INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

February 2025

CS2 - Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) In a two-state survival model with states "Alive (A)" and "Death (D)", the transition rate from state A to state D is given by $\mu(t)$, and the transition probability $p_{AA}(s,t)$ satisfies the backward differential equation:

$$\frac{\partial}{\partial s}p_{AA}(s,t) = -\mu(s)p_{AA}(s,t)$$

If we define the cumulative hazard function $H(t)$ as:

$$H(t) = \int_0^t \mu(u)du$$

which of the following statements are true about $p_{AA}(s,t)$ and its behavior under the given model?

a) $p_{AA}(s,t)$ can be expressed as $e^{-H(t-s)}$, assuming a stationary hazard rate.
b) If $\mu(t) = c$ (a constant), then $H(t) = ct$ and $p_{AA}(s,t) = e^{-c(t-s)}$.
c) The cumulative hazard $H(t)$ reflects the instantaneous probability of transitioning from A to D at time t.
d) $p_{AA}(s,t)$ is always a decreasing function of t, irrespective of $\mu(t)$.
e) For $\mu(t)$ to increase over time, $p_{AA}(s,t)$ must be non-linear in $H(t)$.

A) Options b and d are true
B) Options c and e are true
C) Options b, c and e are true
D) Options d alone is true
E) Options b, c, d and e are true **[3]**

Q.2) Consider the stochastic process

$$X_t = X_{t-1} + Z_t \textit{ where } Z_t \textit{ is defined as}$$
$$Z_t = \begin{cases} +1 \textit{ with a probability of } p \\ -1 \textit{ with a probability of } 1-p \end{cases}$$
$$X_0 = 0$$

If $P(X_{10} = 10) = p^{10}$ *and* If $P(X_2 = 2) = 0$, what does this imply about the stationarity of the process?

A) The process is strictly stationary.
B) The process is weakly stationary but not strictly stationary.
C) The process is neither strictly stationary nor weakly stationary.
D) The process is strictly stationary only for small time lags.
E) The process has a constant mean, so it must be weakly stationary. **[2]**

Q.3) A stochastic process X_t is defined such that:

$X_t = aX_{t-1} + Z_t$, where $Z_t \sim N(0,\sigma^2)$ and $|a| < 1$.

The process starts with $X_0 = 0$.

You are given the following information:

- $\mathbb{E}[X_t] = 0$ for all t.
- The autocovariance function is $\gamma(h) = \frac{\sigma^2 a^{|h|}}{1-a^2}$.

Which of the following statements are true regarding the weak stationarity of X_t ?
a) X_t is weakly stationary because the autocovariance function depends only on $h = |t-s|$.
b) X_t is not weakly stationary because $X_0 = 0$, making the mean time-dependent.
c) X_t is weakly stationary only if σ^2 is constant over time.

d) X_t is not weakly stationary because the variance $\gamma(0)$ grows with t.
e) X_t is weakly stationary only if $a > 0$.

A) Statements (a) and (d) are true
B) Statements (a), (c) and (d) are true
C) Statements (a) alone is true
D) Statements (c) and (e) are true
E) None of the above statements is true. **[2]**

Q.4) Why is it important for an insurance company to check if its current mortality experience is consistent with past experience or published life tables?

I. To ensure the mortality rates increase with age.
II. To adjust investment strategies based on future claims.
III. To verify if premiums are appropriately set for profitability and competitiveness.
IV. To avoid regulatory penalties.
V. To assess if pricing assumptions align with observed mortality trends.

A) Both I and II are correct
B) Both III and V are correct
C) Both III and IV are correct
D) I, III, and V are correct
E) Both II and III are correct **[2]**

Q.5) A tech company introduces a new line of high-end smartwatches, offering a 1-year warranty to cover any hardware defects. Each watch costs Rs. 1,200 to repair or replace. To estimate the cost of providing the warranty, the company tracks 500 smartwatches sold in January 2023. Customers are contacted every quarter for a year to report any hardware defects. Here is the defect report:

- End of Q1 (March 31): 5 defects reported, 25 customers unreachable.
- End of Q2 (June 30): 8 defects reported, 15 more customers unreachable.
- End of Q3 (September 30): 4 defects reported.
- End of Q4 (December 31): 6 defects reported.

