INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2026

CS2 - Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) What is the value of e_x, the curtate expectation of life, if ${}_tp_x = m^t$, where $m < 1$ and ${}_4q_x = 0.5904$?

A. 3.25
B. 3.50
C. 3.75
D. 4.00
E. 4.25

[2 Marks]

Q.2) Giri plans to take the first bus arrives at a station near his home, which can be either an AC or non-AC bus, arrive according to Independent Poisson processes with mean arrival times of 12 minutes and 4 minutes, respectively. Calculate the total expected time of reaching his office (in minutes), if the travel time to his office is 40 minutes by AC bus and 60 minutes for a non-AC bus.

A. 48
B. 40
C. 50
D. 55
E. 58

[2 Marks]

Q.3) A parametric survival model is modelled using force of mortality that follows Gompertz law, with parameter values of $g = \exp\left(-\frac{B}{\mathrm{Ln}(c)}\right) = 0.9828$ and $c = 1.103$. What is the probability of a life aged 60 currently dies between age 65 to 66 years?

A. 0.01268
B. 0.03605
C. 0.01014
D. 0.01521
E. 0.00845

[2 Marks]

Q.4) Assume a controlled cohort of 200 policyholders of an Insurer, who all reached exact age 35 on 1st January 2020, and are observed over the calendar year 2020 (i.e., for age [35,36)).

- 128 policyholders stay active the entire year and successfully reach age 36 on 31st December, 2020.
- 24 policyholders have died exactly at the end of June 2020.
- 48 policyholders withdraw/lapse from the policy exactly at the end of Sep 2020.

Calculate $\hat{m}_{35}$ value using exact calculation of central exposed to risk approach?

A. 0.1579
B. 0.1714
C. 0.1364
D. 0.1277
E. 0.1875

[2 Marks]

Q.5) Calculate the probability of at least 2 more claims to occur by time 10, given that 18 claims have occurred by time 5, assuming occurrence of claims follow Poisson process with intensity, 0.02t, for time t<8 and it is doubled for time t>=8.

A. 0.3046
B. 0.2030

C. 0.2676
D. 0.1329
E. 0.1734

[2 Marks]

Q.6) For a time-inhomogeneous Markov process, $\frac{\partial}{\partial s} p_{ij}(s,t) = -\sum_k q_{ik}(s) p_{kj}(s,t)$,

where $q_{ik}(s)$ = the **instantaneous transition rate** from state i to state k at time s and $p_{kj}(s,t)$= the **transition probability** of being in state j at time t, given that the process is in state k at time s. Identify the name of equation?

A. Kolmogorov forward equation
B. Chapman–Kolmogorov equation
C. Kolmogorov backward equation
D. Master equilibrium equation
E. Fokker–Planck equation

[2 Marks]

Q.7) Assuming annual claim frequency follows Poisson distribution with mean 4 on a Personal Accident policy, with a severity of distribution function $F(200{,}000) = 0.8$, Calculate probability of occurrence of at least one claim with the size more than 200,000 in a year.

A. 0.9817
B. 0.4493
C. 0.5507
D. 0.9592
E. 0.3262

[2 Marks]

Q.8) The ARMA (1,1) model of, $X_t = \alpha X_{t-1} + e_t + \beta e_{t-1}$ follows stationary time series with $\alpha > 0$, which has the value of autocorrelation lag 4 is 0.725 and lag 6 is 0.116.

Where e_t is a white noise process with mean zero and variance σ^2. What is the value of α?

A. 2.5000
B. 0.4000
C. 12.500
D. 0.0256
E. 0.7804

[2 Marks]

Q.9) A time series Y_t which is modelled by, $Y_t = -0.65 + Y_{t-1} + \varepsilon_t$, where $Y_t$is observed value at time t. Given that $Y_{12} = 54$, what is the forecast value for Y_{20}?

A. 48.80
B. 59.20
C. 10.38
D. -10.38
E. -12.86

[2 Marks]

Q.10) For the given below weakly stationary bivariate time series process $(X_t, Y_t)^n$, what are the calculated eigen values?

$$X_t = 0.2X_{t-1} + 0.3Y_{t-1} + \epsilon_t^x$$
$$Y_t = 0.1X_{t-1} + 0.4Y_{t-1} + \epsilon_t^y$$

Where ϵ_t^x, ϵ_t^y are two independent white noise processes.

A. (0.3, 0.4)
B. (0.2, 0.4)
C. (0.1, 0.5)
D. (0.4, 0.1)
E. (0.2, 0.5)

[2 Marks]

Q.11) Operational losses from standard processing errors of a leading Life Insurer, follow a lognormal distribution with $\mu = 6$ and $\sigma = 2.5$ (i.e., $\ln X \sim N(6, 2.5^2)$), where X represents the individual loss from a standard processing error. Fraudulent errors cost four times the standard errors and account for 20% of all incidents. Calculate the probability that an operational loss exceeds 25,000?

