INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

November 2025

Subject CS2 - Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q. 1) In a TV reality show, 10 contestants are required to stay in a closed mansion for a period of 10 weeks. Every week a contestant is voted out by the audience and he has to leave the mansion.

It has been reported by the local media that many of the contestants themselves opt out due to health reasons even when they are not voted out.

You are an actuary and you have decided to observe the number of contestants opting out due to health reasons for a period of first 5 weeks and estimate the probability of contestants being healthy after 5 weeks using Kaplan-Meier estimator i.e. $p_{KM}(5)$ as well as Nelson-Aalen estimator i.e. $p_{NA}(5)$.

You observe that during the 5 weeks under observation, there was not even a single case of opting out due to health reasons. One contestant each was voted out in each of the 5 weeks and at the end of 5 weeks, 5 contestants continue to stay in the closed mansion.

Which of the following is TRUE?

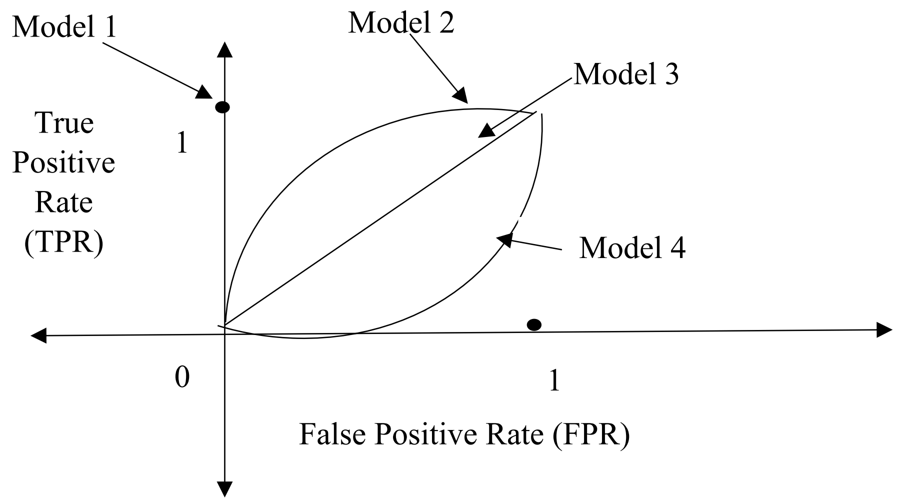
A. $p_{KM}(5) = 0$, $p_{NA}(5) = 1$
B. $p_{KM}(5) = 1$, $p_{NA}(5) = 0$
C. $p_{KM}(5) = 1$, $p_{NA}(5) = 1$
D. $p_{KM}(5) = 0$, $p_{NA}(5) = 0$
E. $p_{KM}(5) = 0.5$, $p_{NA}(5) = 0.5$. **[2]**

Q. 2)

A political party wanted to determine the winnability of its candidates in the country's general election across 500 constituencies. It had assigned this to a firm of psephologists and the firm had developed four different logistic regression models to determine a candidate's probability of winning an election based on multiple factors like education, gender, number of bills presented in the parliament, etc.

Elections happened in 500 constituencies and the election results have been declared.

The FPR (false positive rate) and TPR (true positive rate) for each of these logistic regression models have been calculated. The ROC (receiver operating characteristic) curve for each of these models have been plotted in the below graph:



Which of the following pairs are correctly matched?

	Random Classifier	Perfect Classifier	Good Classifier	Bad Classifier
A.	Model 3	Model 1	Model 2	Model 4
B.	Model 1	Model 2	Model 4	Model 3
C.	Model 4	Model 3	Model 1	Model 2
D.	Model 2	Model 4	Model 3	Model 1
E.	Model 3	Model 2	Model 4	Model 1

[2]

Q. 3) A demographer is undertaking an investigation into the mortality of persons aged between exact ages 55 and 56.

The investigation began on 1st April 2024 and ended on 31st March 2025.

The following table gives details of 5 lives involved in the investigation.

Life	Date of 55th birthday	Date of Death
B	1st July 2023	1st December 2024
D	1st December 2023	-
E	1st February 2024	1st May 2024
H	1st August 2024	1st March 2025
J	1st January 2025	-

Persons with no date of death were still alive at the end of the investigation.

What is the correct value of central exposed to risk (in months)?

