INSTITUTE OF ACTUARIES OF INDIA

SP6 – Financial Derivatives Principles

November 2025 Examination

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

Solution 1:

i) The AR(1) process

$X_t - b = a\,(X_{t-1} - b) + \varepsilon_t$

Can be rearranged to yield the following equation

$X_t - X_{t-1} = (1-a)\,(b - X_{t-1}) + \varepsilon_t$,

This is the discrete – time version of the Vasicek process. It may be recalled that the autocorrelation of X at lag one is a, which translates to $(1 - \beta\, dt)$ in the Vasicek model

(3)

ii) This follows from backwards induction. In the final layer the values are the same. In each previous layer, we assume the result has already been proven for the next layer. At each node the discounted expectation of the next layer must then be at least as much for the American option as for the European option. Taking the maximum with the intrinsic value can only increase the value. So, at each node the value is at least as much. The result now follows by induction back to the initial point. (3)

iii) "C" carries more rights than "A" since in "C" we can exercise two options separately. In terms of price, they will be the same since the two pieces will have the same optimal exercise time. Hence "C" is preferred to "A". (3)

iv) The impact of volatility on the price of an American digital option after the barrier has been breached depends on the option's structure and the state of the underlying.

Case 1: Knock-in digital option (barrier has been breached)

Once the barrier has been hit, the knock-in feature is activated, and the option effectively becomes a plain American digital option. Increasing volatility (σ) increases uncertainty about the underlying asset's path. For a digital call (pays 1 if S_t≥K), more volatility increases the chance that S_t crosses the strike before maturity, so the option value increases. For a digital put (pays 1 if S_t≤K), the same logic applies — the option value also increases with volatility.

Therefore, once the knock-in barrier has been breached, higher volatility increases the option's price, as the option is now live and its payoff depends only on the likelihood of ending in-the-money.

Case 2: Knock-out digital option (barrier has been breached)

If it's a knock-out, then once the barrier is breached, the option ceases to exist — its value instantly drops to zero. Once knocked out, increasing volatility has no effect, since the option is already worthless. Before the barrier is hit, higher volatility would increase the probability of knock-out, thus reducing the option's value. But after the breach → it's already dead, so its value stays at 0, regardless of volatility. Therefore, once the knock-out barrier has been breached, the option price = 0, and volatility changes do not affect it.

(3)

v) The probability for $S_T \geq X$ equals that for $\ln(S_T/S) \geq \ln(X/S)$. Now, this event occurs with probability

$$1-N\left(\frac{\ln\left(\frac{X}{S}\right)-\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)=N\left(-\frac{\ln\left(\frac{X}{S}\right)-\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)=N\left(-\frac{\ln\left(\frac{S}{X}\right)+\left(r-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right)$$

Multiply this by e^{-rT} to get the desired result.

(3)

[15 Marks]

Solution 2:

i) The relationship between vega and gamma is derived as follows

$$\Gamma = \frac{\partial^2 C}{\partial S^2} = \frac{e^{-qT}\,\phi(d_1)}{S\sigma\sqrt{T}}$$

$$\nu = \frac{\partial C}{\partial \sigma} = S\,e^{-qT}\,\phi(d_1)\sqrt{T}$$

$$\boxed{\nu = \Gamma\, S^2\, \sigma\, T}$$

(2)

ii) The given option is deeply in the money. Hence the vega will tend to zero. In other words a change in volatility doesn't really affect the option price. Correspondingly the gamma will also tend to zero as gamma and vega move together under these conditions. (3)

[5 Marks]

Solution 3:

i) The value of the trader's position is $V_t = \phi_t - C_t$.
V_t is risk – free. Further, the coefficient of the random term, dW_t, is zero in a small interval of time dt. The change in the value in the interval dt thus is

$$dV_t = \phi_t dS_t - dC_t = \phi_t(\mu S_t dt + \sigma S_t dW_t) - (\mu_t^C C_t dt + \sigma_t^C C_t dW_t)$$

Since the trader's position does not change, it is therefore risk – free if and only if

$$\phi_t \sigma S_t = \sigma_t^C C_t$$

Thus, the result.

