INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2026

SP6 - Financial Derivatives

Time allowed: 3 Hours 15 Minutes

Q.1)

i) Verify that, under the Black – Scholes model, a European futures option is worth as the same as the corresponding European option on the cash asset if the options and the futures contract have the same maturity. The cash asset has a continuous dividend yield. **[4]**

ii) Why should the result in (i) hold under any model? **[1]**

[5 Marks]

Q.2) Answer the following:

i) Prove that F ≤ S + C, where F is the forward price of a commodity, S is the spot price of the commodity and C is the net carrying cost of the commodity to the delivery date. **[4]**

ii) For a commodity that can be sold short, such as a financial asset, prove that F ≥ S + C – U; where F is the forward price of a commodity, S is the spot price of the commodity, C is the net carrying cost of the commodity to the delivery date and U is the net storage cost for carrying one unit of the commodity to the delivery date. **[4]**

iii) Consider a call option on the maximum of two assets that pay *max(S1(T), S2(T))* at expiration. Replicate this by a position in one of the assets plus an exchange option. Note an exchange option is one which gives the holder the right to exchange one asset for another at expiration and its value at expiration is the maximum of the difference of the two assets and zero. **[4]**

iv) Why is it not optimal to exercise an American put immediately before an ex-dividend date. **[3]**

v) Consider a generalized double barrier option with two barriers defined by functions f_l and f_h where $f_l(t) < f_h(t)$ for $t \geq 0$. Transform it into a double barrier option with constant barriers of 0 and 1 by changing the underlying price process into a new process X_t. Find the drift and volatility of X_t. **[10]**

vi) A binary call pays off a fixed amount Q if the underlying asset finished above the strike price and nothing otherwise. Derive its price in terms of N(.) where N(.) has the usual meaning in terms of the cumulative normal density function. **[6]**

vii) Consider shares of two companies, both of which have a positive correlation with the performance of Nifty and depend on no other stochastic variables. Suppose that first one has an expected return of 20% per annum and a volatility of 25% per annum, and the second one has a volatility of 35% per annum. Assume that the instantaneous risk - free rate of interest is 6% per annum. What is the market price of risk for Nifty? What is the expected return from the second security? **[5]**

[36 Marks]

Q.3)

i) Describe briefly the Vasicek one-factor model of interest rates and its key statistical properties. **[2]**

ii) Let r_t denote the short rate of interest applicable at time t. According to a particular parameterization of this model, the instantaneous forward rate applicable at time t > 0 implied by the market prices at time 0 is found to be:

$$f(t) = r_0 e^{-\alpha t} + r_\infty (1 - e^{-\alpha t}) + k(1 - e^{-\alpha t}) e^{-\alpha t}$$

where a is a positive constant.

a) Show that, if the graph of *f(τ)* has a stationary point, then

$$e^{-\alpha t} = \frac{r_0 - r_\infty + k}{2k}$$

b) Hence describe the shape of the graph of f (t) in each of the following cases

1. $r_0 = 0.04$, $r_¥ = 0.06$, k = 0.1
2. $r_0 = 0.06$, $r_¥ = 0.04$, k = 0.02
3. State a possible explanation for a yield curve with the shape of the graph in (ii-b)(2)

[4]

iii) Describe briefly the main advantages and limitations of the Vasicek model. **[2]**

[8 Marks]

Q.4) Let the short riskless interest rate in the market be 25% per period. Consider a stock, trading in the same market, with the current price being 100\. The stock price either doubles or falls by half in each period. The actual probability of an increase is 60% and that of a decrease is 40%. Consider European and American options with a strike price of 75 and maturity two periods from now. Also, you can assume discrete compounding of the interest rate process.

Answer the following questions based on this information.

i) What are the risk neutral probabilities? **[2]**

ii) What is the European put values at each node? **[3]**

iii) What is the American put values at each node? **[3]**

iv) What is the price of a European call option with strike $75 maturing two periods from now? **[2]**

[10 Marks]

Q.5) You are observing bonds and swaps in a LIBOR based interest rate market. Column A gives the set of zero coupon bond prices that apply currently (each price is for a nominal of 100). Column B gives the prices of the same bonds after the yield curve has instantly shifted up by 10 basis points (0.10%) for all maturities.

	A	B
Maturity (Years)	Current Zero Coupon Bond Price	Shifted Zero Coupon Bond Price
1	95.238	95.147
2	90.488	90.316
3	85.770	85.526
4	81.492	81.183
5	77.796	77.428

The modified duration of a bond priced at par is 4.299 on current rates.

i) Using this information,

a) Calculate the value of a 5-year 10% annual coupon bond at current rates.
b) Hence derive the value of 5-year annual fixed-floating swap with 10% fixed coupon.

[4]

ii) Let the "absolute yield sensitivity" of an instrument be defined as the instrument's value now less its value when rates have risen by a small increment (Δr), all divided by the small increment (Δr).

a) Show how absolute yield sensitivity for a bond is related to its modified duration.
b) Show numerically that the absolute yield sensitivities of the bond and swap in (i) are the same.

[5]

[For the purpose of this computation, assume that a 10 basis point shift is a small increment.]

iii) A "reverse floater" is a bond that pays a coupon equal to a fixed value X less the value of LIBOR at each fixing. So, for example, if LIBOR is 5.75% at the next fixing, the coupon becomes X – 5.75%. Consider a 5-year annual coupon reverse floater where X = 10%.

a) Show how the cashflows of the reverse floater can be decomposed into those of a fixed-floating swap and a zero coupon bond.
b) Hence demonstrate numerically that the modified duration of the reverse floater is approximately twice that of a bond priced at par.

[7]

iv) Explain how you would risk manage, in terms of duration, a portfolio of bonds that includes a reverse floater.

[3]
[19 Marks]

Q.6) Consider a perpetual bond with a continuously compounded yield of *"y"*. Further, the bond pays interest @ Rs. 2 per annum. Assume that interest is paid continuously on the bond, and that *"y"* follows the process:

$dy = \mu(\alpha - y)dt + \sigma y\, dZ$

where μ, α and σ are positive constants and dZ is a standard Brownian motion.

Given this information, answer the following two questions:

i) What is the process followed by the bond price? **[6]**

ii) What is the expected return to the bond holder expressed as a percentage of the bond price?

[4]
[10 Marks]

Q.7)

i) Explain the use and significance of Ito's lemma for the valuation of derivatives based on stochastic processes. **[2]**

ii) Explain what it means for a stochastic process X_t to be a martingale under a probability measure **P**. **[2]**

iii) Consider a standard ***P-Brownian*** motion W_t. Obtain the differential equation for the stochastic process.

$$Y_t = W_t^2 - t,$$

And state with reasons whether it is a martingale.

[4]

iv) Show for what value(s) of the constant a, if any, the stochastic process

$$Z_t = W_t^4 - 6W_t^2 t + at^2$$

is a martingale.

[For parts (iii) and (iv) you may assume that both processes are sufficiently bounded.]

[4]
[12 Marks]

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