INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2026

CM2 - Economic Modelling (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) Please read the statements provided below:

a) Actively managed Mutual funds can consistently generate higher returns than Nifty 50 Index due to the expertise of the fund managers in analysing companies.
b) A stock trader has made super normal profit for past 20 years just using Bollinger-band technique (a well- known technical tool).
c) A trust-worthy newspaper reports that XYZ Mining Corp has larger than expected coal reserve which would increase the Market capitalization by 5% and the share price increased by 5%.
d) A TV reporter traded in a stock and made money on the basis of information in his 'Interview' with the management (before it was broadcast).

Each statement above represents a certain form of Market Hypothesis. Which of the following options are correct?

A. Statement c represents Strong form Market Hypothesis
B. Statement b represents Semi-Strong form Market Hypothesis
C. Statement a & d can represent either inefficient or weak form market hypothesis
D. All the above 3 options are wrong
E. Not Applicable

[2 Marks]

Q.2) Below mentioned are the assumptions underlying the Average cost per claim method:

- The first accident year is fully run off.
- The average cost per claim in each development year is a constant proportion in monetary terms of ultimate average cost per claim for each accident year.
- The number of claims in each development year is a constant proportion of the ultimate number of claims for each development year.
- Inflation is not allowed explicitly, rather it is allowed implicitly as a weighted average of past inflation.

Which of the following options is correct with respect to the assumptions?

A. None of the assumptions are right
B. 1 assumption is right
C. 2 assumptions are right
D. 3 assumptions are right
E. All the assumptions are right

[2 Marks]

Q.3) The number of claims from a portfolio of policies has a Poisson distribution with Parameter 20 per year. The individual claim amount distribution is LogNormal with mean 4 and Variance 1.05. The rate of premium income from the portfolio is 900 per year.

If the insurer has initial surplus of 1000, estimate the probability that the insurer surplus at time 2 will be negative, by assuming that the aggregate claims distribution is approximately normal is approximately

A. 0%
B. 0.5%
C. 1.0%
D. 2.5%
E. 5%

[2 Marks]

Q.4) In the binomial model, it is assumed that:

i) There are no taxes or trading/transaction costs
ii) Assets can be bought and sold at all times
iii) Assets may be held in any amounts
iv) Principle of no arbitrage applied

Which of the above are true

A. i, ii & iii
B. ii, iii & iv
C. i, iii & iv
D. i & iv
E. All of the above

[2 Marks]

Q.5) Which of the following statements about options are true.

a. Early exercise of an American call option on a non-dividend paying share is not optimal.
b. American call option on a dividend paying share is always worth as much as its intrinsic value.
c. European call option on a dividend paying share is worth less than its intrinsic value.
d. Early exercise of an American put option is attractive if risk-free rate of interest increases.

A. All the four
B. a and d only
C. a only
D. d only
E. None of the above

[2 Marks]

Q.6) Consider the following inequalities relating to the probability of ruin for a claims process operating in continuous time: Which of these inequalities are correct?

[Here, u, u1 and u2 represent initial surplus and t, t1 and t2 represent time.]

I. Prob(u,t1) <=Prob(u,t2) if 0 <=t1<=t2

II. Prob(u1,t) <=Prob(u2,t) if 0 <=u1<=u2

III. 0<=Prob(u,t) <= 1

A. I and II only
B. I and III only
C. II only
D. I , II and III
E. None of the above

[2 Marks]

Q.7) Following are a few scenarios given with respect to options:

a. Decrease in underlying share price
b. Increase in dividend rate
c. Reduction in time to expiry
d. Increase in strike price
e. Reduction in risk-free interest rate
f. Increase in volatility of share price

For which of the above-mentioned scenarios, the price of a put option increases in

A. a, b and c
B. a, c and d
C. a, b, c and e
D. a, b, d, e and f
E. All of the above

[2 Marks]

Q.8) An investor uses the utility function $U(w) = \ln(w)$, where $w > 0$, to make investment decisions. The investor plans to invest Rs. 10,000 for one year and is considering two assets:

- **Asset A:** This is a risk-free asset that will retain its value over the year.
- **Asset B:** This asset has a 75% chance of increasing in value by 50% over the year, and a 25% chance of decreasing in value by 50% over the year.

The investor ignores discounting when evaluating investments. They will allocate a proportion x of their wealth to Asset A, and the remaining proportion $(1 – x)$ to Asset B.

Which of the following expressions, in terms of x, correctly represents the investor's expected utility at the end of the year?

