

INSTITUTE OF ACTUARIES OF INDIA

CM1A - Actuarial Mathematics for Modelling

May 2026 Examination

INDICATIVE SOLUTION

Introduction:

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiners have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1

Option D

$$A = 500,000 \cdot (1 + 0.084/12)^{12}$$

$$A = 543,655.3$$

$$B = 500,000(1.087) = 543,500$$

$$B - A = 543500 - 543,655.3$$

$$B - A = -155$$

[2 marks]**Solution 2**

Option C. Bond A has a higher Macaulay duration than Bond B.

[2 marks]**Solution 3**

Option A. 10460

$$V = 1/1.04$$

$$EPV = 250000 \sum_{k=1}^{40} \frac{1}{(1.04)^k} = 250000 \cdot 0.0418390714 = 10459$$

[2 marks]**Solution 4**

Option D 54164

- Initial fund at time 0 = Rs. 50,000
- Force of interest:

$$t = 0.03 + 0.01t$$

$$= 50000 \exp\{(t(0.03t + 0.005t^2))\}_{\text{from } 0-2} = 50000 \exp 0.06 + 0.02 = 50000e^{0.08}$$

$$= 50000 \cdot 1.083287068 = 54164.3534$$

[2 marks]**Solution 5**

Option c

IRR:

$$-1200/(1+i) + 500/(1+i)^2 + 450/(1+i)^3 + 400/(1+i)^4 = 0$$

Equivalently, shifting one period:

$$-1200 + 500/(1+i) + 450/(1+i)^2 + 400/(1+i)^3 = 0$$

Solving gives:

$$i \approx 6.38\%$$

[2 marks]

Solution 6

Option B

$$10412.60 = 700/1.04 + 700/(1.05)^2 + 10700/(1+s)^3$$

$$s \approx 5.53\%$$

[2 marks]

Solution 7

Option C

Here,

$$\Delta P/P \approx -D \cdot \Delta i$$

$$(798-840)/840 = -0.05, \Delta i = 0.005$$

So,

$$D \cdot 0.050 / 0.005 = 10$$

[2 marks]

Solution 8

Option A

$$PV(\text{outgo}) = 500 + 500/1.08 = 962.962963$$

$$\text{Revenue} = 120e^{(0.04t)}$$

$$PV \text{ of revenue} = \int 120 e^{0.04t} \cdot \exp(-0.076961041t) dt = 1003.103128$$

$$NPV = 40.14$$

[2 marks]

Solution 9

Option C

For the combined portfolio:

$$18600 = (FB + FE)_{\text{total}} - 82400(FB + FE)_{\text{total}} = 101000$$

Given ratio A:B=101:20, if block B is 10000, then block A is:

$$10000 \times 101/20 = 50500$$

[2 marks]

Solution 10

Option B

$$PV = 6000 \left(\sum_{k=1}^{\infty} (0.97)^{k-1} v^k \right) k p_{xy}$$

$$v=1/1.05, k_{pxy}=e^{-0.03k} e^{-0.02k} = e^{-0.05k}$$

$$PV=6000(k=1\sum_{\infty})(0.97)^k-1(1.05)^k e^{-0.05k}$$

Using GP,

$$=6000 \times 0.9059327852 \times 8.247739289 = 44831.43899$$

[2 marks]

Solution 11

$$d=i/1+i$$

Given $i=5\%$,

$$d=0.05/1.05=0.047619$$

Now substitute $A_{50}=0.320$:

$$0.320+0.047619 a_{50}=10.047619$$

$$a_{\overline{50}|0.05}=0.680 \quad a_{\overline{50}|0.05}=0.680 \cdot 0.047619=14.28$$

[2 marks]

Solution 12

Option B

$$EPV=25000 \cdot v \cdot p$$

So,

$$p=22800 \times 1.0525000p=0.9576$$

Now, remaining in service means neither death nor resignation occurs.

