

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

**CS1 – Actuarial Statistics
(Paper A)**

May 2025

INDICATIVE SOLUTION

DRAFT

Q.1) Option B [1]

Q.2) Option B [1]

Q.3) Option B [2]

We are given:

- $X \sim \text{Poisson}(11) \rightarrow \text{Var}(X) = 11$
- $Y \sim \text{Poisson}(19) \rightarrow \text{Var}(Y) = 19$
- $\text{Cov}(X, Y) = -24$
- $Z = 2X - 3Y + 4$

Using variance rules for linear combinations:

$$\text{Var}(aX + bY + c) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y)$$

Here, $a = 2$, $b = -3$, $c = 4$

$$\begin{aligned} \text{Var}(Z) &= 2^2 \times 11 + (-3)^2 \times 19 + 2 \times 2 \times (-3) \times (-24) \\ &= 4 \times 11 + 9 \times 19 + 4 \times 72 \\ &= 44 + 171 + 288 \\ &= 503 \end{aligned}$$

Q.4) Option C [3]

A single die roll follows a discrete uniform distribution on $\{1, 2, 3, 4, 5, 6\}$.

The general formulas for the mean and variance of a discrete uniform distribution on $\{1, 2, \dots, k\}$ are:

$$\text{Mean: } \mu = (1 + k) / 2$$

$$\text{Variance: } \sigma^2 = (k^2 - 1) / 12$$

Applying $k = 6$:

$$\mu = (1 + 6) / 2 = 3.5$$

$$\sigma^2 = (6^2 - 1) / 12 = 35 / 12 \approx 2.9167$$

Total mean for 150 rolls (μ) = $150 \times 3.5 = 525.0$

Total standard deviation for 150 rolls (σ) = $\sqrt{150 \times 2.9167} = 20.9165$

Applying continuity correction, we compute $P(449.5 < S < 500.5)$. This corresponds to finding $P(Z < z_2) - P(Z < z_1)$, where:

$$z_1 = (449.5 - 525.0) / 20.9165 = -3.6096$$

$$z_2 = (500.5 - 525.0) / 20.9165 = -1.1713$$

Using the standard normal distribution:

$$P(Z < -1.1713) - P(Z < -3.6096) = 0.1208$$

Q.5)

part (i) Option E

[2]

For a Gamma distribution with shape parameter α and rate parameter λ , the mean is given by:

$$E[X] = \alpha / \lambda$$

In this case, $\alpha = 9$ and $\lambda = 3/\theta$. So,

$$E[X] = 9 / (3/\theta) = 3\theta$$

The Method of Moments sets the population mean equal to the sample mean. That is,

$$\bar{X} = 3\theta$$

Solving for θ gives:

$$\hat{\theta} = \bar{X} / 3$$

The Method of Moments Estimator (MME) of θ is:

$$\hat{\theta} = \bar{X} / 3$$

Part (ii) Option C

[2]

The MME for θ is:

$$\hat{\theta} = \bar{X} / 3$$

We know that for a Gamma distribution with parameters $\alpha = 9$ and $\lambda = 3/\theta$:

$$E[\bar{X}] = E[X] = 3\theta$$

Then,

$$E[\hat{\theta}] = E[\bar{X} / 3] = (1/3) * E[\bar{X}] = (1/3) * 3\theta = \theta$$

Therefore, the bias is:

$$\text{Bias}(\hat{\theta}) = E[\hat{\theta}] - \theta = \theta - \theta = 0$$

The estimator $\hat{\theta}$ is unbiased.

Part (iii) Option B

[2]

The Mean Square Error (MSE) is defined as:

$$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta}) + [\text{Bias}(\hat{\theta})]^2$$

Since the estimator is unbiased, $\text{Bias}(\hat{\theta}) = 0$, so:

$$\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta})$$

$$\text{Now, } \text{Var}(\hat{\theta}) = \text{Var}(\bar{X} / 3) = (1/9) * \text{Var}(\bar{X})$$

For a Gamma distribution, $\text{Var}(X) = \alpha / \lambda^2$

Given $\alpha = 9$ and $\lambda = 3/\theta$,

$$\text{Var}(X) = 9 / (9 / \theta^2) = \theta^2$$

$$\text{Var}(\bar{X}) = \text{Var}(X) / n = \theta^2 / n$$

Thus,

$$\text{Var}(\hat{\theta}) = (1/9) * (\theta^2 / n) = \theta^2 / (9n)$$

Therefore, the Mean Square Error is:

$$\text{MSE}(\hat{\theta}) = \theta^2 / (9n)$$

Q.6) Option D

[3]

The random variables X and Y are related as follows:

$$X|Y=y \sim N(y + 3, 0.01y^2 + 5)$$

Y has a $N(100, 60)$ distribution.