(3)

ii) The condition of no arbitrage means that the drift in the no-risk position must be the short-term interest rate. Thus,

$$\phi_t \mu S_t - \mu_t^C C_t = rV_t = r(\phi S_t - C_t)$$

which on substituting ϕ_t gives

$$\sigma_t^C C_t \mu/\sigma - \mu_t^C C_t = r(\sigma_t^C C_t/\sigma - C_t).$$

Divide both sides by $C_t^C \sigma_t^C$ to obtain the result.

This means that there exists a common risk – premium

$$\lambda = (\mu - r)/\sigma \,,$$

that is same for the asset and all derivatives priced on this asset as an underlying derivatives of it.

(3)

iii) By Ito's lemma,

$$dC_t = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}(dS)^2$$

Note that the other second derivative terms are zero. The only non – zero term in $(dS)^2$ is $\sigma^2 S_t^2 (dW_t)^2 = \sigma^2 S^2 dt$. Therefore,

$$dC = \left(\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2\right)dt + \sigma S\frac{\partial C}{\partial S}dW_t$$

Matching $C_t\mu_t^C$ with the coefficient of dt and $C_t\sigma_t^C$ to the coefficient of dW_t, we get the result.

(4)

iv) From part (ii), we get

$$\mu_t^C = r + \sigma_t^C(\mu - r)/\sigma = r + (S/C)\frac{\partial C}{\partial S}(\mu - r)$$

Substituting this in part (c), we get

$$\mu^C = C^{-1}\left(\frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}\right) = r + (S/C)\frac{\partial C}{\partial S}(\mu - r)$$

The implication of this is that derivatives can be priced without knowing the rate of return on the asset.

(5)

[15 Marks]

Solution 4:

i) <u>**Selling the stock**</u>

- Receives Rs. (650 * (1 – 1%)) = Rs. 643.50
- Loss of dividend of Rs. 45 as on 15th October 2025 whose value as on 1st October 2025 is
  - Rs. 44.90 (if computed @5% for 15 days)
  - Rs. 44.89 (if computed @6% for 15 days)

Since the client will be net lender rather than borrower the applicable rate of interest is 5%. And hence the loss of dividend in present value terms should be Rs. 44.90

<u>**Buying the futures**</u>

- Pay the brokerage: Rs. (594 * 0.005) = Rs. 2.97
- Pay the margin: Rs. (651 * 0.05) = Rs. 32.55
- Pay Rs. (594 – 32.55) = Rs. 561.45 on maturity (31st October 2025) and get the shares.
- The value of this amount as on 1st October 2025 is the present value of Rs. 561.45 @ 5% for 31days = Rs. 559.08

In a nutshell, the expected gain from the strategy today is:
Rs. (643.50 – Rs. 44.90 – Rs. 2.97 – Rs. 32.55 – Rs. 559.08) = Rs. 4.00
Hence, the client can be advised to go ahead with the strategy

(10)

ii) The risks in implementing this strategy are:

- The discount is not as substantial as you may infer by looking at the prices.
- Even if the prices are known for calculating the gain/loss before putting the order on the exchange, the actual transacted price may be different compared to the price used in calculating the gains.
- The company may either reduce or increase the amount of dividend after the transactions. The decrease is beneficial under the strategy but not the increase.
- Taxes have been ignored in the above calculations, and they may impact the benefits.
- The daily settlements have been ignored in the above calculations. This may give rise to further cash-flow management problems and interest gain/losses on the same.
- Insufficient market depth or low trading volumes may make it difficult to enter or exit positions at desired prices, distorting arbitrage margins

(5)

[15 Marks]

Solution 5:

i) Price of a 5-year 10% annual coupon bond at current rates:

$$P = 10\sum_{i=1}^{5} d_i + 100d_5$$

$$= 10(0.95238 + 0.90488 + 0.8577 + 0.81492 + 0.77796) + 100(0.77796)$$
$$= 120.875$$

where d_i are the values of the zero-coupon bonds of length i (per nominal of 1 unit).