The company expects to sell 5,000 smartwatches annually, and the Nelson-Aalen method will be used to estimate the cumulative hazard function.

Use the data provided to estimate the survival probability S(t) for a smartwatch lasting beyond 1 year.

A) 0.9481
B) 0.9517
C) 0.9254
D) 0.9571
E) 0.9456 **[3]**

Q.6) A drone is flying through a city of skyscrapers, represented as rooms connected by walkways. At each walkway, it randomly selects a path to another building. The city is mapped out into six distinct buildings (states), and each building has multiple walkways leading to different locations. The drone starts in Building 1 and its goal is to reach Building 5, where it will land to recharge.

The drone can move between buildings according to the following rules:

- From Building 1, it can only go to Building 3.
- From Building 2, it can only go to Building 3.
- From Building 3, it can move with equal probability to Building 1, 2, 4, or 5.
- From Building 4, it can move with equal probability to Building 3 or 6.
- From Building 5, it can move with equal probability to Building 3 or 6.
- From Building 6, it can move with equal probability to Building 4 or 5.

Is the Markov chain irreducible and aperiodic?

A) The Markov chain is irreducible but not aperiodic because there is no power of PPP where all entries are strictly positive.
B) The Markov chain is neither irreducible nor aperiodic because the drone cannot reach all states from every state.
C) The Markov chain is irreducible and aperiodic because the drone can eventually reach every building and the period of each state is 1.
D) The Markov chain is irreducible but not aperiodic because some states only have access at even-numbered steps.
E) The Markov chain is aperiodic but not irreducible because the drone must pass through Building 3 to reach certain states. **[3]**

Q.7) Suppose an insurance portfolio can generate at most 3 claims per year, where the number of claims N follows the probability mass function:
P(N=0)=0.4, P(N=1)=0.3, P(N=2)=0.2, P(N=3)=0.1

Each claim is either for an amount of 1, 2, or 3, with probabilities P(X=1)=0.5; P(X=2)=0.3; and P(X=3)=0.2 independently of each other and the number of claims.
Determine the approximate value of P(S≤3), where S is the total aggregate claim amount in a year.

A) 0.7525
B) 0.8225
C) 0.8875
D) 0.9225
E) 0.9675 **[3]**

Q.8) An insurance company models the survival function $S(t)$ of a policyholder using a proportional hazards model:

$$h(t \mid Z) = h_0(t)e^{\beta Z}$$

where Z is a covariate and $\beta = 0.4$.
The baseline survival function is $S_0(t) = e^{-0.02}$. Calculate the median survival time for a policyholder with $Z = 2$.
A) 14.9
B) 15.6
C) 12.8
D) 11.1
E) 13.9 **[2]**

Q.9) Consider a process for two variables X_t and Y_t :

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 1 & 0.5 \\ -0.3 & 1 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} 0.4 & 0.1 \\ 0.2 & 0.3 \end{bmatrix} \begin{bmatrix} X_{t-2} \\ Y_{t-2} \end{bmatrix} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{bmatrix}$$

Determine the stability of the system.

A) Stable
B) Marginally Stable
C) Unstable
D) Conditionally Stable
E) Cannot be determined. **[2]**

Q.10) Consider the Gamma(2, 1) distribution and the Pareto(3, 1) distribution. Which of the following describes the limiting density ratio of the two distributions as $x \to \infty$?

A) The limiting density ratio is 1, meaning the tails are of equal thickness.
B) The limiting density ratio is 0, meaning the Gamma distribution has a lighter tail.
C) The limiting density ratio is infinity, meaning the Gamma distribution has a thicker tail.
D) The limiting density ratio is 1, meaning both distributions have equally heavy tails.
E) The limiting density ratio is a positive constant, but not 1. **[2]**

Q.11) Suppose the current mortality rate for lives aged 65 years is $m_{65,0}$=0.008, and the minimum possible mortality rate is believed to be α_{65}=0.0016. It is estimated that 50% of the maximum possible reduction in mortality will have occurred by 15 years. Using the appropriate reduction factor, calculate the projected mortality rate at age 65 years in 30 years.

