INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2026

CM1 - Actuarial Mathematics for Modelling

(Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) An investor is quoted a nominal rate of interest of 8% p.a. convertible half-yearly. Calculate the equivalent rate of discount per annum convertible quarterly, d(4).

A. 7.542%
B. 7.768%
C. 7.843%
D. 8.000%
E. 8.163%

[2 Marks]

Q.2) An investor subject to income tax at 25% and capital gains tax at 30% purchases a bond redeemable at 105% in exactly 10 years, paying annual coupons of 6% p.a. in arrears, to achieve a net effective yield of 5% p.a. Calculate the price per $100 nominal.

A. $92.35
B. $95.12
C. $97.90
D. $100.45
E. $103.28

[2 Marks]

Q.3) Two independent lives, (65) and (60), are subject to mortality under AM92 Ultimate and PFA92C20 respectively, with $q65 = 0.01131$ and $q60 = 0.00440$. Calculate the probability that at least one of the two lives is alive at the end of one year.

A. 0.98869
B. 0.99429
C. 0.99560
D. 0.99995
E. 1.00000

[2 Marks]

Q.4) A whole life assurance issued to (55) has sum assured $50,000 payable at the end of the year of death, level annual premiums of $1,415 payable in advance, and a gross premium reserve of $9,850 at duration 9. Using $q64 = 0.00625$ and $i = 5\%$ p.a. effective, calculate the gross premium reserve at duration 10.

A. $10,988.55
B. $11,235.40
C. $11,588.18
D. $11,902.11
E. $12,015.33

[2 Marks]

Q.5) A liability requires payments of $1,000, $2,000 and $3,000 at the end of years 1, 2 and 3 respectively. Using an effective rate of interest of 6% p.a., calculate the discounted mean term (duration) of the liability, in years.

A. 2.000

B. 2.104
C. 2.301
D. 2.483
E. 2.667

[2 Marks]

Q.6) A loan of $200,000 is repaid by a level annuity payable monthly in arrears over 15 years at an effective rate of interest of 7% p.a. Calculate the monthly instalment.

A. $1,652.30
B. $1,773.70
C. $1,890.15
D. $1,955.80
E. $2,010.44

[2 Marks]

Q.7) In a multiple decrement table, the independent annual rate of mortality is q(d) = 0.008 and the independent annual rate of withdrawal is q(w) = 0.12. Assuming forces of decrement are constant over the year, calculate the dependent rate of withdrawal, (aq)^w.

A. 0.1131
B. 0.11953
C. 0.12000
D. 0.12508
E. 0.13020

[2 Marks]

Q.8) An insurer holds 1,000 term assurance policies in force at the start of a year, sum assured $30,000 each, payable at the end of the year of death. The reserve per policy is $450 at the start of the year and $500 for survivors at the end of the year. Expected mortality was q = 0.006; 8 deaths actually occurred. Interest on the opening reserve should be ignored for the purposes of this calculation. Calculate the mortality profit for the portfolio for the year.

A. Profit of $59,000
B. Loss of $59,000
C. Loss of $29,500
D. Profit of $29,500
E. Loss of $88,500

[2 Marks]

Q.9) A 2-year bond with annual coupons of 4% p.a. in arrears, redeemable at par, is priced at $98.5 per 100 nominal. The 1-year spot rate is 3% p.a. effective. Calculate the 2-year spot rate implied by no arbitrage, to 3 decimal places.

A. 3.842%
B. 4.201%
C. 4.500%
D. 4.841%
E. 5.103%

[2 Marks]

Q.10) A project requires investments of $50,000 at time 0 and $20,000 at time 1, and generates income of $30,000 at the end of each year from year 2 to year 5 inclusive. Calculate the net present value at an effective rate of interest of 10% p.a.

A. $12,450.30
B. $15,780.66
C. $18,269.06
D. $21,003.55
E. $24,118.90

[2 Marks]

Q.11) A permanent health insurance 3-state model (Healthy/Sick/Dead) has constant forces: sickness sigma = 0.10 p.a., healthy-to-dead mu = 0.02 p.a. Calculate the probability that a healthy life remains continuously healthy for the next 2 years.

