

INSTITUTE OF ACTUARIES OF INDIA

CM2A - Economic Modelling

May 2026 Examination

INDICATIVE SOLUTION

Introduction:

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiners have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1: Option C

$$\text{Variance of A} = (18\% \cdot 18\% \cdot 0.37 + 6\% \cdot 6\% \cdot 0.22 + 12\% \cdot 12\% \cdot 0.41) - (18\% \cdot 0.37 + 6\% \cdot 0.22 + 12\% \cdot 0.41)^2$$

$$\text{SD} = \text{Square root of variance} = 4.52\%$$

$$\text{Variance of B} = (4\% \cdot 4\% \cdot 0.37 + 9\% \cdot 9\% \cdot 0.22 + 7\% \cdot 7\% \cdot 0.41) - (4\% \cdot 0.37 + 9\% \cdot 0.22 + 7\% \cdot 0.41)^2$$

$$\text{SD} = \text{Square root of variance} = 1.939\%$$

[2 marks]**Solution 2: Option A**

$$\text{PV of dividend} = 4 \cdot \exp(-4\% \cdot 3/12) = 3.96$$

$$\text{Fair Price} = (200 - \text{PV of dividend}) \cdot \exp(4\% \cdot 6/12) = 200$$

[2 marks]**Solution 3: Option B****[2 marks]****Solution 4: Option C**

$$E(S(2)/S(0)) = S(0) \exp(\mu \cdot t + t \cdot \sigma^2/2) = 1150 \exp(0.14 + .11/2) = 1398$$

$$V(S(2)/S(0)) = \text{Use formula from material and tables} = 1150 \cdot 1150 \cdot \exp(2 \cdot .14 + .11) \cdot (\exp(.11) - 1) = 477^2$$

approx

[2 marks]**Solution 5: Option B****[2 marks]****Solution 6: V All are correct****[2 marks]****Solution 7: IV. Both a and c****[2 marks]****Solution 8: Option I**

$$\begin{aligned} \text{Payoff of call} &= \max(S - K, 0) \\ &= \max(51 - 50, 0) \\ &= \text{Rs. 1} \end{aligned}$$

$$\begin{aligned} \text{Profit/Loss} &= \text{Payoff} - \text{Premium} \\ &= 1 - 2 = -1 \end{aligned}$$

$$\begin{aligned}
 \text{Payoff of put} &= \max(K - S, 0) \\
 &= \max(55 - 52, 0) \\
 &= \text{Rs. 3}
 \end{aligned}$$

$$\text{Profit/Loss} = \text{Premium received} - \text{Payoff} = 3 - 3 = 0$$

$$\text{So overall loss} = -1 + 0 = -1$$

[2 marks]

Solution 9: ii is correct

a) is false since there cannot be a claim until time 2

b) is false since the insurance company could be ruined at time 3 if there is a claim, if U is sufficiently small.

c) is false since the insurance company cannot be ruined in year 4, since by that stage it will have sufficient premiums to cover any loss.

d) is true since if it is not ruined by time 4, the insurance company cannot be ruined

[2 marks]

Solution 10: Option A

[2 marks]

Solution 11: Option C

[2 marks]

Solution 12: Option A

[2 marks]

Solution 13:

According to the Efficient Market Hypothesis (EMH), asset prices instantly incorporate all available information. However, the theory does not specify the exact process by which new data is absorbed. Identifying the precise moment information enters the market is also a challenge, as rumours often precede official disclosures.

Research frequently indicates that markets may overreact or underreact to news, with these errors only correcting over extended periods. If these slow corrections exist, traders could potentially exploit them, contradicting the core principle of market efficiency. Past performance predictability: past winners subsequently underperform

Evidence of Informational Inefficiency

i) Past Performance Predictability: Stocks that have performed well in the past (winners) often go on to underperform.

- ii) **Predictive Power of Accounting Ratios:** Ratios such as Price-to-Earnings (P/E) can signal lower future returns, suggesting prices aren't always accurate.
- iii) **Post-IPO Performance:** Companies often experience poor stock performance in the period following their initial market entry.
- iv) **Delayed Reaction to Earnings:** Stock prices may take significant time to fully adjust to earnings announcements rather than doing so immediately.
- v) **Post-Merger Underperformance:** Acquiring firms frequently see abnormally weak performance following a merger.
- vi) **Post-Demerger Outperformance:** Conversely, companies often show abnormally strong performance after a demerger or spin-off.