Using the law of total variance:

$$\text{Var}(X) = E[\text{Var}(X|Y)] + \text{Var}(E[X|Y])$$

From the given conditional distribution,

$$E[X|Y] = Y + 3$$

$$\text{Var}(X|Y) = 0.01Y^2 + 5$$

So,

$$E[X|Y] = Y + 3 \Rightarrow \text{Var}(E[X|Y]) = \text{Var}(Y) = 60$$

Now compute $E[\text{Var}(X|Y)]$:

$$E[0.01Y^2 + 5] = 0.01E[Y^2] + 5$$

Using the identity: $E[Y^2] = \text{Var}(Y) + (E[Y])^2$

$$E[Y^2] = 60 + 100^2 = 60 + 10000 = 10060$$

$$E[\text{Var}(X|Y)] = 0.01 * 10060 + 5 = 100.6 + 5 = 105.6$$

Thus, $\text{Var}(X) = 105.6 + 60 = 165.6$

Q.7)

Part (i) Option E

[2]

$T \sim \text{Exponential}(\lambda)$ where mean = 5 days $\Rightarrow \lambda = 1/5 = 0.2$

$$P(T > 2) = e^{(-\lambda t)} = e^{(-0.2 * 2)} = e^{(-0.4)} \approx 0.6703$$

Part (ii) Option B**[2]**

The median m satisfies:

$$P(T \leq m) = 0.5 \Rightarrow 1 - e^{(-\lambda m)} = 0.5$$

$$e^{(-\lambda m)} = 0.5 \Rightarrow -\lambda m = \ln(0.5) \Rightarrow m = -\ln(0.5)/\lambda = \ln(2)/0.2 \approx 3.4657 \text{ days}$$

Part (iii) Option B**[3]**

The sum of 15 independent Exponential(λ) variables follows a Gamma distribution:

$$T_{\text{total}} \sim \text{Gamma}(15, \lambda=0.2) \Rightarrow \text{Mean} = 75, \text{Variance} = 375$$

Using Central Limit Theorem approximation:

$$T_{\text{total}} \approx N(75, \sqrt{375})$$

$$\text{Standard deviation} = \sqrt{375} \approx 19.36$$

$$P(T_{\text{total}} > 60) = P(Z > (60 - 75)/19.36) = P(Z > -0.7755) \approx 0.7812$$

Q.8)**Part (i) Option A****[2]**

The probability density function is given by:

$$f(x) = \lambda e^{(-\lambda x)} \text{ for } x \geq 0$$

We are testing the null hypothesis $H_0: \lambda = 1$ against the alternative $H_1: \lambda = 2$.

The rejection region is $x < 0.1$.

Size of the test is the probability of rejecting H_0 when it is true:

$$P(X < 0.1 \mid \lambda = 1) = 1 - e^{(-0.1)} \approx 0.0952$$

Part (ii) Option C**[2]**

Power of the test is the probability of rejecting H_0 when H_1 is true:

$$P(X < 0.1 \mid \lambda = 2) = 1 - e^{(-0.2)} \approx 0.18127$$

Q.9)**Part (i) Option D****[4]**

Data collected from 250 females aged 18-40 with observed and relative frequencies:

Observed: [12, 124, 78, 36]

Relative frequencies: [6p, 60p, 36p, 23p] with $\hat{p} = 0.008$

Expected frequencies: [12.0, 120.0, 72.0, 46.0]

Test statistic = $\sum((O - E)^2 / E) \approx 2.8072$

Part (ii) Option C

[2]

Degrees of freedom = 4 - 1 = 3

Critical value from chi-squared distribution (df=3, $\alpha=0.05$) ≈ 7.815

Since the test statistic ≈ 2.8072 is less than 7.815, we Do not reject the null hypothesis.