A. 0.7290
B. 0.7526
C. 0.7866
D. 0.8106
E. 0.8353

[2 Marks]

Q.12) A 3-year unit-linked policy has projected end-of-year non-unit cashflows per policy in force at the start of each year of (-25, -8, 40). Using a reserving interest rate of 4% p.a. effective, calculate the non-unit reserve required at the end of year 1 to zeroise the negative cashflow projected at the end of year 2.

A. $6.15
B. $6.92
C. $7.69
D. $8.32
E. $8.94

[2 Marks]

Q.13) An investor buys an asset for $10,000 and sells it 3 years later for $12,500. Over the same period a relevant price index rose from 180 to 198. Calculate the annual effective real rate of return achieved, to 2 decimal places.

A. 3.23%
B. 4.35%
C. 4.99%
D. 5.61%
E. 7.72%

[2 Marks]

Q.14) Which of the following correctly defines the par yield for an n-year bond?

A. The coupon rate p.a. such that a bond redeemable at par is itself priced at par when discounted on the current spot-rate curve.
B. The internal rate of return of the bond's cashflows assuming redemption at par.
C. The single spot rate applicable to an n-year bond.

D. The 1-year forward rate applicable from time n-1 to time n.
E. The average of the 1-year through n-year spot rates.

[2 Marks]

Q.15) For a conventional non-profit endowment assurance, under which condition will the prospective gross premium reserve equal the retrospective gross premium reserve at a given duration?

A. Whenever premiums are payable annually in advance.
B. Whenever the sum assured is level and payable at the end of the year of death.
C. Whenever the reserve is calculated using exactly the same mortality, interest and expense basis as was used to derive the original premium, applied consistently.
D. Only when no expenses are allowed for in the premium basis.
E. Never - the two methods are structurally different and cannot coincide.

[2 Marks]

Q.16) A defined benefit pension scheme has a liability consisting of two lump sum outgoes: \$6 million payable in 5 years' time and \$9 million payable in 15 years' time. The scheme's actuary wishes to construct an asset portfolio consisting of a single zero-coupon bond that satisfies Redington's first two conditions for immunisation relative to this liability, using an effective rate of interest of 6% p.a.

i) Calculate, showing all working, the term and nominal amount of the zero-coupon bond required so that the asset portfolio has the same present value and the same discounted mean term (duration) as the liability. **[6]**

ii) Calculate the convexity of the liability cashflows and of the asset (bond) cashflow. Hence determine, with reasons, whether Redington's third condition for immunisation is satisfied by this portfolio. **[6]**

iii) In September 2022, several UK pension schemes using leveraged LDI strategies that were duration-matched to their liabilities still suffered severe losses, triggering forced asset sales. Without further calculation, comment on what your answer to part (ii) suggests about the robustness of this portfolio to a large interest rate shock, and identify one additional real-world risk - not captured by Redington's framework at all - that also contributed to losses of this kind. **[3]**

[15 Marks]

Q.17) GreenCharge Ltd is building a network of solar-powered EV charging hubs. The project is financed by a \$2,000,000 green infrastructure loan, to be repaid by level annual instalments in arrears over 8 years at an effective rate of interest of 5.5% p.a.

i) Calculate, showing all working, the annual instalment, and construct a full loan schedule for years 1 to 3 showing the interest component, capital component, and capital outstanding at the end of each year. **[7]**

ii) The charging network itself is expected to generate income of \$150,000 at the end of year 1, \$300,000 at the end of year 2, \$450,000 at the end of year 3, and then a level \$500,000 p.a. at the end of each year from year 4 to year 8. Calculate, showing all working, the net present value of the project's cashflows at a rate of 8% p.a., taking the \$2,000,000 loan amount as the initial outlay at time 0, and estimate the internal rate of return using linear interpolation between 8% and 12%. **[6]**

iii) Comment, without further calculation, on whether the loan repayment structure in part (i) is well suited to the pattern of the project's cash inflows described in part (ii). **[2]**

[15 Marks]

Q.18) An insurer launches 'FitCover', a critical illness product for a life aged 40 exact, sold in partnership with a wearable fitness device manufacturer. The policy is modelled using the following 3-state continuous-time Markov model: Healthy -(sigma)-> Critical Illness, and Healthy -(mu)-> Dead.