[4 marks]

Solution 14:

We will use the Put Call Parity here

$$C + K * \text{Exp}(-rt) = P + S$$

$$199.08 + 1000 * \text{exp}(-0.25 * r) = 31.73 + S$$

$$85.68 + 1200 * \text{exp}(-0.25 * r) = 114.86 + S$$

Hence, $S = 1150$ and $r = 0.07$ when we solve them

Assumption: rest all the parameters of option pricing remain constant

Hence, using put call parity again we will get the missing value as 134.29

$$\text{Futures Price} = S * \text{exp}(rt) = 1150 * \text{exp}(0.07 * 0.5) = 1190.96$$

[5 marks]

Solution 15:

$$\text{Daily Change} = 1000 * 2\% = 20 \text{ for both assets}$$

$$\text{Variance} = 20 * 20 + 20 * 20 + 2 * 20 * 20 * .5 = 1200$$

$$\text{SD} = \text{Sqrt}(1200) = 34.64$$

$$10 \text{ Day } 99\% \text{ Var} = 2.33 * \text{SQRT}(10) * 34.64 = 255.24$$

[5 marks]

Solution 16:

$$\text{Annual Premium} = 1.25 * 0.18 * (0.15 * 100 + 0.85 * 10) = \text{Rs. } 5.2875 \text{ Crores}$$

All possible events in year 1 and year 2 are mentioned below in the table along with the probability of every event occurring. The probability of ruin is the event in which there is Injury but no recovery.

Year 1	Year 2	Probabilities				Probability Product
No Injury	No Injury	0.82	0.82			0.6724
No Injury	Injury but recovered	0.82	0.18	0.85		0.12546
No Injury	Injury but no recovery	0.82	0.18	0.15		0.02214
Injury but recovered	No Injury	0.18	0.85	0.82		0.12546
Injury but recovered	Injury but recovered	0.18	0.85	0.18	0.85	0.023409
Injury but recovered	Injury but no recovery	0.18	0.85	0.18	0.15	0.004131
Injury but no recovery		0.18	0.15			0.027
					Ruin	0.053271

Probability of ruin = $0.02214 + 0.004131 + 0.027 = 0.053271$

[7 marks]

Solution 17

i) CAPM assumptions

- All investors have the same one-period horizon.
- All investors can borrow or lend unlimited amounts at the same risk-free rate.
- The markets for risky assets are perfect. Information is freely and instantly available to all investors and no investor believes that they can affect the price of a security by their own actions.
- Investors have the same estimates of the expected returns, standard deviations and covariances of securities over the one-period horizon.
- All investors measure in the same “currency” e.g. pounds or dollars or in real or money terms.

[2]

- ii) Beta of each security = $\text{Cov}(R_i, R_m) / \text{Var}(R_m)$ where M is the market and I is the security.
Hence for the market portfolio, $\text{beta} = \text{Cov}(R_m, R_m) / \text{Var}(R_m) = 1$

[2]

- iii) Market portfolio is the weighted average of the risky securities in the market,

$$\text{Cov}(R_i, R_m) = \text{Cov}(R_i, 0.25R_1 + 0.75R_2)$$

$$\text{Cov}(R_1, \text{Market}) = 0.00084$$

$$\text{Cov}(R_2, \text{Market}) = 0.001447$$

$$\text{Var } R_m = 0.001295$$

$$\text{Beta } 1 = 0.648523$$

$$\text{Beta } 2 = 1.117195$$

[2]

iv) $E R_1 = 7.5\%$

$E R_2 = 8.7\%$

Final Return = 8.4%

Hence, using CAPM $R_f = 5.84\%$

[2]
[8 marks]

Solution 18:

The binomial tree looks like this with the bold and underlines numbers representing the up and down rates in the tree

	<u>112%</u>	611.8	<u>115%</u>
	532		
		494.76	<u>93%</u>
475			
	<u>94%</u>	513.475	<u>115%</u>
	446.5		
		415.245	<u>93%</u>

Risk neutral probabilities

At time 1 = 50.25%

At time 2 = 41.00%

Payoff will only at dd scenario and the payoff is $\text{MAX}(0, 450 - 415.245) = 34.755$

Value of option at time 2 up = 0

Value of option at time 2 down = $\exp(-0.02) * (0 + 34.755 * (1 - 41\%)) = 20.099$

Value at time 1 = $\exp(-0.03) * (0 + 20.099 * (1 - 50.25\%)) = 9.70$

Part 2: The recombining binomial tree is assumed to have the size of up-steps and down-steps to be same under all states and across all time intervals. It means $u_t(j) = u$ and $d_t(t) = d$ for all times t and states j , also $d < \exp(r) < u$

Hence the risk neutral probability 'q' is also constant for all times and in all states.

The main advantage of recombining binomial tree is that it greatly reduces the computational time by reducing the number of states.