Given dependent decrements:

$$0.9576=1-0.004-k \quad 0.9576=0.996-k \quad k=0.996-0.9576k=0.0384$$

[2 marks]

Solution 13

Option A

Expected death claims: $1000 \times 0.004=4$ "deaths"

Expected claim outgo: $4 \times 200,000=800,000$

Actual death claims: $3 \times 200,000=600,000$

So mortality profit: $\text{"Mortality profit"} = \text{"Expected claims"} - \text{"Actual claims"} = 800,000 - 600,000 = 200,000$

[2 marks]

Solution 14

Option D

This is the gross premium difference formula. The single payment today should replace all future

premiums, so it must leave the insurer in the same position: $\text{Single payment} = \text{EPV of future premium} - \text{EPV of future benefits} - \text{EPV of future expenses}$

So,

$$\text{Single payment} = 48,000 + 2,500 - 41,200 = 9,300$$

[2 marks]

Solution 15

Option E

Discounted tuition outgo at time 0:

$$\text{PV fees} = 150,000 [1 + (1.10)^{-0.5} + (1.10)^{-1} + (1.10)^{-1.5}]$$

$$\approx 559,402$$

After-tax salary: $10,00,000 \times 0.8 = 8,00,000$ per year

Monthly salary: $8,00,000/12 = 66,666.67$

First payment is at month 25, so check cumulative discounted salary from months 25 onward.

PV of salaries from month 25 to month 34: $\approx 527,551$

Still short of fees by: $559,402 - 527,551 = 31,851$

PV of month 35 salary: $66,666.67 \times 1.1^{-35} \approx 50,487$

So recovery happens during month 35.

[2 marks]

Solution 16

Mortality Rates:

Rajesh (PMA92):

$$q_{\{62\}} = 0.00794; q_{\{63\}} = 0.009189; q_{\{64\}} = 0.010604$$

Meera (PFA92):

$$q_{\{59\}} = 0.004345; q_{\{60\}} = 0.004965; q_{\{61\}} = 0.005667$$

Survival Probabilities:

$$p_{\{62\}} = 0.992053; p_{\{63\}} = 0.990811$$

$$p_{\{59\}} = 0.995655; p_{\{60\}} = 0.995035; p_{\{61\}} = 0.994333$$

Formula:

$$\text{EPV} = 60000[$$

$$v \cdot q_{\{62\}} \cdot p_{\{59\}} + v^2 \cdot p_{\{62\}} \cdot q_{\{63\}} \cdot p_{\{59\}} p_{\{60\}} + v^3 \cdot p_{\{62\}} p_{\{63\}} q_{\{64\}} \cdot p_{\{59\}} p_{\{60\}} p_{\{61\}}]$$

Calculation:

Year 1:

$$60000 \times 0.961538 \times 0.007947 \times 0.995655 = 456.49$$

Year 2:

$$60000 \times (0.961538)^2 \times 0.992053 \times 0.009189 \times 0.995655 \times 0.995035 = 501.00$$

Year 3:

$$60000 \times (0.961538)^3 \times 0.992053 \times 0.990811 \times 0.010604 \times 0.995655 \times 0.995035 \times 0.994333 = 547.68$$

$$EPV = 456.49 + 501.00 + 547.68 = \text{Rs.} 1,505.17$$

[5 marks]

Solution 17

i)

$$\text{disc factor} = e^{-\delta} = v = 1/1.05 = 0.95238101$$

$$\text{death payment disc factor} = \text{Int}(e^{-\delta t dt}) = 1 - 0.95238101 \times 0.048790 = 0.975997$$

$$\text{Probability of remaining in service: } p = 1 - 0.003 - 0.012 - 0.085 = 0.900$$

So

$$\begin{aligned} EPV &= 500000(0.003)(0.975997) + 300000(0.012)(0.952381) + 50000(0.900)(0.952381) = 1463.99 + \\ &3428.57 + 42857.14 \\ &= 47749.70 \end{aligned}$$

[4]

ii) Although no benefit is paid on resignation, resignation reduces the probability that the employee remains in active service throughout the year. Therefore it reduces the value of the survival benefit and must be included.