A. 36
B. 30
C. 28
D. 26
E. 22. **[2]**

Q. 4) GDP growth (X_t) and inflation rate (Y_t) affect each other and a multi-variate time series process (vector autoregression i.e. VAR) is used for modelling the relationship between these two processes:

$$X_t = 0.3X_{t-1} + 0.5Y_{t-1} + e^X{}_t$$
$$Y_t = 0.2X_{t-1} + 0.2Y_{t-1} + e^Y{}_t$$

If the above multi-variate time series is considered to be **stationary**, which of the following represents the correct eigen-values for this time series?

A. $\lambda = 0.57$ and $\lambda = -1.07$
B. $\lambda = 0.57$ and $\lambda = -0.07$
C. $\lambda = -1.57$ and $\lambda = 0.07$
D. $\lambda = 1.57$ and $\lambda = 0.07$
E. $\lambda = -1.57$ and $\lambda = -0.07$. **[2]**

Q. 5) Lee-Carter mortality projection model is used for studying mortality improvements in persons aged 60. Following are the details of the model:

$$m_{60,t} = \exp(a_{60} + b_{60} * k_t)$$

where:

- $m_{60,t}$ is the central mortality rate for a person aged 60 projected at time t. **[2]**

- $a_{60} = -4.70$
- $k_0 = 1.5$ and $k_{40} = -1.5$. **k_t decreases linearly** from **1.5** at **time 0** to **-1.5** at **time 40** such that from period 0 to 40, $\sum k_t = 0$.

What is the central mortality rate for a person aged 60 projected at time t=20?

A. 0.00592
B. 0.00635
C. 0.00760
D. 0.00910
E. 0.01089.

Q. 6) A climate risk actuary is studying the relationship between Air Quality Index (AQI) denoted by random variable Y and daily rainfall (in centimetres) denoted by random variable X using a Gumbel Copula with parameter $\theta = 2$. Following is the joint cumulative distribution function C (u, v) of Gumbel (2) Copula:

$$C(u, v) = \exp\left(-\left[(-\ln u)^2 + (-\ln v)^2\right]^{1/2}\right)$$

AQI of less than or equal to 200 is considered to be a sign of moderate pollution. Higher rainfall is generally said to reduce the AQI by washing out the pollutants from the air.

What is the joint probability of rainfall up to 2 centimetres during a day and AQI being less than or equal to 200 using Gumbel (2) Copula?

You are given that $F_Y(Y=200) = 0.857$ and X is exponentially distributed with parameter 0.20.

A. 0.283
B. 0.326
C. 0.432
D. 0.574
E. 0.819. **[2]**

Q. 7) A social scientist has developed a new model to determine the employee retention rate based on two important factors *viz*. daily working hours and flexible work options. A proportional hazards model has been fit for this purpose.

It has two co-variates:

- $Z_1 = 1$ for working hours > 8 and $Z_1 = 0$ for working hours ≤ 8
- $Z_2 = 1$ for fixed check-in, check-out and $Z_2 = 0$ for flexible check-in, check-out

You have four friends working with different companies under different working conditions, of which two friends Babita and Chinmay have recently resigned and are serving their notice period.
Following data has been collected in respect of these four friends:

Friend	**Ajay**	**Babita**	**Chinmay**	**Danish**
Working hours	10	8	9	7
Check-in/Check-out	Fixed	Flexible	Flexible	Fixed

Using Breslow's approximation, what is the contribution to the partial likelihood in respect of friends who have resigned? Assume $\beta_1 = \beta_2 = 1$.

A. $e^3 / (2 + e^2 + e)^2$
B. $e^2 / (1 + e^2 + 2e)^2$
C. $e^2 / (2 + e^2 + e)^2$
D. $e / (2 + e^2 + 2e)^2$
E. $e / (1 + e^2 + 2e)^2$. **[2]**

Q. 8) A new age entrepreneur has recently started manufacturing of zippers for suitcases. He wants to know the average lifetime (in months) till which a zipper can last.

If the lifetime of a zipper is assumed to follow a Weibull distribution with parameters $c = 0.0004$ and $\gamma = 2$, what is its median lifetime?

A. 35
B. 38
C. 42
D. 45
E. 50. **[2]**

Q. 9) Let T_x be the random future lifetime after age x and let K_x be the curtate future lifetime in years of a life aged x. The curtate expectation of life of age x is denoted by e_x and the complete expectation of life is denoted by $e^0{}_x$.

Consider the following statements:
1. $E(T_x) = e_x$
2. $E(K_x) = e^0{}_x$
3. $E(T_x) \approx E(K_x) - 0.50$
4. $E(T_x) \approx E(K_x) + 0.50$
5. $T_x \leq K_x$

Which of the given statements are correct?

