

INSTITUTE OF ACTUARIES OF INDIA

Subject SP6 - Financial Derivatives

May 2026 Examination

INDICATIVE SOLUTION

Introduction:

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiners have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

- i) It can be replicated as $D(X_0, H_1) + S \{ D(X_i, H_{i+1}) - D(X_i, H_i) \}; i = 1, 2, \dots, n$.

Here $D(X, H)$ represents a *down – and – out option* with strike price X and barrier H . This can be justified as follows: If the nearest barrier H_1 is never hit, then the first down – and – out option provides the necessary payoff and each term in the summation is zero because it contains two options with identical strike price. If H_1 is hit, then $D(X_0, H_1)$ and $D(X_1, H_1)$ are rendered worthless. The remaining /portfolio is: $D(X_1, H_2) + S \{ D(X_i, H_{i+1}) - D(X_i, H_i) \}; i = 2, \dots, (n - 1)$. The rest follows inductively. **[5]**

- ii) The normal model does. The reason is because the normal model has fatter left tails and thinner right tails for the probability distribution in the future. It therefore gives thinner left tails and fatter right tails for the probability distribution of bond prices. **[3]**

- iii) A cap gives the holder an option at each reset date to borrow at the capped rate. An option on a swap, in contrast, offers the holder the one – time option to borrow at a fixed rate over the remaining life of the swap. Clearly, swaptions are less flexible. More rigorously, a swaption can be viewed as an option on a portfolio and a cap is a portfolio of options and a portfolio of options is always more valuable than an option on a portfolio. **[3]**

- iv) Since $X(t)$ is a Brownian Motion, $[X(t) - X(s)] \sim N(m(t - s), s^2(t - s))$. Thus,

$$E[Y(t)|Y(s)] = E[e^{X(t)}|e^{X(s)}] = e^{X(s)}E[e^{X(t)-X(s)}] = Y(s)e^{(t-s)\left(\mu + \frac{\sigma^2}{2}\right)}.$$

[3]

- v) The asset is drift less so when the American pays off there is precisely a 50% chance (in the risk neutral measure) that the European will pay off too. So the European is worth half as much as the American. **[3]**

[17 Marks]**Solution 2:**

The “knock-in, knock-out” call can be thought of as a portfolio of as:

- Buying 2 ordinary up-and-in call with strike 60 and barrier H_1 ,
- Writing 1 ordinary up-and-in call with strike 60 and barrier H_2 .

Also, it may be noted that “up – and – in” call + “up – and – out” call = ordinary call.

Let the price of the ordinary call with a strike 60 be p

$P = 4.0861$, being equal to the Up and Out call with a strike of infinity.

Then,

- The price of the UIC ($H_1 = 70$) is $p - 0.1294$
- The price of the UIC ($H_1 = 80$) is $p - 0.7583$.

Thus, the price of the “knock-in, knock out” call is $2(p - 0.1294) - (p - 0.7583) = 4.5856$.

[7 Marks]

Solution 3:

i)

$$F = (100 - 7.5 (1/1.06 + 1/1.065^2)) 1.06 * 1.065 = 97.8973$$

$$(Or F = 7.5/1.083206 + 7.5/1.083206^2 + 107.5/1.083206^3 = 97.8973)$$

$$s = 0.3 \cdot 8.3206\% \cdot 2.5787 = 6.4369\%$$

$$d1 = (\ln(F/100) + 0.5 * 2 * 0.064369^2) / (\sqrt{2} * 0.064369) = -0.1879$$

$$d2 = (\ln(F/100) - 0.5 * 2 * 0.064369^2) / (\sqrt{2} * 0.064369) = -0.2790$$

$$N(d1) = 0.4255 \text{ and } N(d2) = 0.3901$$

$$\text{Call option} = (1/(1.065^2)) ((97.8973 \cdot 0.4255) - (100 * 0.3901)) = 2.32$$

$$\text{Bond price} = 100 - \text{call option price} = 97.68$$

[7]

ii) A few possible reasons could be:

- The callable bond gives Reserve Bank of India the opportunity to replace, in two years' time, a 3 year 7.5% coupon bond, with one with a lower coupon. They are willing to pay for this option.
- If the yields do not change from the current upward sloping yield curve, the 3-year yield, in 2 years' time, would likely be lower than 7.5%, but the price paid for the option would likely even out this benefit.
- Should interest rates fall, then the three-year yield would be lower than expected by the market, benefiting the Reserve Bank of India with cheaper funding.
- If there is large uncertainty about future inflation, leading to a large term premium, this might cause a currently large 5-year yield, and the 3-year yield might be much lower if inflation fears are brought under control.
- Trying to appeal to a wider range of investors, some of whom may be seeking additional yield (which is not due to credit risk)
- Financial flexibility – Reserve Bank of India may choose to replace those outstanding bonds with bonds of an alternative term (e.g. 10-year bonds) at the end of the 2-year period, to better match their financing needs at the time.