A) 0.0020
B) 0.0025
C) 0.0030
D) 0.0035
E) 0.0040 **[3]**

Q.12) An innovative skyscraper in a metropolitan city has a special elevator that only moves when it has collected 4 passengers. Passengers arrive at the elevator boarding station according to a Poisson process with a rate of λ=1/10 per minute.
What is the probability that the elevator does not move in the first 90 minutes (i.e., fewer than 4 passengers arrive in the first 90 minutes)?
Options:
A) 1.5%
B) 2.1%
C) 3.5%
D) 5.0%
E) 7.0% **[3]**

Q.13) An insurance company in Brazil has been using a standard mortality table since 2012 for its product pricing. After several years of gathering experience data, the company's Chief Actuary wants to perform a chi-squared test at a 5% confidence level to validate the suitability of the standard mortality table for individuals aged 60 to 65. The experience data and the standard mortality table rates are summarized as follows:

Age Group	Central Exposed to Risk	Number of Deaths in Sample	Standard Mortality Rate Used
60-61	47,000	370	0.0080
62-63	43,000	380	0.0095
64-65	40,000	390	0.0110

What is the chi-squared test statistic to determine if the mortality assumptions should be updated?

A) 3.25
B) 5.98
C) 4.62
D) 6.11
E) 7.77

[2]

Q.14) Two risks X and Y have marginal distributions $F_X(x) - 1 - e^{-x}, F_Y(y) - 1 - e^{-2y}$, and are modeled using a Clayton copula:

$$C(u,v) = \left(u^{-\theta} + v^{-\theta} - 1\right)^{-1/\theta}, \theta > 0$$

If $\theta = 2$, calculate the joint probability $P(X \leq 1, Y \leq 0.5)$.

A) 0.632
B) 0.891
C) 0.315
D) 0.499
E) 0.444

[3]

Q.15) Consider a time series given by the following relationship: $X_t=2.5+0.4\epsilon_t+0.3\epsilon_{t-1}+0.2\epsilon_{t-2}+0.1\epsilon_{t-3}$ where ϵ_t denotes white noise with variance σ^2. What is the variance of X_t?

A) $0.32\sigma^2$
B) $0.55\sigma^2$
C) $0.35\sigma^2$
D) $0.30\sigma^2$
E) $0.75\sigma^2$

[2]

Q.16) A bank has developed a machine learning model to predict whether a loan application should be approved or not. After testing the model, the bank generated the following confusion matrix:

	Predicted Yes	Predicted No
Actual Yes	120	30
Actual No	10	40

What is the F1 score of this model?

A) 80.4%
B) 85.6%
C) 78.2%

D) 81.8%
E) 87.4% **[2]**

Q.17) Let $T(t)$ denote the temperature at time t in a specific city. The temperature increments over nonoverlapping time intervals are independent. Suppose further that the process is defined by the following stochastic differential equation (SDE):

$$dT(t) = \mu(T(t), t)dt + \sigma(T(t), t)dW(t)$$

where $W(t)$ is a standard Brownian motion, and $\mu(T(t), t)$ and $\sigma(T(t), t)$ are continuous functions of $T(t)$ and t.
Which of the following statements are correct?
a) $T(t)$ satisfies the Markov property regardless of the choice of $\mu(T(t), t)$ and $\sigma(T(t), t)$.
b) $T(t)$ has stationary increments only if $\mu(T(t), t)$ and $\sigma(T(t), t)$ are independent of $T(t)$.
c) If $\mu(T(t), t) = \alpha T(t)$ and $\sigma(T(t), t) = \beta T(t)$, then $T(t)$ represents a geometric Brownian motion.
d) $T(t)$ represents a standard Brownian motion only if $\mu(T(t), t) = 0$ and $\sigma(T(t), t) = 1$.
e) The process $T(t)$ cannot have stationary increments if $\mu(T(t), t)$ or $\sigma(T(t), t)$ depends on $T(t)$

A) Statements a and e are true
B) Statements a, b, c and e are true
C) Statements b, c and e are true
D) Statements a, b, c, d and e are true
E) Statements b and c are true **[3]**

Q.18) Suppose the survival times T_i follow an exponential distribution with hazard rate $\lambda = 0.05$. If three individuals are censored after $t_1 = 10, t_2 = 12, t_3 = 15$, find the likelihood function for λ.

