)(x+2)=0\Rightarrow x=-1$ or $x=-2$.
The possible intervals are $(-\infty,-2), (-2,-1), (-1,\infty)$.
It is clear that $f'(x)<0$ for $\in(-\infty,-2)$; $f'(x)>0$ for $x\in(-2,-1)$ and $f'(x)<0$ for $x\,\epsilon\,(-1,\infty)$. Hence $f(x)$ is increasing in $(-2,-1)$.

12. A. Let $(x)=\frac{7\sin x}{5+\cos x}$. Then $f(-x)=\frac{7\sin(-x)}{5+\cos(-x)}=-\frac{7\sin x}{5+\cos x}=-f(x)$.

Hence $f(x)$ is an odd function. Therefore $\int_{-\pi/5}^{\pi/5}\frac{7\sin x}{5+\cos x}dx=0$.

13. B. The area bounded by the curve $y=\frac{x^2}{2}$, the lines $y=0, x=1$ and $x=3$ is $\int_1^3\frac{x^2}{2}\,dx=4\frac{1}{3}$.

14. D.
$$\begin{aligned}\int x\log 3x\,dx &= \int \log 3x\, d\left(\frac{x^2}{2}\right)=\left(\frac{x^2}{2}\right)\log 3x-\int\frac{x^2}{2}d(\log\ 3x)+C\\ &=\left(\frac{x^2}{2}\right)\log 3x-\int\frac{x^2}{2}\frac{3}{3x}dx+C\\ &=\left(\frac{x^2}{2}\right)\log 3x-\int\frac{x}{2}dx+C=\left(\frac{x^2}{2}\right)\log 3x-\frac{x^2}{4}+C.\end{aligned}$$

15. C. Given that
$$M=\begin{bmatrix}1 & \cos\theta & \sin\theta\\ -\cos\theta & -1 & 1\\ \sin\theta & 1 & 1\end{bmatrix}$$
$$\begin{aligned}\Rightarrow |M| &= 1(-1-1)-\cos\theta\,(-\cos\theta-\sin\theta)+\sin\theta\,(-\cos\theta+\sin\theta)\\ &=-2+\cos^2\theta+\cos\theta\sin\theta-\cos\theta\sin\theta+\sin^2\theta=-1.\end{aligned}$$

16. B.
$$\begin{bmatrix}xy & 6 & 1\\ z+4 & x+y & 2\\ 3 & 1 & 5\end{bmatrix}=\begin{bmatrix}16 & a & 1\\ -8 & 12 & b\\ c & 1 & d\end{bmatrix}\Rightarrow x+y=12, z+4=-8$$
$\Rightarrow z=-12$.

Thus, $x+y+z=0$.

17. A. Given that $f(x) = x^2 - 2$. $f(1) = -1$ (-ve) and $f(2) = 2$ (+ve). Hence, there exists a positive root between 1 and 2. $f'(x) = 2x$.

The n^{th} iteration: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

We start with the initial value $x_0 = 1$. $x_1 = 1 - \frac{-1}{2} = 1.5$;

$x_2 = 1.5 - \frac{\frac{9}{4}-2}{3} = \frac{17}{12} = 1.41667$; $x_3 = \frac{17}{12} - \frac{\left(\frac{17}{12}\right)^2 - 2}{\frac{17}{6}} = \frac{577}{408} = 1.41426$.

18. D. The difference table is:

x	$f(x)$	$\triangle f(x)$	$\triangle^2 f(x)$	$\triangle^3 f(x)$
50	205			
		20		
60	225		3	
		23		0
70	248		3	
		26		
80	274			

The Newton's forward interpolation formula is:
$f(x) = f(x_0) + t \triangle f(x_0) + \frac{t(t-1)}{2!} \triangle^2 f(x_0) + \cdots ; t = \frac{x - x_0}{h}$.
Here $x_0 = 50; h = 10$ and $x = 54$.
Hence, $f(54) = 205 + 0.4\,(20) + \frac{0.4(0.4-1)}{2}(3) = 205 + 8 - 0.36 = 212.64$.

19. C. Given that $\vec{A} = \vec{x} + \vec{y} + \vec{z}$, $\vec{B} = -\vec{x} + \vec{y} - \vec{z}$ and $\vec{C} = \vec{y} - \vec{z}$.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-\vec{x} + \vec{y} - \vec{z}) - (\vec{x} + \vec{y} + \vec{z}) = -2\vec{x} - 2\vec{z}$,
$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (\vec{y} - \vec{z}) - (\vec{x} + \vec{y} + \vec{z}) = -\vec{x} - 2\vec{z}$.
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -2 & 0 & -2 \\ -1 & 0 & -2 \end{vmatrix} = 2j$. $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{2^2} = 2$.
Hence the area is $\frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{2} = 1$ sq. unit.