[9 marks]

Solution 19:

	1. Cumulative claims table(incurred)						Loss ratio	0.981481		
		Development Year								
	Accident Year	0	1	2	3	Ultimate	Earned Premium			
	2022	4000	4500	5125	5300	5300	5400			
	2023	4400	4900	5500			5900			
	2024	4500	5150				6100			
	2025	5000					6250			
	2. Calculation of development factors									
		F(0,1)	F(1,2)	F(2,3)	F(3,ul)					
		1.127907	1.130319	1.034146	1					
	3. Calculation of cumulative development factors									
		F(0,ul)	F(1,ul)	F(2,ul)	F(3,ul)					
		1.318428	1.168915	1.034146	1					
	Reciprocals	0.758479	0.855494	0.966981	1					
	5. Calculation for initial ultimate liability for each accident year									
	Accident Year	Initial ultimate Liability		Paid out so far						
	2022	5300		5300						
	2023	5790.741		5599.537						
	2024	5987.037		5121.874						
	2025	6134.259		4652.708						
	6. Calculation for emerging liability						7. Calculation of ultimate liability			
	0						5300			
	191.2037						5691.204			
	865.1634						6015.163			
	1481.552						6481.552			
	2537.919						23487.92	Outstanding Reserve		

					Claims till date	20000			
					Required reserve	<u>3487.919</u>			

[9 marks]

Solution 20:

i) We need $E[e^{-rt} B_t | F_s] = e^{-rs} B_s$

If the bond has already defaulted by time s then $e^{-rs} B_s = e^{-rs} 20e^{-r(2-s)} = e^{-rt} B_t$

Otherwise $e^{-rt} B_t = 20e^{-2r}$ if the bond defaults before time t , or

$e^{-rt} B_t = e^{-2r}(20(1 - e^{-\lambda(2-t)}) + 100e^{-\lambda(2-t)})$ otherwise

Then $E[e^{-rt} B_t | F_s] = 20e^{-2r}(1 - e^{-\lambda(t-s)}) + e^{-2r}(20(1 - e^{-\lambda(2-t)}) + 100e^{-\lambda(2-t)}) e^{-\lambda(t-s)}$

$= e^{-2r}(20(1 - e^{-\lambda(2-s)}) + 100e^{-\lambda(2-s)})$

$= e^{-rs} B_s$ hence this is a martingale

[4]

ii)

a) Let the portfolio contain x cash and y bonds.

We need the value at time 2 to be Rs. 25 if the bond has defaulted,

so $xe^{2r} + 20y = 25$

We also need the value at time 2 to be zero if the bond has not defaulted,

so $xe^{2r} + 100y = 0$

Subtracting the equations: $(20 - 100)y = 25 \Rightarrow -80y = 25 \Rightarrow y = -0.3125$

Substituting y back: $xe^{2r} + 100(-0.3125) = 0 \Rightarrow xe^{2r} = 31.25 \Rightarrow x = 31.25 e^{-2r}$

[3]

b) The price at time zero must be the cost of buying this portfolio

$= x + yB_0 = 31.25e^{-2r} - 0.3125e^{-2r}(20(1 - e^{-2\lambda}) + 100e^{-2\lambda})$

$= 25e^{-2r}(1 - e^{-2\lambda})$

[2]

iii) Let the portfolio value be V .

We need the value of the portfolio at time 0 to be $E[e^{-2r} V_2]$ under probability measure P .

$E[e^{-2r} V_2] = 25e^{-2r} * [\text{Probability that bond defaults}]$

$= 25e^{-2r}(1 - e^{-2\lambda})$ hence the requirement holds.

This fits with the fact that being able to hedge a derivative price without arbitrage means we can price it under the Equivalent Martingale Measure.

[3]
[12 marks]

Solution 21:

i) Market Price(MP) of the bond = 100

Strike Price(P) = 100

Time to maturity = 2 years

$D1 = 0.0495$

$D2 = -0.0495$

$N(D1) = 51.97\%$

$N(D2) = 48.03\%$

Call = 3.45

[2]

ii) Long Exposure = $N(D1)/1.07^2 = 45.4\%$ since we need to discount the benefits

Value of the long exposure = $100 * 45.40\% = 45.40$

Cash position = $3.45 - 45.40 = -41.95$

Hence buying 45.4% of the underlying by borrowing 41.95 in cash

[3]

iii) Price of the bond = PV of 19 years of coupons and redemption amount at 7.50% = 95.02

[2]

iv) MP = 95.02

Strike Price(P) = 100

Time to maturity = 1 year

$D1 = -0.6948$

$D2 = -0.7648$

$N(D1) = 24.36\%$

$N(D2) = 22.22\%$

Call = 0.87

[2]

v) Long Exposure = $N(D1)/1.07 = 22.77\%$ since we need to discount the benefits

Value of the long exposure = $95.02 * 22.77\% = 21.63$

Cash position = $-41.95 * 1.07 + 45.40\% 95.02 1.07 - 21.63 = -20.36$

Hence buying 22.77% of the underlying by borrowing 20.36 in cash

[3]

vi) Because this replication is a self financing process hence in money terms the change in cash value position is the same as the money change in the bond position

Change in cash position = $41.95 * 1.07 - 20.36 = 24.522$

Change in bond position = $95.02 22.77\% - 45.40\% 95.02 * 1.07 = 24.52$

Hence the self financing approach is verified

If this would have been a new call option and not a replicated portfolio then the cash borrowed would 0.87-
 $21.63 = -20.77$

The difference in the cash value represents that originally the bond position was purchased when bond price was 100 and sold when the price fell to 95.02 and hence the difference in the cash position.

[5]

[17 marks]