[1]

[5 marks]

Solution 18

Expected profit:

$$\text{Premiums received at start: } 500 \times 1500 = 750000$$

$$\text{Expected initial expenses: } 500 \times 120 = 60000$$

$$\text{Net expected fund at start: } 750000 - 60000 = 690000$$

$$\text{Accumulated to end of year: } 690000 \times 1.05 = 724500$$

$$\text{Expected claims: } 500 \times 0.004 \times 300000 = 600000$$

$$\text{Expected lapses: } 500 \times 0.02 = 10$$

$$\text{Expected policies in force at year-end: } 500 - 2 - 10 = 488$$

$$\text{Expected closing reserve: } 488 \times 1200 = 585600$$

$$\text{Expected profit: } 724500 - 600000 - 585600 = -461100$$

Actual profit:

$$\text{Actual initial expenses: } 500 \times 130 = 65000$$

Net actual fund at start: $750000 - 65000 = 685000$

Accumulated to end of year: $685000 \times 1.05 = 719250$

Actual claims: $1 \times 300000 = 300000$

Actual policies in force at year-end: $500 - 1 = 499$

Actual closing reserve: $499 \times 1200 = 598800$

Actual profit: $719250 - 300000 - 598800 = -179550$

Profit due to mortality: $2 - 1 \times 300000 - 1 \times 1200 = 300000 - 1200 = 298800$

Profit due to expenses: $65000 - 60000 \times 1.05 = 5250$

So expense profit: -5250

Profit due to lapses:

Expected lapses = 10, actual lapses = 0, so 10 extra policies remain in force: $10 \times 1200 = 12000$

So lapse profit: -12000

[5 marks]

Solution 19

i) Let $w = 1/(1.08)^2$.

Then

$$PV = w + 2w^2 + \dots + nw^n + n(w^{n+1} + w^{n+2} + \dots)$$

$$= w(1 - w^n)/(1 - w)^2$$

So

$$35.48 = w(1 - w^n)/(1 - w)^2$$

Hence

$$w^n = 1 - 35.48(1 - w)^2 / w$$

$$\text{Using } w = 1/(1.08)^2 = 0.85733882,$$

$$n = \ln(0.154048) / \ln(0.85733882) = 12$$

[4]

ii) Immediately after the payment at time $2n$, the remaining payments are n every 2 years forever.

So the value at time $2n$ is

$$n(1/(1.08)^2 - 1)$$

Now

$$(1.08)^2 - 1 = 0.1664$$

and using $n=12$,

$$\text{Value at time } 2n = 120.1664 = 72.1154$$

[1]

[5 marks]

Solution 20

i) Amount of each of the first 4 instalments

Using the retrospective method at time 4:

$$(500000)(1.09)^4 - XS_4 = 180000$$

$$S_4 = (1.09)^4 - 1/0.09 = 4.573097$$

So

$$(500000)(1.09)^4 - 4.573097X = 180000$$

$$705792.805 - 4.573097X = 180000$$

$$4.573097X = 525792.805$$

$$X = 114973.97$$

[2]

ii) The 5th instalment is 20% higher than each of the first 4 instalments:

$$\text{5th instalment} = 1.2X = 1.2(114973.97) = 137968.77$$

Outstanding balance immediately after the 5th instalment:

$$180000(1.09) - 137968.77 = 196200 - 137968.77 = 58231.23$$

The 6th instalment clears this balance exactly after one further year:

$$\text{6th instalment} = 58231.23(1.09) = 63472.04$$

[2]

iii) Total capital repaid in first 3 instalments

= Loan-outstanding balance after 3rd instalment

1st instalment

$$\text{Interest: } 500000(0.09) = 45000$$

$$\text{Capital repaid: } 114973.97 - 45000 = 69973.97$$

2nd instalment

$$\text{Outstanding after 1st: } 500000 - 69973.97 = 430026.03$$

$$\text{Interest: } 430026.03(0.09) = 38702.34$$

$$\text{Capital repaid: } 114973.97 - 38702.34 = 76271.63$$

3rd instalment

$$\text{Outstanding after 2nd: } 430026.03 - 76271.63 = 353754.40$$

$$\text{Interest: } 353754.40(0.09) = 31837.90$$

$$\text{Capital repaid: } 114973.97 - 31837.90 = 83136.07$$

Total capital repaid in first 3 instalments: $69973.97 + 76271.63 + 83136.07 = 229381.67$

[2]

iv) If redeemed immediately after the 5th instalment, the borrower pays the outstanding balance at that time.