20. B. Given that $\vec{a} = 2\vec{x} + 4\vec{y} + \vec{z}$, $\vec{b} = 3\vec{x} - \vec{y} + 2\vec{z}$.

The angle between them is $\cos\theta = \frac{\vec{a} \circ \vec{b}}{|\vec{a}||\vec{b}|}$.
$\vec{a} \circ \vec{b} = (2 \times 3) + (4 \times (-1)) + (1 \times 2) = 4$; $|\vec{a}| = \sqrt{4 + 16 + 1} = \sqrt{21}$.
$|\vec{b}| = \sqrt{9 + 1 + 4} = \sqrt{14}$. Hence, $\cos\theta = \frac{4}{7\sqrt{6}}$ and $\theta = \cos^{-1}\frac{4}{7\sqrt{6}}$.

21. C. Correct sum of the observations $= 25 \times 20 - (15 + 17) + (25 + 27) = 520$. Correct mean $= \frac{520}{20} = 26$.

22. B. If weights are changed to $3w_1, 3w_2$ and $3w_3$, new weighted mean $= \frac{12\times 3w_1 + 16\times 3w_2 + 20\times 3w_3}{3w_1+3w_2+3w_3} = \frac{12\times w_1 + 16\times w_2 + 20\times w_3}{w_1+w_2+w_3} = 18$.

23. C. After the said transformations, a number x becomes $y = -0.5\,x + 1.5$. Standard deviation of x is $s_x = 2.1$. Standard deviation of y is $s_y = |-0.5| s_x = 0.5 \times 2.1 = 1.05$.

24. A. Since $3x - 5y = 4,\ 3\,(\text{mode of } x) - 5(\text{mode of } y) = 4$.
Mode of $y = \frac{4-3(\text{mode of } x)}{-5} = \frac{4-3\times 13}{-5} = 7$.

25. C. For a symmetrical distribution, median = (first quartile +third quartile)/2.
So, third quartile = 2 × median – first quartile $= 90 - 30 = 60$.

26. D. Total number of arrangements of the letters of the word "ENGINEERING" $= \frac{11!}{3!3!2!2!}$.

Considering the 3 E's as one letter, there are 9 letters which can be arranged in $\frac{9!}{3!2!2!}$ ways.

So the number of ways three E's come together $= \frac{9!}{3!2!2!}$.

Require probability $= \frac{9!/3!2!2!}{11!/3!3!2!2!} = \frac{9!\times 3!}{11!} = \frac{3}{55}$.

27. A. E = the event that the problem is solved by S_1.
F = the event that the problem is solved by S_2.
Required probability $= P(E \cap \bar{F}) + P(\bar{E} \cap F) = P(E) + P(F) - 2\,P(E \cap F)$
$= P(E) + P(F) - 2\,P(E)P(F) = \frac{1}{3} + \frac{1}{2} - 2 \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{2}$.
Alternatively, since E and $\bar{F}$ are also independent, the required probability is $\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{1}{2}$.

28. D. Let E_i be the event that ball drawn in i^{th} draw is red, $i = 1, 2, 3$.
Balls are drawn one by one with replacement. So the events E_1, E_2, E_3 are independent.

$$P(E_1) = P(E_2) = P(E_3) = \frac{6}{15} = \frac{2}{5}$$

Required probability $= P(E_1 \cup E_2 \cup E_3) = 1 - P\left(\overline{(E_1 \cup E_2 \cup E_3)}\right) = 1 - P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3)$ is
$1 - P(\bar{E}_1)P(\bar{E}_2)P(\bar{E}_3) = 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{125-27}{125} = \frac{98}{125}$.

29. A. Let D_1 be the event that first chip is defective, D_2 the event that the second chip is defective, and D_3 the event that the third chip is defective.

Required probability $=P(D_1 \cap D_2 \cap D_3) = P(D_1)P(D_2|D_1)P(D_3|D_1 \cap D_2) = \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13} = \frac{4}{455}$.