A) 0.951
B) 0.156
C) 0.397
D) 0.049
E) 0.468 **[2]**

Q.19) You are given the following covariance matrix of two features:

$$\mathbf{\Sigma} = \begin{bmatrix} 4 & 3 \\ 3 & 5 \end{bmatrix}$$

Find the eigenvalues of $\mathbf{\Sigma}$.

A) 6.854 and 2.146
B) 6.236 and 2.764
C) 5.400 and 3.600
D) 7.541 and 1.459
E) None of the above **[2]**

Q.20) A company insures three types of vehicles: Cars, Trucks, and Motorcycles. The number of claims per year follows a Poisson distribution with parameter λ, and the claim amounts are exponentially distributed with a mean of 10 units. The value of λ, depending on the vehicle type, is as follows:

Vehicle Type	λ	Proportion of Vehicles
Cars	3.0	0.50
Trucks	4.0	0.30
Motorcycles	5.0	0.20

Let S represent the total claim amount for one vehicle in a year. Which of the following steps is necessary to calculate Var(S)?

A) Use the law of total variance: $Var(S)=Var[E(S|\lambda)]+E[Var(S|\lambda)]$.
B) Use $Var(S)=E(S^2)-E(S)^2$, and calculate both E(S) and $E(S^2)$.
C) Use the variance of the Poisson distribution for $Var(N|\lambda)$
D) Calculate the variance of the exponential claim distribution and sum it across vehicle types.
E) Use $Var(S)=E(\lambda^2)-E(\lambda)^2$.

[2]

Q.21) Imagine a financial market where a trader has access to insider information about a stock price process X_t. The natural filtration F_t represents the information available to the market up to time t. The insider, however, has access to a larger filtration G_t, where $F_t \subset G_t$ for all t. The stock price process is modeled as a geometric Brownian motion under the filtration F_t but may have some additional randomness under G_t.
Which of the following best describes the relationship between the insider's filtration G_t and the stock price's natural filtration F_t?

A) The insider's filtration G_t contains more information than F_t, but this does not impact the stock price distribution.
B) The insider's filtration G_t contains more information than F_t, and this can impact the trader's ability to predict future stock prices.
C) The insider's filtration G_t only impacts the stock price if the stock process is Markovian.
D) The insider's filtration G_t is irrelevant if the stock process has independent increments.
E) The insider's filtration G_t affects the stock price if and only if the process is not adapted to F_t.

[2]

Q.22) Under the Gompertz law of mortality, the force of mortality μ_x at age x is given by:

$$\mu_x = B \cdot e^{Cx}$$

where $B = 0.0005$ and $C = 0.1$.
Calculate the survival probability ${}_5p_{40}$, i.e., the probability that an individual aged 40 survives the next 5 years.

A) 0.8551
B) 0.7607
C) 0.8377
D) 0.6723
E) 0.9405

[2]

Q.23) In a particular country, two terminal diseases, Disease Y and Disease Z, are significant causes of death for men aged 70 and older. The Markov jump model of the health process for males aged 70 shows annual constant transition rates between the states: Healthy (H), Disease Y (Y), Disease Z (Z), and Dead (D).

For a healthy male aged exactly 70, the following transition rates are provided:

- From Healthy to Disease Y: μ_{HY}=0.005
- From Healthy to Disease Z: μ_{HZ}=0.007
- From Disease Y to Dead: μ_{YD}=0.4
- From Disease Z to Dead: μ_{ZD}=0.7
- From Healthy directly to Dead: μ_{HD}=0.014

What is the probability that a healthy male aged exactly 70 will die in the coming year, considering all possible transition paths?