A. 0.0565
B. 0.0621
C. 0.0712
D. 0.0775
E. 0.0668

[2 Marks]

Q.12) Under the Commercial Lines of a Non-Life Insurer, the claim study conducted at the end of year 2020 indicates that claim experience follows Pareto distribution with $\alpha = 3$ and $\theta = 1500$. Subsequently the claims team observed the reported claims were subject to average inflation of 5.482% p.a. Calculate the probability of the individual claim to exceed the policy limit of 5,000 at the end of year 2026?

A. 0.0223
B. 0.0199
C. 0.0250
D. 0.0855
E. 0.0101

[2 Marks]

Q.13) What is the ratio of mean excess of loss function at $x = 4\theta$ to the mean excess of loss function at $x = \theta$, where the losses follow a two-parameter Pareto distribution with ($\alpha > 1$) and θ unspecified.

A. 200%
B. 225%
C. 250%
D. 275%
E. 300%

[2 Marks]

Q.14) A non-life insurance property insurance policy Fire and natural disaster claims, where these aggregate events follow a compound Poisson process:

- **Fire claims**: Occur on average every 8 months in two years' time. Claim cost follows an exponential distribution with mean 2,500.
- **Natural disaster claims**: Occur on average once in two years' time. Claim cost follows an exponential distribution with mean 15,000.

Calculate the probability that a randomly submitted claim in two years' time exceeds 12,500?

A. 0.3276
B. 0.6893
C. 0.4306

D. 0.1137
E. 0.2274

[2 Marks]

Q.15) For a small P&C insurer, which has lower capital base, trying to assess the impact of any catastrophe event to ensure holding adequate capital. Catastrophe event is defined as any claim reported exceeding INR24 lacs. In its current insurance portfolio,

- Claim frequency follows a Poisson process with an annual rate of 500.
- Claim severities are independent and follow an Exponential distribution with a mean of INR3 lacs.

Calculate the median waiting time (in years) until the first catastrophe claim?

A. 8.2650
B. 6.4231
C. 5.7673
D. 4.8962
E. 4.1325

[2 Marks]

Q.16) A medical research team models the time-to-relapse (in days) for patients using a Weibull distribution with the cumulative distribution function:

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\theta}\right)^{0.25}\right], \quad x > 0$$

From a clinical study, four patients relapsed exactly at 5, 10, 15, and 20 days, while two additional patients remained relapse-free when the study window closed at 29 days (right-censored at 29). Determine the Maximum Likelihood Estimate (MLE) of the scale parameter θ. **[5 Marks]**

Q.17) A health insurance company provides coverage to a group of corporate employees. Let N(t) be the number of urgent care visits in t years and N(t) follows a Poisson process with intensity of 0.3 per annum.

Currently the company arrives at the annual premium based on claim experience over a period of last 2 years.

- If there are no hospital visits during the last 2 years, then an annual premium of **INR 1,200** is charged
- If the hospital visits during the last 2 years are greater than or equal to 1, then annual premium of **INR 1,500** is charged

The company is now proposing to revise its premium structure based on claims experience over past 3 years instead of past 2 years. Further it has also decided to make slight modification to grouping of the number of claims to determine the annual premium. Under the new premium structure,

- If the number of hospital visits during the last 3 years is less than or equal to 1, then annual premium of **INR X** will be charged.
- If the number of hospital visits during the last 3 years is greater than 1, then annual premium of **INR1,425** will be charged

Calculate the value of **INR X** required to achieve revenue neutrality for the portfolio under the proposed pricing schedule.

[5 Marks]

Q.18) A highway traffic management system models the flow of vehicles through a series of consecutive monitoring checkpoints (toll gates) on a high-speed corridor. The system categorizes traffic intensity changes at each transition into three operational states:

- **State 1 (High Congestion):** +20 (% traffic increase (a growth factor: 1.20)
- **State 2 (Moderate Flow):** +10 (% traffic increase (a growth factor: 1.10)
- **State 3 (Free Flow):** 0% change (a growth factor: 1.00)

The transition probability matrix T representing how traffic states shift from one checkpoint interval to the next is given by:

$$T = \begin{bmatrix} 0.10 & 0.20 & 0.70 \\ 0.20 & 0.40 & 0.40 \\ 0.30 & 0.60 & 0.10 \end{bmatrix}$$

Starting from an initial baseline traffic index of **100** at **State 3**, Transport Department wants to find the probability that the cumulative traffic index will reach **at least 115** after **two checkpoint transitions**.

[7 Marks]

Q.19) A new general insurer, **Abhaya Aran**, trying to enter the market with the aim of becoming a market leader in the motor insurance sector and exploring different ways to attract customers. As per current industry practice, in general the insurers offer three levels of discount, 0%, 20%, 40%. If a policyholder makes a claim they move one level down, if not at 0% discount level already. Upon completion of claim free year, they move up a discount level in the following year or remain at the maximum 40% level.