Q.10)

Part (i) Option D

[3]

The moment generating function (MGF) of a random variable X is defined as:

$$M_X(t) = E[e^{tX}] = \int_0^\infty e^{tx} * f(x) dx$$

For the given f(x), we compute the MGF as the sum of two integrals:

$$\begin{aligned} M_X(t) &= \int_0^\infty e^{tx} * [(4/5)e^{-x} + (2/5)e^{-2x}] dx \\ &= (4/5) \int_0^\infty e^{(t-1)x} dx + (2/5) \int_0^\infty e^{(t-2)x} dx \end{aligned}$$

Using the formula for integration of e^x :

$$\int_0^\infty e^{-ax} dx = 1/a, \text{ valid for } a > 0:$$

$$\bullet \text{ First integral: } (4/5) \int_0^\infty e^{(t-1)x} dx = (4/5) * [1 / (1 - t)], \text{ for } t < 1$$

$$\bullet \text{ Second integral: } (2/5) \int_0^\infty e^{(t-2)x} dx = (2/5) * [1 / (2 - t)], \text{ for } t < 2$$

Therefore, the moment generating function of X is:

$$M_X(t) = (4/5) / (1 - t) + (2/5) / (2 - t), \text{ valid for } t < 1$$

Part (ii) Option E

[3]

Recall the MGF obtained:

$$M_X(t) = 4 / [5(1-t)] + 2 / [5(2-t)]$$

To find the mean ($E[X]$), we differentiate $M_X(t)$ with respect to t and evaluate at $t = 0$:

$$\begin{aligned} M'_X(t) &= d/dt [4 / (5(1-t))] + d/dt [2 / (5(2-t))] \\ &= (4/5)(1 / (1-t)^2) + (2/5)(1 / (2-t)^2) \end{aligned}$$

Evaluate at $t = 0$:

$$M'_X(0) = (4/5)(1) + (2/5)(1/4) = 4/5 + 1/10 = 9/10$$

So, the mean of X is:

$$E[X] = 9/10$$

Now, differentiate again to get $M''_X(t)$:

$$M''_X(t) = (4/5)(2 / (1-t)^3) + (2/5)(2 / (2-t)^3)$$

Evaluate at $t = 0$:

$$M''_X(0) = (4/5)(2) + (2/5)(2 / 8) = 8/5 + 1/10 = 17/10$$

Now, use the formula:

$$\text{Var}(X) = M''_X(0) - [M'_X(0)]^2 = 17/10 - (9/10)^2$$

$$\text{Var}(X) = 17/10 - 81/100 = 170/100 - 81/100 = 89/100$$

So, the variance of X is:

$$\text{Var}(X) = 89/100$$

Q.11)

Part (i) Option B

[3]

Let X_0, X_1, X_2 be the number of cars with 0, 1, and 2 accidents, respectively.

Define $X = X_0 + X_1 + X_2$ as the number of cars with at most two accidents.

Total number of cars is n .

The likelihood of observing X cars with at most 2 accidents is:

$$P(X) = (7q)^X * (1 - 7q)^{(n - X)}$$

Taking log of the likelihood function:

$$\log L(q) = X * \log(7q) + (n - X) * \log(1 - 7q)$$

To find the MLE, differentiate and set to zero:

$$d/dq [\log L(q)] = 0$$

Solving gives:

$$\hat{q} = X / (7n)$$

Part (ii) Option E

[4]

From the likelihood function, the MLE for q is:

$$\hat{q} = X / (7n)$$

To compute CRLB, we calculate the second derivative of the log-likelihood function:

$$d^2 \log L / dq^2 = [49X / (7q - 1)^2] - [X / q^2] - [49n / (7q - 1)^2]$$

This simplifies to:

$$d^2 \log L / dq^2 = [49(X - n) / (7q - 1)^2] - [X / q^2]$$

The CRLB is given by:

$$\text{CRLB} = -1 / E[d^2 \log L / dq^2]$$

Plugging in expectations where needed, the expression becomes:

$$\text{CRLB} = [q^2(7q - 1)^2] / [X(7q - 1)^2 + 49q^2(n - X)]$$

- MLE of q : $\hat{q} = X / (7n)$

- CRLB of q :

$$\text{CRLB} = [q^2(7q - 1)^2] / [X(7q - 1)^2 + 49q^2(n - X)]$$

Use $E[X] = 7qn$

Substitute $E[X]$ into the detailed CRLB expression:

$$\text{CRLB} = [q^2(7q - 1)^2] / [7qn(7q - 1)^2 + 49q^2(n - 7qn)]$$

$$\text{Denominator} = n[7q(7q - 1)^2 + 49q^2(1 - 7q)]$$

Perform the algebraic expansion and simplification:

$$= n * 7q(1 - 7q)$$

Substitute back into CRLB:

$$\text{CRLB} = [q^2(7q - 1)^2] / [n * 7q(1 - 7q)]$$

Noting that $(7q - 1)^2 = (1 - 7q)^2$, we cancel terms:

$$\text{CRLB} = q(1 - 7q) / (7n)$$

Q.12)