The policy pays $100,000 immediately on diagnosis of a critical illness, and $50,000 immediately on death while Healthy. Premiums of $P p.a. are payable continuously while Healthy, ceasing on diagnosis or death.

Basis: sigma = 0.008 p.a., mu = 0.004 p.a. (both constant), term 20 years, force of interest delta = 5% p.a.

i) Write down, in integral form, expressions for the expected present value of the critical illness benefit and the death benefit. **[5]**

ii) Calculate, showing all working, the annual premium rate P using the equivalence principle. **[8]**

iii) Wearable data suggests that policyholders who consistently meet a daily activity target experience a lower sigma and a lower mu than the standard basis. Without further calculation, explain how the premium in part (ii) would change for such a policyholder, and comment on one practical difficulty an insurer would face in pricing this dynamically. **[2]**

[15 Marks]

Q.19)

i) MetaboliCare is a special endowment-type assurance launched by an insurer in response to emerging clinical evidence that GLP-1 weight-management therapies are associated with meaningfully lower rates of obesity-related mortality among long-term users. The product rewards policyholders who maintain a verified weight-management programme throughout the policy term.

The policy is issued to a life aged 45 exact, term 20 years (maturity age 65). Level annual premiums are payable in advance throughout the term or until earlier death. Benefits:

- $30,000 payable at the end of year 10, if the policyholder is alive at that date (a 'programme milestone' benefit). The policy remains in force after this payment, with premiums continuing to be payable as before.
- $70,000 payable at maturity (end of year 20), if alive at that date
- - $50,000 payable at the end of the year of death, provided death occurs after the end of year 5. If death occurs during the first 5 years, no death benefit of $50,000 is payable - only a return of premiums paid to date, without interest, is payable instead. Candidates are not required to value this return-of-premium feature.

Basis: constant annual mortality rate q = 0.006 at all durations; i = 5% p.a. effective; initial expenses $600 at outset; renewal expenses 5% of each premium from the second policy year onward. The same basis (q = 0.006, i = 5% p.a.) is used for both premium calculation and reserving throughout this question.

Note on the mortality basis: for tractability within this question, the mortality improvement associated with the programme is represented by a constant annual rate of q = 0.006 throughout the term, rather than a rate that varies by duration since enrolment or by attained age. In practice, an insurer observing genuinely improving mortality with programme adherence would likely investigate a duration-dependent (select-type) structure rather than a flat rate - candidates are not required to do so here, but this simplification is revisited in part (iv).

a) Calculate, showing all working, the gross annual premium for this policy. **[8]**

b) Explain briefly, without further calculation, why the net premium for this policy would be smaller than the gross premium calculated in part (a). **[2]**

You are given that the gross premium reserve per policy in force at the end of the 11th policy year is $19,431.37.

ii) Calculate, showing all working and using the recursive relationship between successive reserves - noting that the premium P and the associated renewal expense are both payable at the start of the 12th policy year - the gross premium reserve per policy in force at the end of the 12th policy year. **[5]**

At the start of the 12th policy year there are 500 policies in force. During the 12th policy year: the actual investment return earned was 5.3% p.a. (against an assumed 5% p.a.); 4 actual deaths occurred (against an expected 3.0 based on the valuation basis); actual renewal expenses were exactly in line with the reserving basis.

iii) Calculate, showing all working, the total profit for the 12th policy year arising from this group of policies, and the amount of this profit attributable to mortality experience. Use the reserve you calculated in part (ii) as the year-end reserve per surviving policy. **[7]**

iv) The insurer had expected policyholders adhering to the programme to experience lower mortality than the general population, yet actual deaths exceeded the expected number this year. Comment on possible explanations for this outcome, on the sources of profit or loss identified in part (iii), and on whether the constant-mortality-rate simplification noted above could itself be masking a genuine duration effect in the underlying experience. **[3]**

[25 Marks]

****