The probability of an accident depends on the discount level found to be at,

Discount Levels	Probability of accident
0%	0.30
20%	0.20
40%	0.15

For the above mentioned NCD method offered as per current industry practice,

i) Determine the transition matrix between different states. **[1]**

ii) Calculate the proportion of policyholders at each discount level, assume that the system has reached the stable state. **[3]**

iii) Assuming the size of yearly premium payable without any discount is INR2,000/-, calculate the average premium paid when the system is at stable state. **[2]**

The new insurer feels that the current discounts levels, doesn't offer good incentive for good driving history and at the same time, drivers with poor claim history causing overall losses to the insurers in general, as there is less penalty applied on poor claim history.

Hence, it has conducted a market study, and they found increasing more discount levels and offer better terms at higher discounts for better driving history could attract good customers and dissuade drivers having poor claim history. Considering the recommendations, it proposes to offer new NCD method as given below:

There are five discount levels proposed, 0%, 20%, 35%, 45%, 50% in the NCD system:

For discount levels less than 50%:

- Policyholder moves up one level upon claim free prior year.
- Similarly moves down one level upon making one claim in prior year.
- Irrespective of the current discount level, if insured make more than one claim in any given prior year, they move to 0% discount level straight.

At 50% discount level: Once reaching the top discount level (i.e., 50% discount level),

- Policyholder will remain at the highest discount level if not more than one claim is made in last two consecutive prior years.
- Policyholder will move down one level upon making one claim in each of the last two consecutive prior years
- On any other cases, policyholder will move to 0% discount level straight



Based on the study conducted, the expected probability of an accident depends on the discount level as given below:

Discount Levels	Probability of one accident in a year	Probability of more than one accident in a year
0%	0.157	0.095
20%	0.128	0.069
35%	0.117	0.050

45%	0.100	0.034
50%	0.080	0.020

iv) Determine the transition matrix for the proposed NCD by the insurer. **[4]**

v) Calculate the proportion of policyholders at each discount level when the system reaches a stable state, under the proposed NCD method. **[6]**

vi) Assuming the size of yearly premium payable without any discount is INR2,000/-, calculate the average premium paid once the system reaches the stable state. **[2]**

vii) Compare and comment the transition probabilities & average premiums receivable by insurer between the insurer's proposed NCD method against what is currently being offered in the industry. **[2]**

[Hint: treat the transitions within 50% discount levels as separate states]

[20 Marks]

Q.20) Calculate the expected value and the auto-covariance function of the below stationary time series,

$Y_t = 0.2 + 0.4Y_{t-1} + 0.6e_{t-1} + e_t$, where e_t is a white noise process with variance σ^2.

[7 Marks]

Q.21) For the given time series model: $Y_t = 1 + 0.5Y_{t-1} + 0.3Y_{t-2} + \epsilon_t$, where ϵ_t is a white noise process with variance σ^2. Calculate the auto-correlation value of ρ_4 and first three partial auto-correlation values of the given time series. **[7 Marks]**

Q.22) Under a health insurance policy where losses expected follow a generic density function,

$$f(x) = \frac{10125*10^6}{(x+1500)^4}, x > 0$$

As the current year claim experience is way too higher than what was originally anticipating, it has decided to introduce few changes in the policy such that expected payment per loss to be settled improves materially. The planned policy changes to be made are, introduce a deductible of 250, a policy excess limit of 3,000. In addition to that it also proposed to apply coinsurance factor of 80% on the claims to be settled. Using the above information, calculate the following:

i) Calculate the expected payment per loss prior implementing proposed changes? **[1]**

ii) Calculate the expected payment per loss post implementing proposed changes? **[6]**

iii) Using results of (i) & (ii), estimate the reduction in payout, if the proposed changes get implemented in the policy? **[1]**

[Hint: try identify the type of the given distribution, mapping to one of the well-known distribution family]

[8 Marks]

Q.23) You are given the following information regarding a Pareto distribution with parameters α and θ:

- its 80th percentile is 70.9976
- its 96th percentile is 192.4020

Calculate the mean residual life value at X=40 of the distribution?

[Hint: Try identify ways to equate the given two information]

[5 Marks]

Q.24) A Cloud Engineering service provider uses two types of system, A and B for running its IT operations. Each system consists of two independent components (X & Y). The lifetime of each component follows an exponential distribution with mean 2.5 years.

- System A operates only when both components are functioning.
- System B operates as long as at least one component is functioning.

Let LT_A & LT_B denote the lifetime of System A and B respectively.

i) Calculate the failure rate and survival probability of a single component. **[2]**

ii) Calculate the Expected Lifetime of System A (LT_A). **[2]**

iii) Calculate the Expected Lifetime of System B (LT_B). **[2]**

[6 Marks]

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