A) 0.0102
B) 0.0166
C) 0.0089
D) 0.0140
E) 0.0204

[4]

Q.24) You are analyzing a financial time series, where the data from n=600 realizations has been observed. The following summary statistics are given:

- $\sum_{i=1}^{600} x_i = 14{,}520.60$
- $\sum_{i=1}^{600} (x_i - \bar{x})^2 = 4{,}256.20$
- $\sum_{i=1}^{599} (x_i - \bar{x})(x_{i+1} - \bar{x}) = 3{,}122.50$

The time series follows the model:
$X_t=\mu+\alpha_1(X_{t-1}-\mu)+\epsilon_t$ where ϵt is a white noise process with variance σ^2.
Estimate the parameters μ, α_1 , and σ^2 using the data provided.

A) μ=24.20, α_1=0.725,σ^2=2.234
B) μ=24.20, α_1=0.733,σ^2=1.876
C) μ=24.20, α_1=0.733,σ^2=1.964
D) μ=23.90, α_1=0.720,σ^2=2.145
E) μ=24.10, α_1=0.733,σ^2=1.884

[3]

Q.25) An association provides group life insurance to its employees, consisting of two types of members who are entitled to different benefit amounts:

	Number of Members	Benefit Amount	Probability of Dying During the Year
Regular Members	1,500	INR 40,000	0.010
Special Members	300	INR 25,000	0.015

Calculate the coefficient of skewness based on your results.

A) 0.272
B) 0.285
C) 0.293
D) 0.305
E) 0.256

[4]

Q.26) You are tasked with creating a mortality table for pensioners aged 65 to 95 in 15 years' time. Two types of models are being considered: an age-period model and an age-cohort model. The following data are available:

- The organization has 10 years of past mortality data for pensioners aged 60 to 90.
- Mortality rates are generally expected to improve over time, but there is uncertainty regarding how improvements will affect different cohorts.

Which model is likely to provide more accurate mortality projections for pensioners aged 65 to 95 in 15 years' time, and why?

A) The age-period model will provide more accurate projections because it focuses on calendar time, which is most important for short-term forecasts.
B) The age-cohort model will provide more accurate projections, as it allows you to capture cohort-specific mortality improvements, which are more relevant for pensioners.
C) The age-period model is preferred because the available 10 years of data may not be sufficient to assess cohort effects accurately.
D) The age-cohort model will provide more accurate projections if additional data from younger cohorts are collected.
E) The three-factor model (age, period, and cohort) will provide more accurate projections, but requires overcoming the issue of factor dependence. **[2]**

Q.27) In a factory, the time until breakdown of a boiler is exponentially distributed with parameter λ, depending on how many times the boiler has broken down before:

$$\lambda = \begin{cases} \frac{1}{6} \text{ if the boiler has not previously broken down,} \\ \frac{1}{4} \text{ if the boiler has broken down once previously} \\ \frac{1}{2} \text{ if the boiler has broken down m}\textit{ore than}\text{ once previously} \end{cases}$$

Once a boiler has broken down 5 times, it is scrapped. If it breaks down fewer than 5 times, it is repaired immediately.

Calculate the expected lifetime of a new boiler before it is scrapped.

A) 13 years
B) 14 years
C) 16 years
D) 12.5 years
E) 15 years **[3]**

Q.28) Consider a time series X_t . Express $X_{t+3}-4X_{t+1}+X_{t-2}$ in terms of second-order differences.

A) $X_{t+3}-4X_{t+1}+X_{t-2}=\Delta^2X_t+3\Delta X_t$
B) $X_{t+3}-4X_{t+1}+ X_{t-2}=\Delta^2X_{t+1}+3\Delta X_{t-1}$.
C) $X_{t+3}-4X_{t+1}+ X_{t-2}=2\Delta^2X_t+\Delta X_{t-2}$
D) $X_{t+3}-4X_{t+1}+ X_{t-2}=2\Delta^2X_t+3\Delta X_t$
E) $X_{t+3}-4X_{t+1}+ X_{t-2}=3\Delta^2X_t-2\Delta X_t$ **[2]**

Q.29) A company tracks the daily net profit from its stock trading activities. The profit X_t on day t follows a discrete-time stochastic process defined as:
$X_t=X_{t-1}+I_t$

where I_t is a random variable that takes the value +1 with probability 0.4 (profit) or -1 with probability 0.6 (loss). Assume the initial profit $X_0=100$.
Does this process exhibit independent increments?
Does this process have the Markov property?

