

**INSTITUTE OF ACTUARIES OF INDIA**  
**EXAMINATIONS**

**February 2025**

**CM2 - Financial Engineering and Loss Reserving**  
**(Part A)**

**Time allowed: 3 Hours 15 Minutes**

**Total Marks: 100**

**INDICATIVE SOLUTION**

1. I (2)
2. III (2)  
III. a, b and c is correct open here as Statement d. applied to inefficient markets and not efficient ones
3. At the end of one year the investment will be worth  $1000(x + (1 - x) * 1.5) = 1500 - 500x$  with probability 0.6  
or  $1000(x + (1 - x) * 0.5) = 500 + 500x$  with probability 0.4  
Therefore, the expected utility is  $0.6 * \ln(1500 - 500x) + 0.4 * \ln(500 + 500x)$   
So, "IV" is the correct answer (2)
4. I (2)  
I.  $ct \leq St$  is correct as value of call option cannot be greater than share price and time in question is t and not t+1 or t-1
5. III (2)  
III. a, b and c is correct as "Unlimited short selling (that is, negative holdings) is allowed." Under Black-Scholes model and point d. says "Limited short selling (that is, negative holdings) is allowed"s
6. II (2)  
II. only b and c is correct as statements in points a and d are opposite and not true for price of an European put option on a dividend-paying share
7. II (2)  
The value of 10,000 put options is  $\$1.963 * 10000 = \$19,630$
8. I (2)  
Probability still solvent =  $0.95^3 = 0.8574$
9. IV (2)  
IV) A , B and C is correct as all statements are in favor of a stochastic model.
10. I (2)  
I. a is correct as Loss Ratio = the ratio of incurred claims to earned premiums over a defined period
11. III (2)  
III. Both A and B is correct as Variance is not just a method of downside risk
12. I (2)  
N has a Normal [1,1] distribution, so R has a Normal distribution with mean 150,000 and variance  $100,000^2$ , ie  $R \sim N: (150,000, 100,000^2)$  . So, the variance of return is  $100,000^2 = 10^{10}$

13. II (2)

Any normal distribution is symmetrical about its mean, so that the downside semi-variance of return is equal to half of the variance, ie  $5 \times 10^9$

14. III (2)

The shortfall probability below 50,000 is:

$$\begin{aligned} P(R < 50,000) &= P\left(\frac{R - 150,000}{100,000} < \frac{50,000 - 150,000}{100,000}\right) \\ &= \Phi\left(\frac{50,000 - 150,000}{100,000}\right) \\ &= \Phi(-1) \\ &= 1 - \Phi(1) = 1 - 0.84134 = 0.15866 \end{aligned}$$

15. I (2)

From the tables:

$$\Phi(-1.6449) = 0.05$$

So, there is 5% chance of the investment return 5 having value less than:

$$\begin{aligned} R_{5\%} &= \mu_R - 1.6449 \sigma_R \\ &= 150,000 - 1.6649 \times 100,000 \\ &= -14,490 \end{aligned}$$

So, the Value at Risk at the 95% confidence level is 14,490

16. I (2)

Equation of security market line is:  $E_P = r + (E_M - r)\beta_P$

17. III (2)

III. both b and c is correct because for the Binomial Model:

- Assets may be held in any amount
- There are no taxes or transaction costs.

18. III (2)

19. I (2)

20. I (2)

21. II (3)

We can express the portfolio proportions as functions of the portfolio expected return:

$$E = x_A E_A + x_B E_B = 6x_A + 11(1 - x_A)$$

$$x_A = \frac{11 - E}{5} \text{ and } x_B = \frac{E - 6}{5}$$

The two assets are independent, so the portfolio variance is:

$$V = x_A^2 V_A + x_B^2 V_B$$

Substituting in the portfolio proportions we get:

$$V = 5E^2 - 70E + 265$$

Finally, we square root to get the equation for the opportunity set in expected return-standard deviation space:

$$\sigma = \sqrt{5E^2 - 70E + 265}$$

The efficient frontier is the part of this curve above the point at which the variance is minimised. To find this point we differentiate:

$$\frac{dV}{dE} = 10E - 70 = 0$$

So the efficient frontier is the part of the opportunity set where  $E \geq \frac{70}{10} = 7\%$ , i.e.

