INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2025

SP6 - Financial Derivatives

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1)

i) Connect the AR (1) process to the Vasicek process. (3)

ii) Prove that the price of an American option implied by a tree will always be as much as the price of a European option with the same parameters priced on the same tree. (3)

iii) Suppose there are two American options, "A" and "B". Further, let us assume that "B" has half the notional of "A" but is otherwise identical. Let us now consider a portfolio "C" consisting of two contracts of type "B". Show "C" is better than "A". (3)

iv) What happens to the price of an American digital option when the volatility is increased, and the barrier has been breached? (3)

v) A binary call pays off INR 1 if the underlying asset finished above the strike price and nothing otherwise. Show that its price equals $e^{-rT} N(x - \sigma\sqrt{T})$; where N(.) has the usual meaning in terms of the cumulative normal density function. (3)

[15]

Q.2)

i) Derive the relationship between Gamma and Vega of a call option. (2)

ii) XYZ technologies is currently trading at 1200 per share. A call option of XYZ due to expire in a month has a strike price of 800. Based on the relationship between gamma and vega derived in part (i) comment on the gamma of this option. (3)

[5]

Q.3) Let S_t denote the price of non-dividend paying asset at time t, satisfying the stochastic equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t,$$

where the drift μ and the volatility σ are both assumed constant and W_t is the Brownian motion. Let C_t be the price of a derivative based on this asset satisfying

$$dC_t = \mu_t^C C_t dt + \sigma_t^C C_t W_t.$$

i) If at time t, a trader is long ϕ unit of the asset and short one unit of the derivative, what is the value of trader position? Show that the position is risk free only if

$$\phi_t = \frac{\sigma_t^C C_t}{\sigma S_t}.$$

(3)

ii) Assume that the short-term interest rate has a constant value r. Show that the principle of no arbitrage implies that

$$\frac{\mu_t^C - r}{\sigma_t^C} = \frac{\mu - r}{\sigma}.$$

Also, explain the financial significance of this relation. (3)

iii) Assume that the derivative price can be expressed as Ct = C(S, t) where the function C(S, t) has a continuous second derivative. Use Ito lemma to show that

$$
\begin{aligned}
C_t\mu_t^C &= \frac{\partial C}{\partial t} + \mu S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} \\
\text{and} \quad C_t\sigma_t^C &= \sigma S\frac{\partial C}{\partial S}.
\end{aligned}
$$

(4)

iv) Show that the no arbitrage condition implies that C(S, t) satisfies the following equation

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} = r\left(C - S\frac{\partial C}{\partial S}\right).$$

Explain why μ does not appear in this equation?

(5)

[15]

Q.4) You are an investment actuary offering consulting services specializing in derivatives. Suppose it is October 1, 2025, today and a client comes to you for advice. He currently holds 1000 shares of Actuarial Corporation of Wonderland (ACW) in his portfolio. Further, he observes that the futures contract on ACW is trading at a substantial discount to the market. The client wants to sell the stock and buy the futures.

Before you offer your advice or proceed further, you do some research on ACW prices and observe the following (as of October 1, 2025):

- ACW spot price: Rs. 650 / Rs. 652
- ACW 1-month futures (i.e. expiring on October 31, 2025): Rs. 590 / Rs. 594
- Interest rate for all terms: 5% / 6%
- Dividend per share: Rs. 45
- Ex – Dividend Date: 15th October 2025
- Brokerage on Spot Transaction: 1% of the spot value
- Brokerage on Futures Transaction: 0.5% of the future price
- Margin required at any point in time: 5% of the underlying value as at the close of the day (assume Rs. 651)

Further, for the purpose of this question assume that futures will NOT be settled daily and margin will be released at the maturity along with the settlement.

Based on the above information,

i) What will be your advice with respect to the soundness of the client's strategy of selling the stock and buying futures? (10)

ii) Describe five major risks that the client will be exposed to while implementing the strategy? (5)

[15]

Q.5) You are observing bonds and swaps in a SOFR (Secured Overnight Financing Rate) based interest rate market. Column A gives the set of zero-coupon bond prices that apply currently (each price is for a nominal of 100). Column B gives the prices of the same bonds after the yield curve has instantly shifted up by 10 basis points (0.10%) for all maturities.

	A	B
Maturity (Years)	Current Zero-Coupon Bond Price	Shifted Zero-Coupon Bond Price
1	95.238	95.147
2	90.488	90.316
3	85.770	85.526
4	81.492	81.183
5	77.796	77.428

The modified duration of a bond priced at par is 4.296 on current rates.
Using this information,

i) Calculate the value of a 5-year 10% annual coupon bond at current rates. Hence derive the value of 5-year annual fixed-floating swap with 10% fixed coupon. (4)

Let the "absolute yield sensitivity" of an instrument be defined as the instrument's value now less its value when rates have risen by a small increment (Δr), all divided by the small increment (Δr).

ii) Show how absolute yield sensitivity for a bond is related to its modified duration. Show numerically that the absolute yield sensitivities of the bond and swap in (i) are the same.

[For the purpose of this computation, assume that a 10-basis point shift is a small increment.]

(5)

A "reverse floater" is a bond that pays a coupon equal to a fixed value X less the value of SOFR at each fixing. So, for example, if SOFR is 5.75% at the next fixing, the coupon becomes X – 5.75%.
Consider a 5-year annual coupon reverse floater where X = 10%.

iii) Show how the cashflows of the reverse floater can be decomposed into those of a fixed-floating swap and a zero-coupon bond. Hence demonstrate numerically that the modified duration of the reverse floater is approximately twice that of a bond priced at par. (7)

iv) Explain how you would manage risk, in terms of duration, a portfolio of bonds that includes a reverse floater. (4)

[20]

Q.6) Show that the value of a European option under the case of known dividend yields equals $(1 - \delta)^m$ times that of a European option on a non-dividend paying stock with strike price $(1 - \delta)^{-m}$ X. **[4]**

Q.7) Sinfosix Technologies is a large IT services company listed on the National Stock Exchange (NSE) and is a major constituent of many indices offered by the NSE. Sinfosix futures and options are available for trading and reasonably liquid. Sinfosix has now announced a stock buyback through a tender offer at a price which is 20% above the current market price wherein they will be using the surplus cash in the balance sheet to purchase and extinguish nearly 10% of the floating stock from the company's shareholders.

i) Explain the impact of the buyback announcement on Sinfosix futures? (5)

ii) Explain the impact of the buyback announcement on the Sinfosix options? (5)

iii) Stock exchanges usually make specific adjustments to the derivatives when a company announces a large sized special dividend. What could be these adjustments and why are they made? (5)

iv) Stock exchanges don't make any specific adjustments on the derivatives for buybacks unlike special dividends. Explain the rationale. (5)

[20]

Q.8) Company A and Company B have entered into three OTC derivative contracts under a single ISDA Master Agreement:

Contract	**A's Position (₹ million)**	**B's Position (₹ million)**
Interest Rate Swap	+20	−20
Currency Forward	−15	+15
Equity Swap	+10	−10

Company B defaults on its obligations.

i) Explain how close-out netting under the ISDA Master Agreement will be applied. (2)

ii) Compute the net exposure of Company A to Company B after close-out netting. (2)

iii) Briefly state why this mechanism is critical for managing counterparty risk. (2)

[6]

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