The value of a 5-year annual fixed-floating swap with 10% fixed coupon is the difference between a 10% 5-year bond and an FRN. But the latter is valued at 100.

Hence swap value = 120.875 – 100 = 20.875.
[This is a receiver swap – the payer swap is of opposite sign.]

(4)

ii) $Modified\ Duration, M = -\frac{1}{P}\frac{\partial P}{\partial r}$

Where, r is the level of rates (or yields).

$Absolute\ Yield\ Sensitivity = \frac{P_r - P_{r+\Delta r}}{\Delta r}$

So, Absolute Yield Sensitivity of the Bond = (Modified Duration * Price of the Bond), if Δr is small and the entire yield curve shifts in parallel.

Using the same technique as in (i), we get:
Bond price (shifted) = 10 (4.296) + 100 (0.77428) = 120.388.
Hence Absolute Yield Sensitivity of the bond = [(120.875 – 120.388) / 0.10] = 4.87.
Similarly, the swap values (shifted) = 120.388 – 100 = 20.388.
Hence Absolute Yield Sensitivity of the swap = [(20.875 – 20.388) / 0.10] = 4.87.

(5)

iii) The cash flows of the reverse floater are:

Coupons:	Years 1 to 5: $10 - f_i$	i = 1 to 5	(A)
Redemption:	+ Year 5: 100		(B)

But the coupons on an annual fixed – floating swap are the same as in (A) and the 5-year zero coupon bond pays 100 at the end of year 5 as in (B).
Hence, the decomposition as required.

Price and absolute yield sensitivity are additive.
So, the price of the reverse floater = swap + 5 year zero = 20.875 + 77.796 = 98.671.
Also, the absolute yield sensitivities = swap + 5 year zero = 4.873 + {(77.796 – 77.428) / 0.10}

= 4.873 + 3.680 = 8.553

So, Modified Duration of reverse floater = [(8.553 * 100) / 98.671] = 8.668, which is roughly twice the Modified Duration of the par yield bond (4.299).

(7)

iv) There are two possible alternative answers, both equally valid.

Bond world

In the world of bonds, the portfolio is risk managed by ensuring that modified durations match. The total duration is the sum of the individual bonds, weighted by nominal amount. Longer bonds will have greater convexity. Swaps are treated as a long (or short) fixed bond position and an equal short (or long) floating bond position, and their durations obtained separately. The reverse floater is decomposed into a swap and zero-coupon bond using the analysis in (iii) above.

Swap world

In a swap world, risk management is performed by sensitivity analysis to the underlying yield curve. The bonds are mapped to the relevant yield curve points by taking their price sensitivity to each of the components of the curve in turn, i.e. 1 year zero, 2 year zero, etc. Swaps are decomposed into fixed and floating legs, and the inverse floater is decomposed as in (iii).

(4)

[20 Marks]

Solution 6:

Given that we know the dividends, we can compute the option value by replacing thee current stock price S with S(1 - δ)m. This effectively imply a payoff function of

$$\max((1-\delta)^m S - X) = (1-\delta)^m max(S - (1-\delta)^{-m} X).$$

[4 Marks]

Solution 7:

i) Impact on futures

- Stock buyback happening at a premium would usually lead to an increase in stock price as buyback is happening at a good premium to market price. Near term futures price also may rise but lesser than the rise in stock leading to basis compression. Existing holders of the stock may prefer to hold on to the stock to tender the shares leading to a lower liquidity which will support higher stock price and adding to the basis compression.