Total payment at time 5: $137968.77 + 58231.23 = 196200$

Let the actual yield be j . Then: $500000 = 114973.97 a_4(j) + 196200(1+j)^{-5}$

Substituting $j=0.09$: $114973.97 a_4(0.09) + 196200(1.09)^{-5} = 500000$

So the lender's effective annual rate actually earned is: 9% per annum

[2]

v) If the rate of interest increases while the instalment remains unchanged, a larger part of the first instalment will be used to pay interest and a smaller part will go towards repaying capital.

Therefore, less capital is repaid in the first instalment, so the outstanding balance immediately after the first instalment will be higher.

[2]

[10 marks]

Solution 21

$$\begin{aligned} \text{i) NPV} &= -920000 - 400000(1.08)^{-0.75} + 230000(1.08)^{-1} + 400000(1.08)^{-2} + \\ &470000(1.08)^{-3} + 350000(1.08)^{-4} + 520000(1.08)^{-5} \\ &= -920000 - 377565.39 + 212962.96 + 342935.53 + 373101.15 + 257260.45 + 353903.26 \\ &= 242597.97 \end{aligned}$$

So NPV = Rs. 242,598 approximately.

[5]

ii) Cumulative discounted cash flow after year 4 = -111305.29

Discounted inflow in year 5 = 353903.26

Discounted payback period = $4 + 111305.29 / 353903.26 = 4.31$ years approximately.

Alternate Solution

using AV:

Outflows:

- $920,000(1.08)^5 = 1,351,3149$
- $400,000(1.08)^{4.25} = 555,600$
- Maintenance:
 $50,000[(1.08)^4 + (1.08)^3 + (1.08)^2 + (1.08)] = 243,330$
- Total Outflows (AV) = 2,150,244

Similarly: Total Inflows (AV) = 2,507,341

Cumulative AV:

- Year 4: 1,987,341
- Year 5: 2,507,341

payback occurs between Year 4 & 5:

$$4 + (2,150,244 - 1,987,341) / 520,000 = 4.31$$

[3]

iii) The project should be accepted on financial grounds because the NPV is positive.

Limitations:

discounted payback ignores cash flows after payback;

both measures depend on assumptions about cash flows and discount rate;

they do not capture qualitative or strategic factors.

[2]

[10 marks]

Solution 22

i) By the equivalence principle:

$$\text{EPV of premiums} = P(1 + v p_{50} + v^2 p_{50} p_{51})$$

$$\text{EPV of benefits} = 200000(v q_{50} + v^2 p_{50} q_{51} + v^3 p_{50} p_{51})$$

$$\text{EPV of expenses} = 1000 + 100 v p_{50} + 100 v^2 p_{50} p_{51}$$

So:

$$\begin{aligned} &P(1 + v p_{50} + v^2 p_{50} p_{51}) \\ &= 200000(v q_{50} + v^2 p_{50} q_{51} + v^3 p_{50} p_{51}) \end{aligned}$$

$$\bullet \quad 1000 + 100 v p_{50} + 100 v^2 p_{50} p_{51}$$

Substituting values:

$$P = 61012.91$$

Therefore gross annual premium = Rs. 61,012.91.

[6]

ii) Reserve at end of policy year 1, immediately after premium and renewal expense at that time have been paid:

$$\begin{aligned} &\text{Future benefits} \\ &= 200000(v q_{51} + v^2 p_{51}) \end{aligned}$$

$$\begin{aligned} &\text{Future expenses} \\ &= 100 v p_{51} \end{aligned}$$

$$\begin{aligned} &\text{Future premiums} \\ &= P v p_{51} \end{aligned}$$

Therefore:

$$V1 = 200000(v q_{51} + v^2 p_{51}) + 100 v p_{51} - P v p_{51}$$

$$= 123582.04$$

[4]

[10 marks]