Part (i) Option B

[3]

$$\begin{aligned} p(y | \alpha) &= [\alpha^{(y-1)}] / (1 + \alpha)^y \\ &= e^{\{(y-1)\log(\alpha) - y \log(1 + \alpha)\}} \\ &= e^{\{y \log(\alpha) - \log(\alpha) - y \log(1 + \alpha)\}} \\ &= e^{\{y \log(\alpha / (1 + \alpha)) - \log(\alpha)\}} \end{aligned}$$

Part (ii) Option A

[2]

A distribution belongs to the exponential family if it can be written as:

$$p(y | \theta) = h(y) * \exp\{\eta(\theta) * T(y) - A(\theta)\}$$

We rewrite our expression to match this form:

$$p(y | \theta) = \exp\{y \log(\alpha / (1 + \alpha)) - \log(\alpha)\}$$

So we can identify the components as:

$$\text{Natural parameter } (\theta) = \log(\alpha / (1 + \alpha))$$

$$T(y) = y$$

$$A(\theta) = \log(\alpha)$$

Note: $\alpha = e^\theta / (1 - e^\theta)$, solving for α in terms of θ .

Let:

$$\theta = \log(\alpha / (1 + \alpha))$$

Then:

$$\alpha = e^\theta / (1 - e^\theta)$$

Now:

$$b(\theta) = -\log(\alpha) = -\log(e^\theta / (1 - e^\theta)) = -\theta + \log(1 - e^\theta)$$

- Natural parameter: $\theta = \log(\alpha / (1 + \alpha))$
- $b(\theta) = -\theta + \log(1 - e^\theta)$

Q.13)

Part (i) Option D [2]

Part (ii) Option B [2]

Q.14) Option C [4]

$$\text{Mean of Group A } (\bar{x}) = \Sigma x / n_1 = 205.45 / 35 = 5.87$$

$$\text{Mean of Group B } (\bar{y}) = \Sigma y / n_2 = 120.81 / 30 = 4.03$$

$$\text{Standard Error (SE)} = \sqrt{(\sigma_1^2/n_1 + \sigma_2^2/n_2)} = \sqrt{(1.5^2/35 + 1.0^2/30)} = 0.3124$$

$$\text{Z-value for 95\% confidence} = 1.96$$

$$\text{Difference in sample means } (\bar{x} - \bar{y}) = 1.84$$

$$\text{Confidence interval} = 1.84 \pm 1.96 \times 0.3124$$

$$\text{95\% Confidence Interval for } \mu_1 - \mu_2: (1.23, 2.46)$$

Q.15)

Part (i) Option B [2]

$$S_{xx} = \Sigma x^2 - (\Sigma x)^2 / n$$

$$S_{yy} = \Sigma y^2 - (\Sigma y)^2 / n$$

$$S_{xy} = \Sigma xy - (\Sigma x)(\Sigma y) / n$$

Then the correlation coefficient is:

$$r = S_{xy} / \sqrt{S_{xx} \times S_{yy}}$$

Using the given values:

$$S_{xx} = 86.5 - (14^2 / 8) = 62.0000$$

$$S_{yy} = 31.11 - (9.5^2 / 8) = 19.8287$$

$$S_{xy} = 34 - (14 \times 9.5 / 8) = 17.3750$$

$$r = S_{xy} / \sqrt{S_{xx} \times S_{yy}} = 0.4955$$

Part (ii) Option A

[4]

We test whether the true correlation coefficient ρ between tariff % change (x) and trade deficit change (y) is smaller than 0.5.

- Null Hypothesis (H_0): $\rho = 0.5$
- Alternative Hypothesis (H_1): $\rho < 0.5$

We transform both r and the null value $\rho_0 = 0.5$ using the Fisher Z-transform:

- $z(r) = 0.5 \times \ln((1 + r)/(1 - r)) = 0.5434$
- $z(\rho_0) = 0.5 \times \ln((1 + 0.5)/(1 - 0.5)) = 0.5493$
- $z = (z(r) - z(\rho_0)) \times \sqrt{n - 3}$
- $z = (0.5434 - 0.5493) \times \sqrt{(8 - 3)} = -0.0132$

We compute the left-tailed p-value for this one-sided test:

- p-value = $P(Z < -0.0132) = 0.4947$

At a significance level of 5% ($\alpha = 0.05$):

Since p-value = $0.4947 \geq 0.05$, we do not reject the null hypothesis.