30. B. Let E_i be the event that a job is estimated by engineer i, $i = 1, 2, 3$.
Given that $P(E_1) = 0.3, P(E_2) = 0.20$ and $P(E_3) = 0.50$.

$$P(\text{Error}) = P(\text{Error}|E_1)P(E_1) + P(\text{Error}|E_2)P(E_2) + P(\text{Error}|E_3)P(E_3)$$
$$= 0.01 \times 0.30 + 0.03 \times 0.20 + 0.02 \times 0.5 = 0.019.$$
$$P(E_1|\text{Error}) = \frac{P(\text{Error}|E_1)P(E_1)}{P(\text{Error})} = \frac{0.01 \times 0.30}{0.019} = \frac{3}{19}.$$

31. C. X is uniformly distributed on $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$.

$$P(X = x) = \frac{1}{11}, x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5.$$

$E|X| = \frac{1}{11}(|-5| + |-4| + |-3| + |-2| + |-1| + 0 + |1| + |2| + |3| + |4| + |5|) = \frac{30}{11}$.

32. D. Average accident cost over a year would be Rs. $5 \times 12 \times 15{,}000 =$ Rs. $900{,}000$.

33. B. $P(X = k) = P(X = k + 1) \Rightarrow e^{-8}\frac{8^k}{k!} = e^{-8}\frac{8^{k+1}}{(k+1)!} \Rightarrow \frac{1}{k!} = \frac{8}{(k+1)k!} \Rightarrow k = 7$.

34. C. The probability density function of X is $f(x) = \frac{1}{50}, 150 \le x \le 200$.
The probability that steel sheets produced by the machine to be scrapped is
$P(X \le 160) = \int_{150}^{160} \frac{1}{50}\, dx = \frac{10}{50} = 0.2$.

35. A. $E(e^{0.75X}) = \int_0^\infty e^{0.75x} \times e^{-x} dx = \int_0^\infty e^{-0.25x}\, dx = 4$.

36. B. $P(X \ge \mu) = 0.50$, since μ is the median of the distribution.
$P(\mu - \sigma \le X \le \mu + \sigma) = P\left(\frac{\mu-\sigma-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{\mu+\sigma-\mu}{\sigma}\right) = P(-1 \le Z \le 1), Z \sim N(0, 1)$.
$P(-1 \le Z \le 1) = \int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$.
$P(X \le \mu) = 0.5, P(X \le \mu + \sigma) > 0.5$.
$P(X \ge \mu) = 0.5, P(X \ge \mu - \sigma) > 0.5$.

37. D. M is the median of the distribution.

Then $P(X \le M) = 0.5 \Rightarrow 1 - e^{-\theta M} = 0.5 \Rightarrow \theta = -\frac{1}{M}\ln(0.5)$
Mean = $\int_0^\infty x \times \theta e^{-\theta x} dx = \frac{1}{\theta} = -\frac{M}{\ln(0.5)}$

38. C.
$$P(X_1 + X_2 = 1) = P(X_1 = 1, X_2 = 0) + P(X_1 = 0, X_2 = 1)$$
$$= P(X_1 = 1)P(X_2 = 0) + P(X_1 = 0)P(X_2 = 1)$$
$$= n_1p_1(1 - p_1)^{n_1-1}(1 - p_2)^{n_2} + (1 - p_1)^{n_1}n_2p_2(1 - p_2)^{n_2-1}$$
$$= [n_1p_1(1 - p_2) + n_2p_2(1 - p_1)](1 - p_1)^{n_1-1}(1 - p_2)^{n_2-1}.$$

39. D. $\text{Var}(XY) = E(X^2Y^2) - \left(E(XY)\right)^2$. X and Y are independent, so $E(XY) = E(X)E(Y)$ and $E(X^2Y^2) = E(X^2)E(Y^2)$.
$E(XY) = E(X)E(Y) = 5 \times 0 = 0$.

$E(X^2Y^2) = E(X^2)E(Y^2) = (5^2 + 2^2) \times 1 = 29.$
Var$(Y) = 1.$
Cov$(Y, XY) = E(XY^2) - E(Y)E(XY) = E(XY^2) = E(X)E(Y^2) = 5 \times 1 = 5.$
Therefore, the correlation is $\frac{5}{\sqrt{29}}$.

40. A. Regression coefficient of u on v is $b_{uv} = \frac{cov(u,v)}{var(v)}$

$u = \frac{1}{2}x + \frac{5}{2},\ v = \frac{1}{3}y - \frac{7}{6}.$ Var$(v) = \left(\frac{1}{3}\right)^2 \times var(y) = \frac{1}{9}var(y)$

Cov$(u, v) = cov\left(\frac{1}{2}x + \frac{5}{2},\ \frac{1}{3}y - \frac{7}{6}\right) = \frac{1}{2} \times \frac{1}{3} \times cov(x, y)$