$$\sigma = \sqrt{5E^2 - 70E + 265}, E \geq 7\%$$

22. IV (3)

$$\begin{aligned} (20/80)*5\% + (40/80)*10\% + (20/80)*x &= RM \\ x - 3\% &= 2(RM - 3\%) \Rightarrow x = 2*RM - 3\% \\ \Rightarrow 20*5\% + 40*10\% + 20*(2*RM - 3\%) &= 80*RM \\ \Rightarrow RM &= (20*5\% + 40*10\% - 20*3\%) / 40 = 11\% \end{aligned}$$

23. IV (3)

$$\begin{aligned} \text{We know that } E_i &= r + \beta_i * (EM - r) \\ \text{So } 3.5 &= r + 1.5 * (EM - r) \\ \text{And } 2.2 &= r + 0.2 * (EM - r) \\ \text{Subtracting the second equation from the first:} \\ 3.5 - 2.2 &= 1.3 * (EM - r) \\ \text{So } EM - r &= 1 \\ \text{Also subtracting 7.5 times the second equation from the first:} \\ 3.5 - 7.5 * 2.2 &= -6.5r \\ \text{So } r &= 2\% \end{aligned}$$

24. III (3)

$$\text{Semi-variance (2P\%)} = (6-0)^2 \times e^{-3} + (6-2)^2 \times e^{-3} \times \frac{3^1}{1!} + (6-4)^2 \times e^{-3} \times \frac{3^2}{2!} = 0.0508\% = 5.08\%$$

25. II (3)

$$\text{ProbB (5P < 5)} = \text{ProbB (P < 1)} = \text{ProbB (P = 0)} = 0.13534$$

26. I (3)

There are three possible stock prices at t=2

$$1. = \$80 \times 1.25 \times 1.25 = \$125$$

$$2. = \$80 \times 1.25 \times 0.8 = \$80$$

$$3. = \$80 \times 0.8 \times 0.8 = \$51.2$$

The corresponding payoffs are:

$$1. \$100$$

$$2. \$0$$

$$3. \$100$$

So the option price is:

$$V_0 = 100 \times (1.04)^{-2} \left[ \left( \frac{8}{15} \right) \left( \frac{8}{15} \right) + \left( \frac{7}{15} \right) \left( \frac{7}{15} \right) \right] = 46.4$$

27. II (3)

Cumulative claim amounts:

	0	1	2	3
2021	3620	6340	11400	14900
2022	4440	5600	7250	
2023	4870	6820		
2024	5180			

Development Factors:

$$DF(0,1) = (6340 + 5600 + 6820) / (3620 + 4440 + 4870) = 1.4509$$

$$DF(1,2) = (11400 + 7250) / (6340 + 5600) = 1.5620$$

$$DF(2,3) = 14900 / 11400 = 1.3070$$

Completed Cumulative Claims:

	0	1	2	3
2021				
2022				9476
2023			10653	13923
2024		7516	11739	15343

Outstanding claim amount for 2022: = (9476 – 7250) = 2226

28. III

(3)

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	0	1	2	3
2021				
2022				9476
2023			10653	13923
2024		7516	11739	15343

Outstanding claim amount for 2023: = (13923 – 6820) = 7103

29. III

(3)

$$\text{Cov}(1,M) = 60/200 \cdot \text{var}(1) + 40/200 \cdot \text{cov}(1,2) + 100/200 \cdot \text{cov}(1,3) = 0.00052$$

$$\text{Cov}(2,M) = 60/200 \cdot \text{cov}(1,2) + 40/200 \cdot \text{var}(2) + 100/200 \cdot \text{cov}(2,3) = 0.00178$$

$$\text{Cov}(3,M) = 60/200 \cdot \text{cov}(1,3) + 40/200 \cdot \text{cov}(2,3) + 100/200 \cdot \text{var}(3) = 0.00277$$

$$\begin{aligned}\text{Var}(M) &= (60/200)^2 \cdot \text{var}(1) + (40/200)^2 \cdot \text{var}(2) + (100/200)^2 \cdot \text{var}(3) + \\ & 2 \cdot (60/200) \cdot (40/200) \cdot 0.0006 + 2 \cdot (60/200) \cdot (100/200) \cdot 0.0005 + \\ & 2 \cdot (40/200) \cdot (100/200) \cdot 0.0026 \\ & = 0.001897\end{aligned}$$

$$\text{Beta}(1) = \text{cov}(1, M) / \text{var}(M) = 0.274$$

$$\text{Beta}(2) = \text{cov}(2, M) / \text{var}(M) = 0.938$$

$$\text{Beta}(3) = \text{cov}(3, M) / \text{var}(M) = 1.460$$

$$\text{Beta}(M) = 0.3 \cdot \text{Beta}(1) + 0.2 \cdot \text{Beta}(2) + 0.5 \cdot \text{Beta}(3) = 1$$