[4]

[11 Marks]

Solution 4:

i) The forward swap has two legs:

- One leg is characterized as pay notional at the start, receive the fixed rate, and receive the notional at maturity
- The other leg is characterized as paying notional at start, receiving the floating rate, receiving the notional at maturity. Economically this leg is the same as cash.

These two legs are similar to a long forward bond investment and a short forward cash position, and in combination this is equivalent to a forward bond, so a short position is a good hedge.

[2]

ii) $I = \text{Income present value} = 10/1.06 + 10/1.07^2 = 18.16835$

$$F = (B - I) / P(0, T) = (108 - 18.16835) * 1.07^2 = 102.84826$$

[2]

iii) Optimal hedge ratio $h = r (\text{vol of spot} / \text{vol of forward}) = 0.9 (0.05 / 0.06) = 0.75$

Therefore, volume = AED 1bn * 0.75 = AED 750m nominal bond.

[2]

iv)

- Cross-hedging (or Basis) risk: The swap fixed leg and forward bond are different instruments. The swap is exposed to the swap curve while the forward bond is exposed to the bond curve.
- The correlation and volatilities may change with approaching maturity (3-year rate may have a different behaviour to 5-year rates) and/or over time, necessitating adjustment to the hedge ratio

[2]

v)

- Without a clearing house:
 - Any payment under the forward bond would be paid to/from the bank only at expiry.
- With a clearing house:
 - Registering the forward bond would mean initial and daily margining is payable.
 - Initial margin would be small as the initial transaction cost is zero.
 - Any changes in value of the forward bond would require daily variation margin payments.

[2]

[10 Marks]

Solution 5:

i) Fund = EUR 275 million, Index = 1,100. Therefore, Fund = EUR 250,000 times the index.

Fund falls 5% iff index falls 5% (To 1045 from the current level of 1100).

So, puts of EUR 250,000 times the index with strike 1,045 are required

Apply Black Scholes with:

$S_0 = 1,100$; $K = 1,045$; $T = 1$; $r = 5\%$ (risk free rate); $s = 25\%$; $y = 3\%$ (dividend yield)

Further,

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - y + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

This implies, $d_1 = 0.41017$ and $d_2 = 0.16017$. Thus, $N(-d_1) = 0.34084$ and $N(-d_2) = 0.43657$.

Value of one put = $K e^{-rT} N(-d_2) - S_0 e^{-yT} N(-d_1) = 69.9236$

Thus, the total cost of insurance = EUR (250,000 * 69.9236) = EUR 17,481,000

[4]

ii) From put-call parity (or by re-arranging the Black Scholes equations for valuing puts and calls) we have

$$V_{Put} = V_{Call} - S_0 e^{-yT} + K e^{-rT}$$

This shows that a put option can be created by shorting $e^{-qT} X$ the index, buying a call option (of same term and strike as the put) and investing the remainder in the risk free asset.

For fund manager under question, this involves:

- Selling EUR 275_m_ e^{-yT} = EUR 266,872,500 of stock
- Investing EUR 250,000 X the index of calls with strike 1,045 and a term of 1 year.
- Investing the remainder in 1-year risk – free zero coupon bonds yielding 5%.

[4]

iii) The delta of one put option is

$$e^{-yT}(N(d_1) - 1) = e^{-0.03 \cdot 1}(0.65916 - 1) = -0.33077$$

Hence, this implies that 33.077% of the portfolio (i.e. EUR 90.962m) needs to be sold (shorted) and the proceeds invested in risk free assets.

[3]

iv) The delta of a 9 – month index futures contract is

$$e^{(r-y)T} = e^{(0.05 - 0.03)0.75} = 1.0151.$$

The short position required is (90.962 million / 1100) = 82,693 times the index.

Hence a short position involving (82,693 / (1.0151 * 250)) = 325.85 (rounded to 326) futures contracts are needed.

[3]

[14 Marks]

Solution 6:

- i) A Credit Default Swap (CDS) is a derivative contract where the protection buyer pays periodic premiums to the protection seller. In return, the seller compensates the buyer if the reference entity defaults. The payoff equals the loss given default (LGD), calculated as:

$$\text{Payoff} = (1 - \text{Recovery Rate}) \times \text{Notional}$$

This allows the buyer to transfer credit risk to the seller.

[2]

ii) Expected Payoff in Default

- Notional = \$100 million
- Recovery rate = 40% → LGD = 60%
- Payoff =

$$0.60 \times 100,000,000 = 60,000,000$$

Thus, the protection buyer receives \$60 million if default occurs.

[3]

iii) Fair CDS Spread Calculation

Default probabilities per year:

- Year 1: 0.04
- Year 2: 0.0384
- Year 3: 0.0369

- Year 4: 0.0354
- Year 5: 0.0340

Discount factors (3% risk-free):

- Year 1: 0.9709
- Year 2: 0.9426
- Year 3: 0.9151
- Year 4: 0.8885
- Year 5: 0.8626

PV of protection leg:

$$PV_{\text{protection}} = 60,000,000 \times (0.0388 + 0.0362 + 0.0337 + 0.0315 + 0.0293)$$

$$= 60,000,000 \times 0.1695 \approx 10,170,000$$

PV of premium leg: Survival probabilities: 0.96, 0.9216, 0.8847, 0.8493, 0.8154 Weighted sum with discounting = 4.068

$$PV_{\text{premium}} = S \times 100,000,000 \times 4.068$$

Equating legs:

$$S \times 100,000,000 \times 4.068 = 10,170,000$$

$$S = 0.0250 \text{ (2.5\% per annum)}$$

Fair CDS spread = 2.5% per year.