Solution 23

i) Unit fund values

End of year 1:

$$F1 = 28000 \times 1.06 \times 0.99 = 29383.20$$

End of year 2:

$$F2 = (29383.20 + 38000) \times 1.03 \times 0.99 = 68710.65$$

End of year 3:

$$F3 = (68710.65 + 38000) \times 1.03 \times 0.99 = 108812.85$$

Answer:

$$F1 = 29383.20$$

$$F2 = 68710.65$$

$$F3 = 108812.85$$

[4]

ii) Expected cost of death benefit guarantee

Guarantee shortfalls:

$$\text{Year 1: } 120000 - 29383.20 = 90616.80$$

$$\text{Year 2: } 120000 - 68710.65 = 51289.35$$

$$\text{Year 3: } 120000 - 108812.85 = 11187.15$$

Per policy in force:

$$\text{Year 1: } 90616.80 \times 0.0015 = 135.93$$

$$\text{Year 2: } 51289.35 \times 0.0017 = 87.19$$

$$\text{Year 3: } 11187.15 \times 0.0019 = 21.26$$

Per policy issued:

$$\text{Year 1: } 135.93$$

$$\text{Year 2: } 0.9985 \times 87.19 = 87.06$$

$$\text{Year 3: } 0.9985 \times 0.9983 \times 21.26 = 21.19$$

Difference:

Per policy in force is conditional on being alive at the start of the year.

Per policy issued allows for survival to that year.

[4]

iii) Non-unit profit signature

Non-unit premium inflows:

At outset: 30% of 40000 = 12000

At starts of years 2 and 3: 5% of 40000 = 2000

Expenses:

At outset = 15000

At starts of years 2 and 3 = 3000

Fund management charges:

Year 1 = $0.01 \times 28000 \times 1.06 = 296.80$

Year 2 = $0.01 \times (29383.20 + 38000) \times 1.03 = 694.05$

Year 3 = $0.01 \times (68710.65 + 38000) \times 1.03 = 1099.12$

Year 1 profit:

$P1 = (-3000)(1.07) + 296.80 - 135.93$

$P1 = -3049.13$

Year 2 profit:

$P2 = 0.9985 \times (-1000) + 0.9985 \times 694.05 - 87.06$

$P2 = -462.45$

Year 3 profit:

$P3 = 0.9985 \times 0.9983 \times (-1000) + 0.9985 \times 0.9983 \times 1099.12 - 21.19$

$P3 = 7.84$

Answer:

$P1 = -3049.13$

$P2 = -462.45$

$P3 = 7.84$

Expected present value at time 0:

$EPV = P1/1.07 + P2/(1.07)^2 + P3/(1.07)^3$

$EPV = -3247.17$

[4]

iv) EPV of profit without Non-Unit Cashflows:

$(-3049.13/1.07) + (-462.45/(1.07)^2) + (7.84/(1.07)^3)$

$= -3247.18$

EPV of profit with Non-Unit Cashflows:

$(-3049.13-431.55)/1.07 + (0/(1.07)^2) + (7.84/(1.07)^3) = (-3480.68/1.07) + 0 + (7.84/(1.07)^3)$
(0.5)

$= -3246.57$

Comment:

The EPV of profit after considering the non-unit reserve is higher compared to EPV of profit without considering non-unit reserve as non-unit reserve is not only making the zeroization of the profit wherever it is negative

but also earning interest on the non-unit reserves leading to slightly higher EPV of profit compared to the EPV of profit without non-unit reserves.

[3]

v) Additional benefit = 50000 at end of year of first death, for two independent lives aged 35.

Probability of first death in year 1:

$$1 - (0.9985)^2 = 0.00299775$$

Probability of first death in year 2:

$$(0.9985)^2 \times [1 - (0.9983)^2] = 0.00338693$$

Probability of first death in year 3:

$$(0.9985)^2 \times (0.9983)^2 \times [1 - (0.9981)^2] = 0.00377215$$

EPV at time 0:

$$\text{EPV} = 50000[(0.00299775)/1.07 + (0.00338693)/(1.07)^2 + (0.00377215)/(1.07)^3]$$

$$\text{EPV} = 441.96$$

Comment:

The rider increases expected non-unit outgo, so it increases the non-unit reserve and increases new business strain.

[5]

[20 marks]