30. I (3)

$$\begin{aligned}E(B_s B_t) &= E(B_s ((B_t - B_s) + B_s)) \\ &= E(B_s)E(B_t - B_s) + \text{Var}(B_s) + (E(B_s))^2 \\ &= s\end{aligned}$$

31. IV (3)

32. I (3)

Black Scholes formula

$$f(t, S_t) = S_t \varphi(d_1) - k e^{-r(T-t)} \varphi(d_2)$$

Substituting values  $S_0, K, r, T$  [1]

$$f(0, S_0) = 1100 * \varphi(d_1) - 1200 e^{-0.02(1-0)} \varphi(d_2)$$

Calculating  $d_1, d_2$  - correct formula and substitution

$$d_1 = (\log(1100/1200) + (0.02 + 0.5\sigma^2)) / \sigma$$

$$d_2 = d_1 - \sigma$$

Guessing and Interpolation

With interpolation the final sigma = 7.96%.

33. II (3)

Options are priced by relative valuation techniques (i.e. risk neutral valuation).

This approach is equivalent to building a hedging strategy for the option and does not take account of the expected return on the share.

Since the hedging strategy involves holding some shares, the drop in price will result in a drop of the value of the option even though the expected future share price has remained the same

34. III (3)

- Call options with lower strike are more valuable  $A > C$
- American calls more valuable than European, if they have same strike and expiry,  $A > D$
- $B > E$
- American calls with longer expiry are more valuable,  $A > B$
- Possible to clearly rank  $A > B > E$ , others not so straight forward

35. I (3)

Factors affecting option price increases	American Call	American Put
Share price	Increase	Decrease
Exercise price	Decrease	Increase
Time to expiry	Increase	Increase
Volatility of share price	Increase	Increase
Risk free rate of interest	Increase	Decrease

36. II (3)

$$0.7610 = e^{-r}q/p \text{ if } S_1 = S_0u$$

$$1.5220 = e^{-r}(1 - q)/(1 - p) \text{ if } S_1 = S_0d$$

Solving the 2 equations with 2 unknown gives

$$e^r * .7610 * p = (1 - 1.5220(1 - p)) * e^r$$

$$= 1 - 1.5220 * e^r + 1.5220 * p * e^r$$

$$1.5220 * e^r - 1 = p * e^r * (1.5220 - .7610)$$

$$\text{This gives } p = 0.7748$$

37. I (3)

$$\text{Market price of risk} = \frac{E(R_m) - R_f}{\sigma_m}$$

$$R_f = 6\% - (4\% \times 20\%) = 6\% - 0.8\% = 5.2\%$$

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

$$E(R_i) = 5.2\% + 0.4\% = 5.6\%$$

38. IV (3)

$$\sigma_i = \frac{\text{Cov}(R_i, R_m)}{\rho \sigma_m}$$

Substitute the given values:

$$\text{Cov}(R_i, R_m) = 0.5 \times (0.2)^2 = 0.5 \times 0.04 = 0.02$$

Now, calculate the standard deviation:

$$\sigma_i = \frac{0.02}{0.7 \times 0.2} = \text{approx } 14\%$$

The standard deviation of the return on the company's share is approximately 14% per annum.

39. I (3)

$$U'(w) = w - 0.5 > 0 \text{ for } w > 0 \text{ – Not satiated}$$

$$U''(w) = -0.5w - 1.5 < 0 \text{ for } w > 0 \text{ – Risk Averse}$$

$$R(w) = w * (-U''(w)/U'(w)) = 0.5$$

$$== -w * (-0.5w - 1.5) / w - 0.5 = 0.5 \text{ - constant relative risk aversion}$$

40. II (3)

$$E(e^{tX}) = \int_0^{\infty} e^{tx} \left[ \frac{4}{2} e^{-4x} + \frac{7}{2} e^{-7x} \right] dx$$

$$\frac{4}{2(4-t)} + \frac{7}{2(7-t)}$$

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