A. Statement 1 only
B. Statement 2 only
C. Statement 3 only
D. Statement 4 only
E. Statement 5 only. **[2]**

Q. 10) A habitat becomes unsuitable for elephants when the environmental resources and conditions no longer meet the ecological and physiological needs of elephants. A habitat transitions from suitable to unsuitable at a rate of $\lambda(t) = 0.01t$ where t is time in years.

What is the probability that a particular habitat remains suitable for elephants till 10 years?
A. 0.61
B. 0.67
C. 0.73
D. 0.81
E. 0.89. **[2]**

Q. 11) Factory manager of a factory which manufactures and stores mineral water bottles has recently complained about the issue of theft of bottles from their central storage facility. A total of 300 bottles were stolen from the facility during financial year 2024-25.

A rate of theft is being calculated for this financial year. At the beginning and at the end of the financial year, the facility had exactly 10,000 bottles in stock. However, there were **[2]**

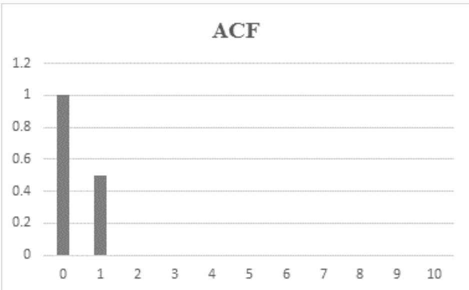
9,500, 12,500 and 7,500 bottles in stock at the end of June, September and December respectively.

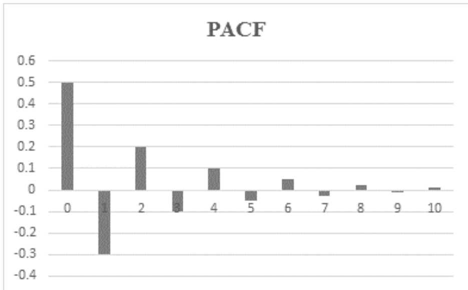
What is the rate of theft for financial year 2024-25?

A. 2.40%
B. 2.75%
C. 3.29%
D. 3.15%
E. 3.04%.

Q. 12) Noise in digital and analog communication channels has been modelled using a time series process X_t.

Following are the plots of ACF (autocorrelation function) and PACF (partial autocorrelation function) based on observed data relating to this process for lags 0 to 10:





Which ARIMA time series would fit the observed data?

A. ARIMA (0, 1, 1)
B. ARIMA (1, 1, 1)
C. ARIMA (1, 0, 0)
D. ARIMA (0, 0, 1)
E. ARIMA (1, 0, 1). **[2]**

Q. 13) The income distribution in a developing economy is modelled using Burr distribution with parameters $\lambda = 1$, $\alpha = 1$ and $\gamma = 1$.

Which of the following expression denotes the hazard rate for this distribution?

A. $(2 + x) / (1 + x)$
B. $(1 + x) / (2 + x)$
C. $(2 + x) / (1 + x)^2$
D. $(1 + x) / (2 + x)^2$
E. $(2 + x)^2 / (1 + x)^2$. **[2]**

Q. 14) A medical practitioner is conducting a study to investigate the number of tuberculosis patients who drop out / discontinue from DOTS (Directly Observed Treatment Short-Course). The DOTS course of medicines is to be followed for a period of 12 months after prescription of the anti-tuberculosis drugs.

Just before the 12th month, the Kaplan-Meier estimate of the number of patients continuing in DOTS is estimated to be 0.80. During the 12th month, 3 more patients have discontinued DOTS. At the end of 12 months, only 7 patients have finally completed the 12-months DOTS course.

What is the revised Kaplan-Meier estimate of the number of patients continuing the DOTS regimen at the end of 12 months? Assume there is no censoring in the data.

A. 0.38
B. 0.49
C. 0.56
D. 0.64
E. 0.75.

[2]

Q. 15) An oil cartel is studying the joint behaviour of prices of oil and natural gas. It is observed that during low demand periods, there could be an extreme fall in both oil and natural gas prices due to strong lower tail dependence between the two.

Which copula would be the most natural baseline for modelling the joint behaviour of oil and natural gas prices?

A. Gumbel Copula
B. Clayton Copula
C. Gaussian Copula
D. Frank Copula
E. Product Copula.

[2]

Q. 16) A non-life insurer specialises in cyber liability insurance which covers data breaches, ransomware attacks and business interruption from cyber risks.

Annual aggregate claims on this portfolio are modelled using a compound Poisson distribution with –

- Number of claims (N) follow a Poisson distribution with a parameter $\lambda = 100$
- Individual claim sizes (X_i) are independent and identically distributed with continuous uniform distribution over 0 to 200.