- Buyback has an impact on the cash futures basis especially on the futures price for the longer term i.e. beyond the buyback date. The basis depends on market perception about the stock. Buyback is often a strong message from the management that they perceive the stock to be undervalued. If market expectation is similar the basis may continue to remain stable. However, if market perceives the stock price increase to be temporary and it will fall after the buyback closes the basis may narrow and it may even lead to backwardation where the futures price can slip to a discount compared to spot.
- The basis may also be impacted by actions of arbitrageurs who may buy stock and sell futures to take part in the buyback by creating a net neutral position. This depends on their perception of acceptance ratio vis-à-vis the acceptance ratio implied by the market.
- Future price depends on the expected dividend $F = e^{(r-q)t}$. If the cash outflow is going to be significant and the company is expected to recalibrate its future dividend payout then the market participants may also recalibrate their dividend expectations leading to an impact on the futures price.

(5)

ii) Impact on options

- The announcement tends to increase the spot price, hence call option premiums generally rise and put option premiums generally fall.
- Immediately after the announcement, speculative activity and uncertainty may spike short-term implied volatility immediately. As the tender date approaches, the buyback price acts as a pivot, reducing short-term downside risk and causing near-term implied volatility to decline.
- The buyback may reduce the trading volumes in spot as investors holding the stock may prefer to tender the shares rather than sell it in the market. Market makers may widen bid–ask spreads and temporarily adjust implied volatility upward considering lower spot liquidity.
- Arbitrageurs may become active as they exploit temporary option mispricing through volatility arbitrage, such as buying straddles pre-event and unwinding post-event.
- Before the tender closes, implied volatility rises sharply due to event risk associated with the tender offer — namely, uncertainty about the acceptance ratio, the exact number of shares the company will repurchase, and the potential price impact if the offer is oversubscribed or undersubscribed. Upside is limited because the offer sets a fixed purchase price, while downside remains uncertain if the market anticipates post-tender selling.
- After the tender concludes, the discrete event risk is resolved: the stock adjusts to market supply–demand equilibrium. Consequently, implied volatility drops sharply as uncertainty disappears, and option prices recalibrate to reflect the lower post-event risk.

(5)

iii) Impact of special dividends

- A large special dividend mechanically reduces the stock price by the dividend amount on the ex-dividend date.
- If adjustments are not done, holders of call options can benefit because they can exercise at the unchanged strike against a mechanically lower stock price, effectively receiving part of the dividend indirectly.

- Conversely, Put holders lose value because the underlying stock price falls mechanically, reducing the intrinsic payoff of the put relative to what would have existed without the dividend.
- Similarly if no adjustment is done the future price will also drop creating disadvantage for investors who are long on futures.
- Stock exchanges typically adjust futures price and strike prices for options by the special dividend to ensure calls and puts remain economically neutral post-dividend.
- Adjustments are done only for dividends exceeding a pre-specified threshold say 5% of stock price and so on

(5)

iv) Adjustments for buyback

- A dividend produces an immediate and predictable price drop on a known date, whereas a buyback is a discretionary market transaction spread over weeks or months, with an uncertain price impact.
- Buybacks affect prices through market demand and sentiment, not through a mechanical adjustment to the underlying. The magnitude and timing of the price change are uncertain, making mechanical adjustments difficult and potentially distorting derivative pricing. Consequently, their impact is economic and uncertain, and should therefore not be neutralized by any derivative adjustment.
- Exchanges hence treat buybacks as normal market events. Any resulting price movement should naturally flow into derivative valuations through the standard pricing relationship between spot and futures or options.
- In case of buybacks, arbitrageurs and market makers incorporate the tender offer premium into options and futures, maintaining market efficiency.

(5)

[20 Marks]

Solution 8:

i) Close-out netting

- All outstanding contracts between the two parties are terminated and valued at current market prices.
- Positive and negative MTMs are netted into a single amount payable by one counterparty.

(2)

ii) Computation

- A’s net position = +20 − 15 + 10 = +15 million.
- Company B owes ₹15 million to Company A after netting.

(2)

iii) Counterparty risk management

- Limits counterparty exposure to a single net amount instead of gross notional exposures.
- Reduces systemic risk and credit exposure in the event of default.

(2)

[6 Marks]

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