Regression coefficient of x on y is $b_{xy} = 4$

$$b_{uv} = \frac{\frac{1}{2}\times\frac{1}{3}\times cov(x,y)}{\frac{1}{9}\times var(y)} = \frac{3}{2} \times \frac{cov(x,y)}{var(y)} = \frac{3}{2} \times b_{xy} = \frac{3}{2} \times 4 = 6.$$

41. C. Minimum possible mean score occurs when all the students in each interval scores the lowest values in that interval (that is, the left limit of the interval). Thus the minimum possible mean is

$$\frac{10 \times 6 + \cdots + 90 \times 6}{200} = \frac{9960}{200} = 49.8.$$

42. A. The percentage of students scoring 60% and above but less than $80\% = \frac{30+16}{200} \times 100 = 23.$

43. D. If the pass mark is 40%, the percentage of students who failed in the subject is $\frac{6+12+16}{200} \times 100 = 17.$

44. C. It is evident from the diagram that South Korea won maximum number of bronze medals.

45. B. It is evident from the diagram that Kazakhstan won the maximum number of medals among the four countries.

46. A. The distribution lifetime of light bulbs of M1 is positively skewed, as the median is closer to the lower quartile and the whisker is shorter at the lower end.

47. B.
   - A- The middle line of the box represents the median. The median lifetime of light bulbs of M1 is less than that of M2
   - B- The median lifetime of M2 is greater than 100 hrs.
   - C- The lifetime distribution of light bulbs of M2 is positively skewed.
   - D- Let $\bar{x}$ and m are the mean and the median of the data from M1. Then $0 = \sum_i (x_i - \bar{x}) = \sum_{Q_1}(x_i - \bar{x}) + \sum_{Q_2}(x_i - \bar{x}) + \sum_{Q_3}(x_i - \bar{x}) + \sum_{Q_4}(x_i - \bar{x})$, where the four sums in the last expression are made over the four quarter parts of the ordered data. Note that for M1 $\sum_{Q_1}(m - x_i) + \sum_{Q_2}(m - x_i) < \sum_{Q_4}(x_i - m)$, since it is evident from the Box plot that each summand on the right side of the inequality is more than TWO TIMES each summand of each sum on the left side (even though the total number of summands on the left side is greater). Therefore, $\sum_{Q_1}(x_i - m) + \sum_{Q_2}(x_i - m) + \sum_{Q_3}(x_i - m) + \sum_{Q_4}(x_i - m) > 0$. It follows that $\bar{x} > m$.

48. D. In Plot 3, there is no linear association between x and y. The correlation coefficient between x and y is near zero.

49. B.
   - A- In Plot 1, the correlation coefficient between x and y is nearly equal to 1, because there is a perfect and positive linear association between x and y.
   - B- In Plot 4, the correlation coefficient between x and y is positive, because there is a positive linear association between x and y.
   - C- In Plot 2, x and y are negatively associated and there is perfect linear association between x and y. The correlation coefficient between x and y is -1.

D- In Plot 2, x and y are negatively associated and there is perfect linear association between x and y. The correlation coefficient between x and y is -1.

50. A. The percentage of time intervals with at least 5 vehicles arrived is

$$\frac{65 + 30 + 20 + 8 + 4 + 2}{200} \times 100 = 64.5.$$

51. C. Median $=$ (100^{th} obs + 101^{th} obs)$/2 = (5 + 5)/2 = 5$.

52. B.
53. A.
54. A.
55. C.
56. D.
57. C.
58. D.
59. B.
60. C.
61. B.
62. C.

63. D. 51 Days as 2024 will be a leap year, so 17 days in January, 29 days in February and 5 days in March.

64. B.

65. B.

66. A. Cousin.

67. A. The watch gains 5 seconds in 3 minutes so it will be 100 seconds in 1 hour. From 8 AM to 10 PM on the same day, the total time passed is 14 hours. Hence, in 14 hours the clock would have gained 1400 seconds or 23 minutes 20 seconds. So, when the correct time is 10 PM, the watch would show 10 hours 23 minutes 20 seconds.

68. B. If all Turns are Good and some Turns are Cans, then some Cans are Good.

69. D. The seating order is EIDCAFHGB.
B is at the end and F is at third place from B so this makes the arrangement F_ _B.
A is left to F, which makes the pattern AF_ _B.
H is adjacent to G and F. Let us add that to the pattern now: _ _ _ _ AFHGB.
C is at the third place to E from the right which puts E at the other end. Now C is to the immediate right of D which makes the final pattern like this: EIDCAFHGB.

70. C. A motorbike is a system with two tires in series. The renal system of a human being is a system with two kidneys in parallel. Entry into an email account is a system in which the username and the password are two components in series. A committee that is supposed to keep its report confidential till submission is a system with all its members in series.

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