Th insurer has a practice of loading 25% of the expected claim costs in respect of an individual claim. These loadings provide an allowance for operating expenses and profits. Premium in respect of an individual risk is determined by adding these loadings to expected claim costs.

Insurer is considering two alternative reinsurance arrangements for its cyber liability insurance portfolio:

Proportional Reinsurance Arrangement with Stat Re:

Quota share arrangement where 80% of the individual claim amount is retained by the insurer and remaining 20% is ceded in the favour of Stat Re.

Stop Loss Reinsurance Arrangement with Math Re:

- A reinsurance claim with Math Re in respect of an individual claim event is triggered only when it breaches the attachment point of 80% loss ratio (where loss ratio is the ratio of claim to premium).
- Coverage limit with Math Re is up to 100% of the loss ratio.

i) Calculate the reduction in the expected aggregate claim amounts by entering into the quota share reinsurance arrangement with Stat Re. (2)

ii) Show that under the stop loss reinsurance arrangement, the expected number of claims to be ceded in favour of reinsurer Math Re is equal to 50. (4)

The company is evaluating both the reinsurance arrangements by comparing the expected savings in aggregate annual claim amounts.

iii) Which of these arrangements leads to higher savings in expected aggregate annual claims? (3)

An actuarial intern who has recently joined the insurance company has shown keen interest in understanding the *modus operandi* of stop loss reinsurance arrangements.

In the course of discussions, he once commented that "an arrangement with an attachment point of 70% and coverage limit up to 90%" is equivalent to "an arrangement with an attachment point of 60% and coverage limit up to 80%". He feels that in both cases, the reinsurer will cover claims equal to 20% of the premiums.

iv) Clearly point out the error in the comment made by the intern. You are NOT required to perform any additional calculations. (1)

[10]

Q. 17) Market prices for a particular blue-chip stock are modelled using a geometric random walk process with drift as follows:

$$P_t = P_{t-1} * \exp(\mu + e_t)$$

where:

- $e_t \sim N(0, \sigma^2)$ are independent and identically distributed random variables
- μ is a constant and μ is independent of e_t

Ram, an investment actuary who feels that P_t is non-stationary, has defined a logarithmic returns process R_t to analyse the performance of this stock. He expects this time series process R_t to be stationary.

$$R_t = \log(P_t / P_{t-1})$$

i) Show that R_t is MA (0) time series process. (1)

ii) By calculating the mean, variance and auto-covariance for the process R_t, check whether this time series process is weakly stationary or not. (3)

Shyam, a technical analyst having a decade-long experience in analysing stock prices, believes that instead of modelling using a moving average process, it would be appropriate to model logarithmic-returns on the stock as an autoregressive process with conditional heteroscedasticity (ARCH). In this process, the error terms e_t do not have a constant variance. Instead, they have a conditional variance which is also an autoregressive process dependent on the past values of the error terms.

He has defined a new time series process for logarithmic returns on the blue-chip stock as follows:

$$X_t = b * X_{t-1} + e_t$$

where:

- $e_t \sim N(0, \sigma^2_t)$ are independent and identically distributed random variables
- $\sigma^2_t = a_0 + a_1 * e^2_{t-1}$
- e_t is independent of X_{t-1}
- $b < 1$, $a_0 > 0$, $a_1 > 0$.

iii) Show that the variance for this newly defined process X_t is given by the following expression:

$$\text{Var}(X_t) = \frac{a_0}{(1-a_1)(1-b^2)}.$$

Hint: [*Note that* $Var(e_t) = E(\sigma^2_t)$.] (4)

iv) Determine the autocorrelation function at lag 1 for this newly defined process X_t. (2)

[10]

Q. 18) A music composer specialised in composing film songs is approached by a film producer for composing **exactly two songs** per week.

The music composer believes in quality work and can compose a **minimum** of **zero** and a **maximum** of **two** songs in a week.

The music composer is also an ornithophile and likes to watch different types of exotic birds in his vicinity. He draws inspiration from these exotic birds and it gets reflected in the melodious compositions that he is able to create.

His success at composing a film song is dependent on the number of exotic birds that he is able to watch during the week.

- If he is not able to spot even a single exotic bird during the week, he cannot compose even a single song during that week.
- If he is able to spot a single exotic bird during the week, he is able to compose only one song during that week.
- If he is able to watch two or more exotic birds during the week, he is able to compose two songs during that week.