Q.16)

Part (i) Option D

[3]

We are given:

$$\Sigma x = 89, \Sigma x^2 = 478, \Sigma y = 348.2, \Sigma y^2 = 4681.13, \Sigma xy = 1323.8, n = 30$$

$$\text{Mean of } x = 2.9667, \text{ Mean of } y = 11.6067$$

$$S_{xx} = \Sigma x^2 - (\Sigma x)^2/n = 213.9667$$

$$S_{xy} = \Sigma xy - (\Sigma x)(\Sigma y)/n = 290.8067$$

$$\beta = S_{xy} / S_{xx} = 1.3591$$

$$\alpha = \bar{y} - \beta \bar{x} = 7.5746$$

$$\text{Fitted regression line: } \hat{y} = 7.5746 + 1.3591x$$

Part (ii) Option A**[4]**

$$MSE = SSE / df_{res} = 8.7911$$

$$SE(\beta) = \sqrt{MSE / S_{xx}} = 0.2027$$

$$t\text{-critical} (df=28, 95\%) = 2.0484$$

$$CI = \beta \pm t * SE = (0.9439, 1.7743)$$

Part (iii) Option D**[2]**

$$\text{Residual Sum of Squares (SSE)} = SST - SSR = 641.44 - 395.29 = 246.15$$

Degrees of Freedom:

$$\text{Regression } df = 1$$

$$\text{Residual } df = 28$$

$$\text{Total } df = 29$$

Mean Squares:

$$MSR = SSR / df = 395.29 / 1 = 395.29$$

$$MSE = SSE / df = 246.15 / 28 = 8.7911$$

Source	DF	SS	MS
Regression	1	395.29	395.29
Residual	28	246.15	8.79
Total	29	641.44	

Part (iv) Option B**[2]**

$$R^2 = SSR / SST = 0.6163$$

Part (v) Option D**[4]**

H_0 : No linear relationship between x and y ($\beta = 0$)

H_1 : A linear relationship exists ($\beta \neq 0$)

$$F = MSR / MSE = 44.9649$$

Critical value at $\alpha = 0.05$ with $df_1 = 1$ and $df_2 = 28$ is $F_{0.05} = 4.1960$

Since $F = 44.9649 > F_{critical} = 4.1960$, we reject the null hypothesis.

Conclusion: There is a significant linear relationship between x and y.

Q.17)**Part (i) Option D****[2]****Part (ii) Option A****[2]**

Part (iii) Option C**[2]**

- Traffic Congestion (X_1): 3 categories $\rightarrow$ 2 dummy variables
- Temperature (X_2): 3 categories $\rightarrow$ 2 dummy variables
- Interaction ($X_1 * X_2$): $3 \times 3 = 9$ combinations $\rightarrow$ 8 dummy variables
- Parameters in A: 1 (Intercept only)
- Parameters in B: 3 (Intercept + 2 for Temperature)
- Degrees of Freedom = $3 - 1 = 2$
- Change in Deviance = $220 - 180 = 40$
- Chi-square critical value ($df=2, \alpha=0.05$) = 5.991

Conclusion: Since $40 > 5.991$, Model B is significantly better than Model A.

- Parameters in B: 3
- Parameters in C: 5 (2 from Temp + 2 from Traffic + Intercept)
- Degrees of Freedom = $5 - 3 = 2$
- Change in Deviance = $180 - 130 = 50$
- Chi-square critical value ($df=2, \alpha=0.05$) = 5.991

Conclusion: Since $50 > 5.991$, Model C is significantly better than Model B.

- Parameters in C: 5
- Parameters in D: 9 (8 for interaction + Intercept)
- Degrees of Freedom = $9 - 5 = 4$
- Change in Deviance = $130 - 125 = 5$
- Chi-square critical value ($df=4, \alpha=0.05$) = 9.488

Conclusion: Since $5 < 9.488$, Model D is not significantly better than Model C.