A. $0.75 \times \ln(15000 – 5000x)$
B. $0.25 \times \ln(15000 – 5000x) + 0.75 \times \ln(5000 + 5000x)$
C. $0.25 \times \ln(15000 – 5000x)$
D. $0.75 \times \ln(15000 – 5000x) + 0.25 \times \ln(5000 + 5000x)$
E. Not Applicable

[2 Marks]

Q.9) Which of the following statements describe the assumptions made in the Black-Scholes option pricing model?

a) The price of the underlying stock evolves according to a geometric Brownian motion process.
b) There are no opportunities for risk-free arbitrage in the market.
c) The risk-free interest rate remains constant, applies to all maturities, and is the same for both borrowing and lending.
d) Short selling is permitted, but only to a limited extent (i.e., negative holdings are allowed).

A. a & b only
B. a & c only
C. a, b and c
D. a, b, c & d
E. None of the above

[2 Marks]

Q.10) A company that focuses on long-term insurance policies holds reserves for these contracts and is looking to develop a model for its future investment returns. Why might the company choose to use a stochastic model?

A. A stochastic model addresses this by considering a variety of possible outcomes.
B. This approach is especially valuable for insurers due to the long-term nature of their contracts.
C. With a deterministic model, selecting which set of input assumptions to use can be difficult.

A. A & B only
B. A & C only
C. B only
D. A, B & C

E. None of the above

[2 Marks]

Q.11) Claims are reported on a group of insurance policies following a Poisson process with rate λ. Each claim is for a fixed amount d, and premiums are collected continuously. The insurer starts with an initial surplus U (where U is less than d), and receives annual premium income equal to $1.8 \times \lambda \times d$. What is the probability that the insurer will be ruined by the very first claim?

A. $1 - \exp(-(1/1.8) \times d \times (1 - U/d))$
B. $1 - \exp(-(1/1.8) \times (1 - U/(d\lambda)))$
C. $1 - \exp(-(1/1.8) \times \lambda \times (1 - U/d))$
D. $1 - \exp(-(1/1.8) \times (1 - U/d))$
E. Not Applicable

[2 Marks]

Q.12) Let Bt (for $t \geq 0$) represent a standard Brownian motion process starting at $B0 = 0$. Under what condition does the expected value E(BsBt) equal s?

A. When s is less than t
B. When t is less than s
C. In neither of the above cases
D. When s equals 0
E. Not Applicable

[2 Marks]

Q.13) Suppose W_t is a Wiener process. Which of the following statements are correct?

A. W_t behaves as a martingale.
B. The covariance between W100 and W150 is a martingale and equals 150.
C. The covariance between W100 and W150 is a martingale and equals 100.
D. The process X_t squared minus t ($Xt^2 - t$) is a martingale.

a) Only Statements A and B are correct.
b) Only Statements C and D are correct.
c) Only Statements A, B, and D are correct.
d) Only Statements A, C, and D are correct.
e) Not Applicable

[2 Marks]

Q.14) In a Black-Scholes framework, consider a European call option with a strike price of Rs. 1000 and a current stock price of Rs. 900. The option expires in 2 years, and the continuously compounded risk-free interest rate is 2% per annum. The current price of the option is Rs. 27.55.

What is the implied volatility?

A. 7.86%
B. 6.43%
C. 8.96%
D. 10%
E. 12%

[2 Marks]

Q.15) Consider a Poisson process for insurance claims, where the security loading is 2/5 and the claim size has the following density function

$f(x) = 2\exp(-4x) + 7/2 \exp(-7x)$

The Moment Generating Function (MGF) for the claim size distribution is:

A. $4/(4-t) + 7/(7-t)$
B. $4/2*(4-t) + 7/2*(7-t)$
C. $(4/(4-t) + 7/(7-t))*(2/5)$
D. $(4/2*(4-t) + 7/2*(7-t)) *(2/5)$
E. None of the above

[2 Marks]

Q.16) A customer has applied to a bank for a loan to be repaid in a single lump sum after 4 years, with interest compounded annually. The bank estimates that there is a 6% probability each year that the customer will become insolvent. If insolvency occurs before repayment, the bank receives nothing.

What is the probability that the customer becomes insolvent and is not able to repay the loan at maturity?

[2 Marks]

Q.17) Suppose you have two independent assets, Asset A and Asset B, with annual expected returns of 8% and 10%, and annual standard deviations of 6% and 12%, respectively. Let xi represent the fraction of the portfolio invested in Asset i. If you can only invest in these two assets, what is the equation for the efficient frontier in the space of expected return versus standard deviation?

[3 Marks]

Q.18) A company has issued a 5 year debt with a nominal face value of 15 cr. The overall asset value of the company is 45 cr. Assuming we use the Merton model to value the debt issued by the company. Calculate the fair price of the debt issued assuming the annualized volatility of the company's assets over the 5 year period is 25% and risk free force of interest is 7% per annum.