If the number of songs pending to be composed at the end of a week exceed two, then he informs the producer about his inability to compose the excess and returns the excess number of songs to the producer. Thus, the **number of pending songs** at his end are **restored to two** after returning the excess.

The number of exotic birds he is able to watch during a week is assumed to follow a binomial distribution. There are exactly 5 exotic birds in his vicinity and the probability of spotting an exotic bird in a week is 0.50.

i) Calculate the probabilities of composing –

- no song during the week,
- exactly one song during the week,
- exactly two songs during the week and
- three or more songs during the week. (3)

Music composer's friend is a student actuary and he has constructed the following transition matrix for the number of songs pending to be composed at the end of a week.

		Number of songs pending to be composed at the end of the week		
		0	**1**	**2**
Number of songs pending to be composed at the beginning of the week	**0**	$p_{00} = 0.31250$	$p_{01} = 0.15625$	$p_{02} = 0.03125$
	1	$p_{10} = 0.31250$	$p_{11} = 0.81250$	$p_{12} = 0.18750$
	2	$p_{20} = 0.15625$	$p_{21} = 0.31250$	$p_{22} = 1.00000$

Out of the nine values of one-week transition probabilities as determined by the student actuary in the above matrix, four values are erroneous.

ii) You are required to identify the four erroneous values, rectify the errors and restate the correct values of these transition probabilities. You are NOT required to reproduce the above table in the answer book.

Hint: [*No. of pending songs at end of week =* ***MIN*** *(No. of pending songs at beginning of week + No. of new songs received from producer during the week – No. of songs composed during the week,* ***2****).*] (5)

iii) If the music composer has no songs pending to be composed at the beginning of the week, calculate the probability that he will have exactly two songs pending to be composed after two weeks. Use the rectified one-week transition probabilities to calculate the required probability. (2)

[10]

Q. 19) A credit rating agency is examining default probabilities of corporate bonds over 1 to 15 years. Crude default rates are observed to be highly irregular.

Default rates have been graduated using two methods:

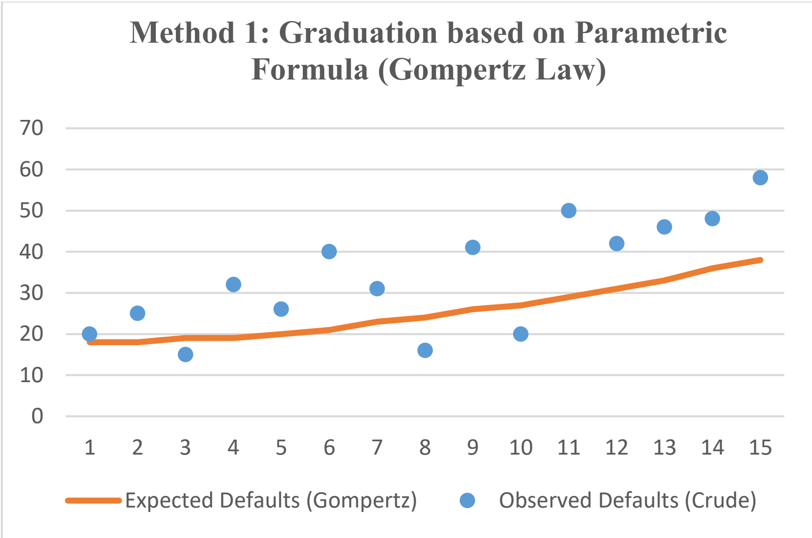
- *Method 1: Graduation using parametric formula (Gompertz Law)*

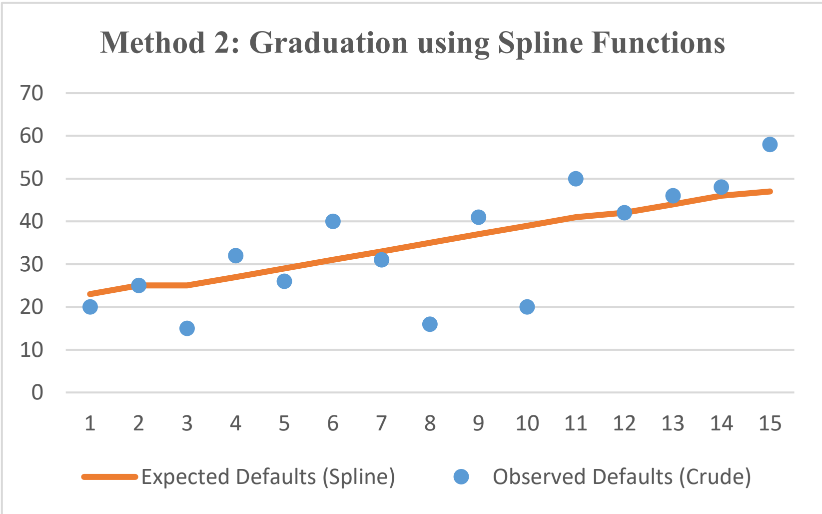
  Under this method, a parametric formula is used and **two parameters** have been fitted for graduating default rates.