[7]

iv) Practical Pricing Factors

- Liquidity: Wider spreads in illiquid markets.
- Counterparty risk: Seller's creditworthiness affects pricing.
- Systemic risk: Correlation with broader credit markets drives spreads.
- Supply/demand: Investor hedging/speculation influences CDS costs.
- Regulation: Clearing rules and capital requirements impact spreads.

Final Answer

The fair CDS spread for XYZ Ltd. is 2.5% per annum, with a protection payoff of \$60 million in case of default.

[3]

[15 Marks]

Solution 7:

- Moneyneess is the relative position of the current price (or future price) of an underlying asset (e.g. stock) with respect to the strike price of a derivative.

First, we estimate the forward swap rate for duration 15 to 20 for the given forward bond prices. This is

$$\frac{BondPrice(15) - BondPrice(20)}{Sum(BondPrice15to20)} = \frac{0.65 - 0.49}{0.65 + 0.62 + 0.59 + 0.56 + 0.53 + 0.49} = \frac{0.16}{3.44} = 4.65\%$$

Hence, it is in the money.

[3]

- ii) We calculate the risk free rate at time $t = 15$ from the bond price to be equal to 2.9%. We can then use Black's formula

$$P_{PS} = NA[i_F \Phi(d_1) - i_S \Phi(d_2)]$$

Where

$$A = \sum_{i=1}^n e^{-it_i}(t_i - t_{i-1}), \quad d_1 = \frac{\ln \frac{i_F}{i_S} + \frac{1}{2} \sigma_F^2 T}{\sigma_F \sqrt{T}}, \quad d_2 = \frac{\ln \frac{i_F}{i_S} - \frac{1}{2} \sigma_F^2 T}{\sigma_F \sqrt{T}} = d_1 - \sigma_F \sqrt{T}$$

Using the estimate of the market volatility to be 25%, we get:

Intermediate Calculations:

Swap Tenor (years):	5
Swap Rate Period (years):	15
Dt (Volatility * sqrt(15)):	0.9682458
d1:	0.4826274
d2:	- 0.4856185
F(d1):	0.6853198
Fd2:	0.3136188
A:	4.4525463
Number of payments/ year:	1
Principal Amount:	Rs. 1000 crores
15 year forward swap rate:	4.65%
Volatility of the forward rate:	25.00%
Risk Free rate:	2.90%
Price:	Rs. 42 crores

[5]

- iii) Use the implied volatility estimated from the earlier part as 25%. The new forward rate can be estimated from the bond price which is equal to 7% and the new risk free interest rate is 3.5%.

The new price can again be calculated using Black's model

Dt (Volatility * sqrt(15)):	0.9682458
d1:	1.002194
d2:	0.0339488

The new price is Rs. 85 crores

[6]

- iv) Market is given higher price than that can be estimated from implied volatility as the implied volatility is not the same when interest rates go up. This is referred to as the volatility smile where the underlying implied volatility changes with the moneyness of the option.

[2]

[16 Marks]

Solution 8:

- i) The parametric livestock mortality product is a contingent derivative written on a non financial index (here, a satellite derived vegetation index). The payoff is a deterministic function of the index path over a 6-month window (e.g., binary or scaled payout once the index crosses a threshold), rather than actual farm level losses. A binary parametric policy of period T will have a payoff is given by $A \cdot X$ where A is the payout and X is a binary function which takes the value of 1 when index $I_t \leq K$ where K is the predefined threshold and $0 \leq t \leq T$.
- ii) Calibration: Historical satellite data is fitted to stochastic processes, such as Ornstein-Uhlenbeck models, to capture the mean-reverting behavior of seasonal vegetation.

[4]

Jump Risk: Sudden ecological shocks, such as rapid-onset flash droughts, are incorporated using jump-diffusion parameters to accurately price sudden index drops.

Climate trends: the model should ensure that trends in data if any is captured properly so that there is no risk of underpricing. For this, historical drift parameters must be adjusted to account for increasing frequency of severe weather events.

[3]

- iii) Since a direct market for vegetation index derivatives may not exist, we must rely on cross-hedging with standard financial instruments. A proxy hedge using traded temperature and precipitation futures or options can be used, provided these variables demonstrate a strong statistical correlation with the targeted vegetation index. Furthermore, extreme systemic risks, such as widespread regional droughts affecting multiple farming communities simultaneously, can be offloaded to capital markets by issuing catastrophe-linked securities or weather-based catastrophe bonds.

[3]

[10 Marks]