Q.18)**Part (i) Option C****[2]**

Assumptions:

- θ is the probability of a claim for a single policy in one month.
- There are 3 independent policies observed for n months $\Rightarrow$ total number of trials = $3n$.

- The number of claims observed over this period is X .
- Prior distribution: $\theta \sim \text{Beta}(\alpha, \beta)$
- Likelihood: $X \sim \text{Binomial}(3n, \theta)$

Likelihood function (ignoring constant):

$$L(\theta) \propto \theta^X (1 - \theta)^{(3n - X)}$$

Prior distribution (ignoring constant):

$$\pi(\theta) \propto \theta^{\alpha - 1} (1 - \theta)^{\beta - 1}$$

Posterior distribution:

$$\begin{aligned} \pi(\theta | X) &\propto \theta^X (1 - \theta)^{(3n - X)} \times \theta^{\alpha - 1} (1 - \theta)^{\beta - 1} \\ &= \theta^{(\alpha + X - 1)} (1 - \theta)^{(\beta + 3n - X - 1)} \end{aligned}$$

$\Rightarrow$ Posterior:

$$\theta | X \sim \text{Beta}(\alpha + X, \beta + 3n - X)$$

Part (ii) Option B

[2]

The Bayesian estimate is the mean of the posterior distribution:

$$E[\theta | X] = (\alpha + X) / (\alpha + \beta + 3n)$$

Part (iii) Option A

[2]

We write the Bayesian estimate in the credibility form:

$$E[\theta | X] = Z \times \hat{\theta}_{\text{MLE}} + (1 - Z) \times \hat{\theta}_{\text{Prior}}$$

Where:

- $\hat{\theta}_{\text{MLE}} = X / 3n$
- $\hat{\theta}_{\text{Prior}} = \alpha / (\alpha + \beta)$

Match this to:

$$E[\theta | X] = (\alpha + X) / (\alpha + \beta + 3n)$$

This gives the credibility factor Z as:

$$Z = 3n / (\alpha + \beta + 3n)$$

Part (iv) Option A

[2]

Given:

- $\alpha = 2, \beta = 4, n = 12 \Rightarrow 3n = 36$

Using the formula:

$$Z = 3n / (3n + \alpha + \beta)$$

$$Z = 36 / (36 + 2 + 4)$$

$$Z = 0.8571$$

Q.19)

Part (i) Option D [2]

Part (ii) Option B [2]

Part (iii) Option E [2]

Number of years (n) = 5

Categories (I) = 3

Category Totals:

- Category 1: $\Sigma X = 239$, $\Sigma X^2 = 11,875$
- Category 2: $\Sigma X = 268$, $\Sigma X^2 = 14,788$
- Category 3: $\Sigma X = 139$, $\Sigma X^2 = 4,111$

Calculate Individual and Overall Means

- Mean of Category 1 = $239 / 5 = 47.8$
- Mean of Category 2 = $268 / 5 = 53.6$
- Mean of Category 3 = $139 / 5 = 27.8$
- Overall mean = $(239 + 268 + 139) / 15 = 43.07$

Between-Group Variance (B)

$$B = (1 / (I - 1)) \times \Sigma [n(\bar{X}_i - \bar{X})^2]$$

$$= (1 / 2) \times [5(47.8 - 43.07)^2 + 5(53.6 - 43.07)^2 + 5(27.8 - 43.07)^2]$$

$$= 915.225$$

Within-Group Variance

- Category 1: $S^2 = 11,875 - 239^2/5 = 454.8$
- Category 2: $S^2 = 14,788 - 268^2/5 = 432.8$
- Category 3: $S^2 = 4,111 - 139^2/5 = 246.8$

$$\text{Average } S^2 = (454.8 + 432.8 + 246.8) / (3 \times 4) = 94.53$$

Credibility Factor Z

$$Z = n / (n + (S^2 / B)) = 5 / (5 + 94.53 / 915.225) = 0.9797$$

Final Expected Claims for Year 6

$$\text{Using: } \hat{\mu}_i = Z \times \bar{X}_i + (1 - Z) \times \bar{X}$$

- Category 1: $0.9797 \times 47.8 + 0.0203 \times 43.07 = 47.57$
- Category 2: $0.9797 \times 53.6 + 0.0203 \times 43.07 = 53.12$
- Category 3: $0.9797 \times 27.8 + 0.0203 \times 43.07 = 28.50$

Category	Expected Claims (Year 6)
Category 1	47.57
Category 2	53.12
Category 3	28.50