- *Method 2: Graduation using spline functions.*

  Under this method, cubic spline functions have been used for graduating default rates. For the purpose of graduation, **three knots** have been used.

  Following graphs show the outcome of both the methods:





i) Based on the graphs, which method can be considered to be better fit to the data? Why? (1)

ii) Write down the formula for calculating the graduated default rates at duration x (μ_x) under both the methods. (2)

Values of chi-square test statistic under methods 1 and 2 have been determined to be 86.34 and 33.09 respectively.

iii) Perform an overall goodness of fit test to test the overall consistency between expected defaults and observed defaults under both the graduation methods. Clearly state the null hypothesis, alternative hypothesis, degrees of freedom, critical value and the conclusion of the test at 0.5% level of significance under both the methods. (5)

The chi-square test for checking the overall fit of graduation suffers from some deficiencies.

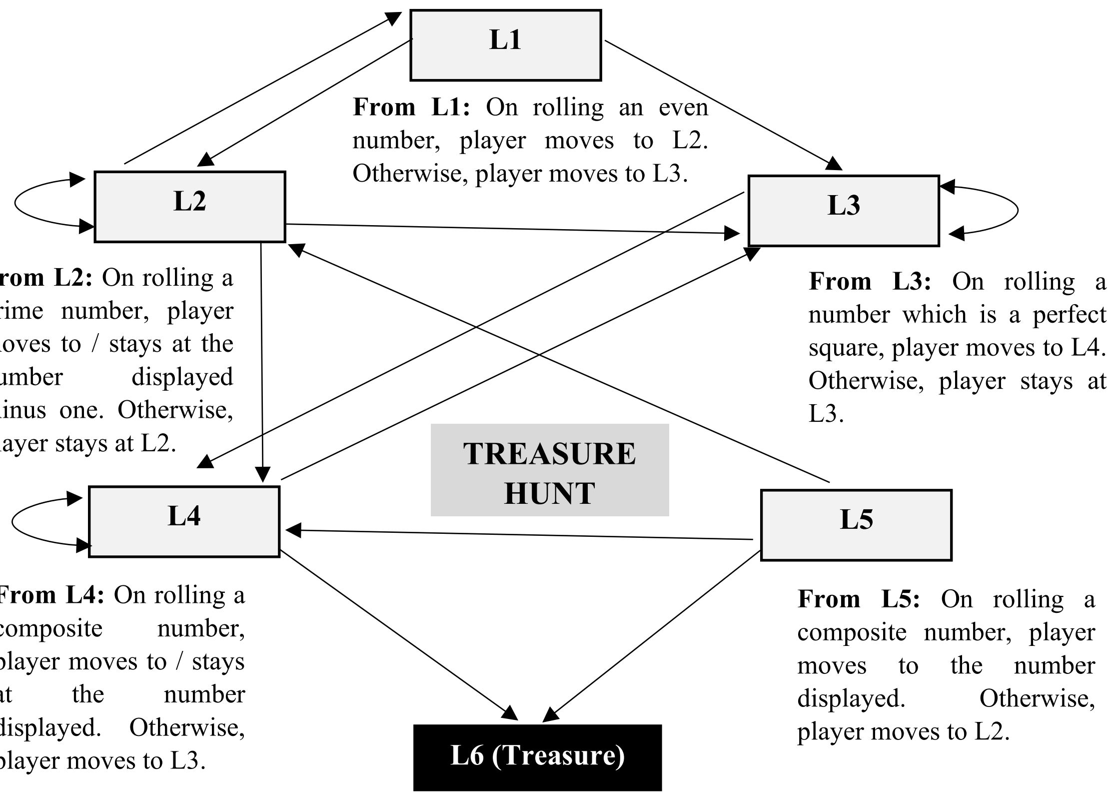
iv) Name the alternative test(s) which can address the following deficiencies of the chi-square test. You are NOT required to perform these alternative tests or to explain how the deficiency is being addressed by these alternative tests:

- The chi-square test fails to detect the effect of excessively large deviations. A few large deviations could be offset by a lot of very small deviations.
- The chi-square test fails to detect a consistent bias arising because of the imbalance between positive and negative deviations. (2)

[10]

Q. 20) A game of treasure hunt is being arranged by a group of friends who are mathematicians by profession. Following are the details of the game.