A) Yes, the process has independent increments; hence, it satisfies the Markov property.
B) No, the process does not have independent increments because each increment depends on the previous one, but it still has the Markov property.
C) Yes, the process has independent increments, but it does not satisfy the Markov property due to its time dependence.
D) No, the process neither has independent increments nor satisfies the Markov property.
E) Yes, the process has independent increments, and as a result, it only partly satisfies the Markov property. **[2]**

Q.30) An actuary is modelling temperature variations using the model:
$(1-B^3)(1-(\alpha+\beta)B+\alpha\beta B^2)X_t=e_t$
where B is the backward shift operator, and e_t is a white noise process. After appropriate seasonal differencing, assume $\alpha=0.5$ and $\beta=0.3$
Write down the Yule-Walker equations for this model.

A) $\rho1-0.8+0.15\rho1=0$; $\rho2-0.8\rho1+0.15=0$
B) $\rho1-0.6+0.2\rho1=0$; $\rho2-0.7\rho1+0.25=0$
C) $\rho1-0.9+0.15\rho1=0$; $\rho2-0.9\rho1+0.25=0$
D) $\rho1-0.6+0.15\rho1=0$; $\rho2-0.8\rho1+0.15=0$
E) $\rho1-0.7+0.25\rho1=0$; $\rho2-0.8\rho1+0.15=0$ **[2]**

Q.31) A portfolio consists of 5,000 independent claims, where each claim amount X follows an exponential distribution with mean 10,000. The insurer has a deductible d=5,000 and proportional reinsurance with 60% retention. Compute the expected claim payment by the insurer for a single claim.

A) 2,264
B) 3,000
C) 3,324
D) 2,192
E) 3,639 **[3]**

Q.32) A clinical trial is conducted to evaluate a new drug. The trial begins with 20 patients, and their survival times are recorded over 18 months. The following events are observed:

- 3 months: 2 deaths
- 6 months: 1 patient censored
- 9 months: 3 deaths
- 12 months: 2 deaths
- 15 months: 1 patient censored
- 18 months: 1 death

What is the Kaplan-Meier estimate of the survival probability at 12 months?
Options:

A) 0.537
B) 0.635
C) 0.568
D) 0.685
E) 0.742 **[3]**

Q.33) A company uses three machine learning models in an ensemble to predict whether a customer will default on a loan. Each model has an individual accuracy as follows:

- Model 1: 85%
- Model 2: 90%
- Model 3: 80%

Assuming that predictions by each model are independent, what is the probability that the ensemble correctly predicts a non-default if the final decision rule is "at least two models must agree on the prediction"? Options:

A) 88.4%
B) 92.6%
C) 90.3%
D) 91.4%
E) 89.2% **[3]**

Q.34) A time series consisting of monthly sales data is modeled using an ARMA(2,1) process. The sample autocorrelation coefficients of the residuals of the model are provided for the first 6 lags. The Ljung-Box test is performed to verify if the residuals follow white noise.

Lag	SACF Estimate
1	0.05
2	-0.10
3	0.07
4	-0.04
5	0.03
6	-0.02

There are 300 observations in total.
Calculate the Ljung-Box test statistic for 6 lags and determine whether the residuals can be considered white noise. Use a 5% significance level. Options:

A) Test statistic = 5.72, reject the null hypothesis of white noise.
B) Test statistic = 6.98, cannot reject the null hypothesis of white noise.
C) Test statistic = 8.21, reject the null hypothesis of white noise.
D) Test statistic = 4.45, cannot reject the null hypothesis of white noise.
E) Test statistic = 7.32, cannot reject the null hypothesis of white noise. **[3]**