[5 Marks]

Q.19) A Bond investor does not know the Price to pay to a Zero-coupon Bond of a new company 'A' that has come for issuance. The term to expiry is 8 years.

However, a similar Company 'B' that had issued a Zero-coupon bond is trading now. Total number of Bonds that are being traded is 10 lakhs.

The financial details of Company 'B' are given below: The current value of liability valued using risk-free rate is Rs 17 crore with maturity of 6 years. The value of the assets of the company is Rs 22 crore with an annual volatility of 35%. The company is expected to be wound up after 6 years when the assets will be used to pay off the bond holders with the remainder being distributed to the equity holders. A constant risk-free rate of return is 6% p.a. (compounded continuously).

Find the Price for 100 nominal Bond that the Investor may be happy to pay if he puts a constant credit spread of 120 basis points p.a. higher than that of spread of bonds of Company B.

[8 Marks]

Q.20)

i) What is iso-elastic utility function? **[1]**

ii) Prove that the function U(x) = (x^6α – 1) /12α is an iso-elastic utility function. **[4]**

iii) An individual is considering purchasing an insurance policy to protect against the random loss of Rs 600 that occur with the probability of 0.6. The utility function of the individual is given by U(x) = Log(x).

The initial wealth of the individual is Rs 1200. Calculate the insurance premium for the individual. **[4]**

[9 Marks]

Q.21)

i) Explain arbitrage opportunity. **[2]**

ii) Define law of one price. **[2]**

iii) Let pt be the price of 3-month European put option on a share with current share price Rs 1250 and strike price of Rs 1200. The call options on the same share are priced at Rs 300 and the current risk- free force of interest is r= 5% p.a.

a) If dividends are payable continuously at a rate of q= 15% p.a., calculate Pt. **[2]**
b) Explain the strategy for arbitrage profit if, instead, the price of the put option is Rs 230. **[4]**

[10 Marks]

Q.22) A Pension Scheme invests in 3 asset classes, namely, Government securities, corporate bonds and Equities in the proportion 25%, 35% and 40% respectively. Assuming investors have been contributing for the past 5 years.

The returns in each of the financial years are given in the below table:

FY ending March	Government	Corporate bonds	Equities
2021	4.20%	5.00%	5.20%
2022	6.10%	7.30%	-10.60%
2023	6.70%	8.00%	28.90%
2024	7.20%	7.60%	-2.70%
2025	6.90%	8.10%	33.40%

Note: Return in each asset class include capital gains/losses and income.

i) Calculate the Variance of the return of each of the asset classes and the portfolio (Note: Round up to 5 digits after decimal at each step, Use 'n' in the denominator). **[6]**

ii) Find the Value at risk for Equities and the portfolio for the next 1 year, if the next 1 year return of Equities and the portfolio follow normal distribution with Mean and Variance as obtained from the Table. Interpret the results. **[4]**

[10 Marks]

Q.23) You have been provided a three-step recombining risk-neutral binomial tree to value a three-year American Put option with strike 1000 for the price of on a non – dividend paying share with a spot price of 1000. Risk-free interest rates are 5% per annum at all durations, continuously compounded. The option is to be priced using a volatility of 25%.

Price of the underlying security at each node (time in years) are:

Time 0	Time 1	Time 2	Time 3
			2762.875
		2121	
	1628.25		1628.25
1250		1250	
	959.625		959.625
		736.625	
			565.5

At each node of the tree, the risk-neutral probabilities are 0.53 for an upward movement and 0.47 for a downward movement.

i) By deriving the expectation and standard deviation of the share price at time 1 assuming the lognormal distribution, both from the tree and theoretically, check whether the tree has been calibrated appropriately at Time 1? **[3]**

ii) Use the tree to determine the price of a three-year European Put option with a strike of 1225. **[3]**

iii) Use the tree to determine the price of a three-year American Put option with a strike of 1225. **[3]**

iv) State the reason for the difference in the pricing of an American & European Put option. **[1]**

[10 Marks]

Q.24) Suppose a market consists of shares of 3 companies *A*, *B* and *C only* with capitalization of Rs 1000 crores, Rs 2000 crores and Rs 3000 crores respectively. Annual returns on the three shares (*RA*, *RB* and *RC*) have standard deviation of 25%, 20% & 15% respectively.

The expected rate of return on the market portfolio is 14% p.a. The correlation between the returns on each pair of shares is 0.75. Risk free rate is 6%.

i) Find the expected return on A, B & C if CAPM is assumed to hold. **[9]**

ii) Split the variances of returns into systematic and specific risk for the three companies using the values provided & the values obtained in (i) above. **[4]**

[13 Marks]

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