- There are total of six locations *viz*. L1, L2, L3, L4, L5 and L6.
- The game is to be played by a rolling an unbiased six-faced die.
- The starting location is L1.
- The treasure is located in L6.
- The player who reaches the final treasure in minimum number of throws of the die is considered to be the winner.
- Following diagram represents the rules relating to moving forward from L1 to L6.



i) Write down the missing values A to J in the one-step transition matrix as given below for the game of treasure hunt. You are NOT required to reproduce the below table or show any calculations relating to determination of probabilities.

	L1	L2	L3	L4	L5	L6
L1	0	A	B	0	0	0
L2	C	D	0	E	0	0
L3	0	0	F	G	0	0
L4	0	0	H	I	0	J
L5	0	2/3	0	1/6	0	1/6
L6	0	0	0	0	0	1

(5)

Let us define N_i to represent the expected number of throws of die required to reach the final treasure from the current state 'i' where i = L1, L2, L3, L4, L5, L6.

For example,

- When the person is in state L6, no more throws are required. So, $N_{L6} = 0$.

- When a person is in state L4, he has to make one throw to either go to L3 or L6 or stay at L4 with the respective probabilities.

  So, $N_{L4} = 1 + p_{46} * N_{L6} + p_{44} * N_{L4} + p_{43} * N_{L3}$.

ii) For a player who has started playing the game from location L1, calculate N_{L1} i.e. the expected number of throws of die required to reach the final treasure. (5)

[10]

Q. 21) Quantum computers rely on qubits to store and process information. Qubits are fragile and they lose their quantum state over time due to decoherence and environmental noise.

A statistician has fit a Cox proportional hazards model for estimating the rate of decoherence H_i for a qubit i in time t. Following are the specifications of the model:

$$H_i(t) = H_0(t) * \exp(\beta_1 * Z_1 + \beta_2 * Z_2 + \beta_3 * Z_3)$$

where:

- $H_0(t)$ is the baseline hazard

- Three covariates are being considered:

  - Qubit type,
  - Error correction and
  - Cooling method.

Data has been collected relating to the covariates and symmetric 95% confidence intervals for the regression parameters have been obtained.

Covariate	*95% Confidence interval*
Z_1: Qubit type:	
▪ Trapped ion qubits	0
▪ Superconducting qubits	(0.45, 0.55)
Z_2: Error correction:	
▪ No	0
▪ Yes	(-0.8, -0.6)
Z_3: Cooling method:	
▪ Dilution fridge method	0
▪ Cryogenic method	(-0.5, -0.1)

i) Define the values that each covariate can take and state the regression parameters of the fitted model for each covariate. (2)

ii) Describe the class of qubits to which the baseline hazard applies. (1)

iii) Which of the following statements are TRUE in relation to the three covariates of the proportional hazards model? Justify your answers based on the regression parameters of the fitted model:

- Superconducting type qubits contribute to faster decoherence as compared to trapped ion qubits.
- Error correction improves qubit survival.
- Cryogenic cooling increases the hazard as compared to dilution fridge. (3)

A trapped ion qubit which was subject to error correction and cryogenic cooling has a 60% chance of survival in 4 milliseconds.

We have to find the chance of survival in 4 milliseconds for a superconducting qubit not subject to error correction, but subject to cooling using dilution fridge. Following workings have been done for this purpose:

Probability of survival for a trapped ion qubit which was subject to error correction and cryogenic cooling in 4 milliseconds
$= \exp(-\int_0^4 . H_0(t) * \exp(\beta_1 * Z_1 + \beta_2 * Z_2 + \beta_3 * Z_3)\, dt)$
$= \exp(-\int_0^4 . H_0(t) * \exp(\mathbf{A})\, dt)$
$= (\exp(-\int_0^4 . H_0(t)\, dt))^{\mathbf{B}}$

Let $\exp(-\int_0^4 . H_0(t))\, dt = x$
$x^{\exp(-1)} = 0.60$

Probability of survival for a superconducting type qubit which was not subject to error correction but subject to dilution fridge cooling in 4 milliseconds
$= \exp(-\int_0^4 . H_0(t) * \exp(\beta_1 * Z_1 + \beta_2 * Z_2 + \beta_3 * Z_3)\, dt)$
$= \exp(-\int_0^4 . H_0(t) * \exp(\mathbf{C})\, dt)$
$= \mathbf{D}^{\exp(1.50)}$.

iv) State the values of the missing terms A, B, C and D. You are NOT required to show any calculations or reproduce the workings in the answer sheet. (4)

[10]

Q. 22) A public sector bank is embarking on adopting artificial intelligence (AI) in its day-to-day operations. AI adoption could be carried out in two phases.