Q.35) A data analyst is studying the relationship between the daily returns of two financial assets, X and Y, over time. The analyst introduces a third variable W, which represents market-wide factors, and wishes to check the covariance between the sum of X and Y and the new variable W.
Under what conditions is the equation $Cov(X+Y,W)=Cov(X,W)+Cov(Y,W)$ true?
Options:

A) The equation will be true only if W is weakly stationary.
B) The equation will always be true, regardless of whether W is stationary.
C) The equation will not be true under any circumstances.
D) The equation will be true only if W is strictly stationary.
E) The equation will only hold when X, Y, and W are mutually independent. **[2]**

Q.36) If $p_{WW}^{t,x}$ represents the probability that an individual is still working after t years, what is the correct expression for this probability, assuming constant decrement rates μ and ν (retirement rate)?

A) $p_{WW}^{t,x} = e^{-(\mu+\nu)t}$
B) $p_{WW}^{t,x} = e^{(\mu+\nu)t}$
C) $p_{WW}^{t,x} = e^{-(\mu-\nu)t}$
D) $p_{WW}^{t,x} = 1 - e^{-(\mu+\nu)t}$
E) $p_{WW}^{t,x} = e^{-(\mu\nu)t}$ **[2]**

Q.37) Consider a stochastic process X_t defined as follows:
$X_t = e_t + 0.5e_{t-1} + 0.2e_{t-2}$
where e_t is white noise with mean 0 and variance σ^2.
Which of the following statements about X_t is true?

A) X_t is a white noise process with zero mean and constant variance.
B) X_t is a stationary process but not white noise.
C) X_t has no memory of past values and is purely random.
D) X_t is a non-stationary process because it depends on lagged terms.
E) X_t is a Markov process of order 2. **[2]**

Q.38) SL is developing a new product for couples that pays a death benefit upon the second death. The survival probability for each life is ${}_{10}p_{70}=0.65$. If deaths are assumed to be independent, what is the probability of paying the death benefit within 10 years? Additionally, if the dependence between the two lives is modelled using a Clayton copula with α=0.5, what is the change in the probability of paying the benefit compared to the independent case?

A) Independent: 0.1225, Clayton: 0.1448, Increase: 0.0223
B) Independent: 0.2375, Clayton: 0.2622, Increase: 0.0247
C) Independent: 0.4225, Clayton: 0.4401, Increase: 0.0176
D) Independent: 0.1225, Clayton: 0.1347, Increase: 0.0122
E) Independent: 0.2375, Clayton: 0.2568, Increase: 0.0193 **[3]**

Q.39) An insurance company collects data on deaths for individuals aged 60 nearest birthday during the calendar year 2023. These deaths occurred for individuals aged between 59½ and 60½. Using this information:

1. What is the rate interval for estimating the force of mortality, μ?
2. What is the age at which the force of mortality, μ, can be estimated?
3. What formula should be used to estimate the probability of death, q, for this group?

A) Rate interval: [59, 60], Age for μ: 59.5, $q \approx 1-e^{-\mu_{60}}$
B) Rate interval: [59.5, 60.5], Age for μ: 60, $q \approx 1-e^{-\mu_{59.5}}$
C) Rate interval: [59, 60], Age for μ: 60, $q \approx 1-e^{-\mu_{59}}$
D) Rate interval: [59.5, 60.5], Age for μ: 60.5, $q \approx 1-e^{-\mu_{59.5}}$

E) Rate interval: [60, 61], Age for μ: 60, $q \approx 1 - e^{-\mu 60}$ **[2]**

Q.40) Suppose a linear regression model is trained with L2 regularization (ridge regression). The loss function is:

$$L(\beta) = \sum_{i=1}^{n} \left(y_i - \mathbf{x}_i^{\top}\beta\right)^2 + \lambda\|\beta\|_2^2$$

where $\lambda - 5$. The design matrix **X** and target vector **y** are given as:

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Find the optimal ridge regression coefficients β.

A) 0.120 and 0.430
B) 0.218 and 0.302
C) 0.254 and 0.587
D) 0.010 and 0.042
E) 0.462 and 0.842 **[3]**

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