- In the first phase, artificial intelligence would be used partially. Chatbots would be developed for customers. AI would be used for analysing data relating to customers and borrowers.

- In the second phase, the bank would progress to full-blown AI implementation. In this phase, artificial intelligence would be fully deployed across multiple processes, decision-making and strategy.

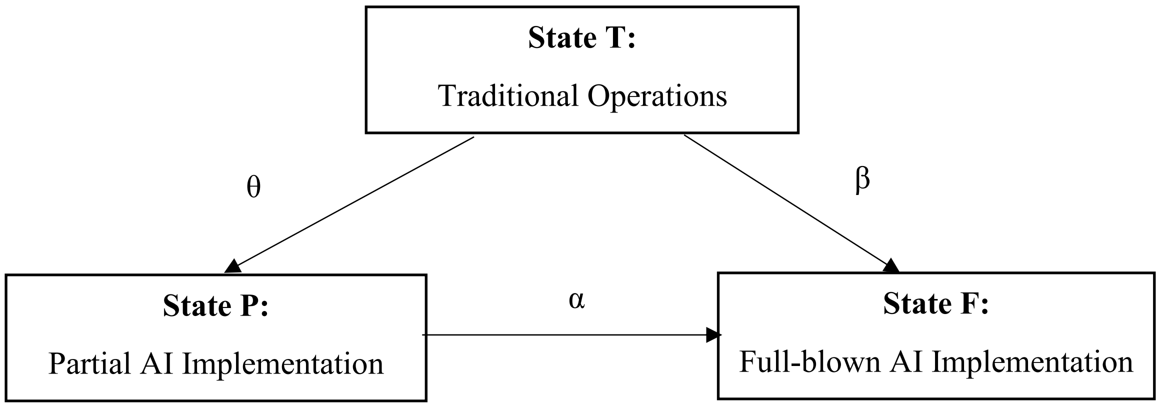
An AI adoption consultant has been appointed by the bank and he expects that on an average; **to commence the first phase** it would take around **2 years**.
He also expects that it would take **5 more years** on an average after phase one **to commence** with **phase two** i.e. full-blown AI implementation.

At the senior management level, it has been decided that once the bank goes ahead with phase one of AI adoption, there would be no going back to the traditional mode of operations. Once, phase one is implemented, the bank cannot even stop at that stage, it has to transition to full-blown AI adoption (phase two). Similarly, once full-blown AI implementation is initiated, there would be no going back to partial adoption (phase one).

AI adoption consultant has also presented an alternative proposal to the senior management that instead of going phase-wise, the bank could directly plan for **full-blown** AI implementation. If the bank goes with this **approach**, then it is expected that it would take just **2 years to commence** with end-to-end AI adoption.

The senior management of then bank is divided in opinion about which approach should be chosen. Half of the members are willing to go for phase-wise adoption and half of them are willing to go for full-blown AI adoption. At this stage, it can be considered that the bank is equally likely to go either with a phase-wise approach or a direct approach.

Following time homogenous Markov jump process has been developed by the bank:



The transition matrix for the Markov jump process is given below:

	State T	State P	State F
State T	0	0.50	0.50
State P	0	0	1
State F	0	0	1

i) Determine the average waiting times in State T and State P. Also, state the statistical distribution associated with these waiting times. (2)

ii) Calculate the values of the transition intensities θ, α and β. (2)

Let the probability of moving from State T to State P be denoted by p_{TP}.

It is given that the probability of occupancy in State T i.e. $p_{TT}(t) = e^{-0.5t}$.

iii) State the Kolmogorov's forward differential equation $\frac{d}{dt}(p_{TP}(t))$ using the transition intensities calculated in part (ii).

You are NOT required to derive this equation. (2)

iv) While solving the differential equation arrived at in part (iii) –

(4)

a) State the integrating factor to be used.

b) State the value of the integration constant c.

c) The final solution of this differential equation takes the form:

$$p_{TP}(t) = \frac{5}{6} * (e^{\mathbf{A}t} - e^{\mathbf{B}t})$$

State the values of A and B in the above expression.

You are NOT required to show any calculations for sub-parts (a), (b) and (c).

[10]

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