

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

February 2025

CM2 - Financial Engineering and Loss Reserving
(Part A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Q.1) Which of the following is true for weak form of Efficient Market Hypothesis?

- I. market prices reflect all of the information contained in historical price data.
- II. market prices reflect all publicly available information
- III. market prices reflect all information, whether or not it is publicly available.
- IV. market prices reflect some of the information contained in historical price data

[2 marks]

Q.2) Which of the following statement(s) is(are) correct for “Active Investment management not being justified according to Efficient Market Hypothesis”

- a. It is impossible to exploit the mispricing of securities in order to generate higher expected returns
- b. Even if price anomalies exist, then the costs of identifying them and then trading will outweigh the benefits arising from the additional investment returns
- c. If active investment management cannot be justified, then a more appropriate investment strategy might be simply to match or ‘track’ the market
- d. We would expect active managers with above average skill to perform better than passive managers

- I. a & b only
- II. a & c only
- III. a, b and c
- IV. a, b, c & d

[2 marks]

Q.3) An investor makes decisions using the utility function $U(w) = \ln(w)$ where $w > 0$. The investor is going to invest \$1000 now for a period of 1 year, and has identified the following two assets to invest in:

- Asset A is risk-free and will not change in value over the year.
- Asset B will increase in value by 50% over the year with probability 0.6 or decrease in value by 50% over the year with probability 0.4.

The investor does not make any allowance for discounting when making investment decisions. They are going to invest a proportion, x , of their wealth in Asset A and the remaining proportion, $(1 - x)$, in Asset B.

Which of the following is the correct formula, in terms of x , for their expected utility at the end of the year

- I. $0.6 * \ln(1500 - 500x)$
- II. $0.4 * \ln(1500 - 500x) + 0.6 * \ln(500 + 500x)$
- III. $0.4 * \ln(1500 - 500x)$
- IV. $0.6 * \ln(1500 - 500x) + 0.4 * \ln(500 + 50x)$

[2 marks]

Q.4) A non-dividend paying share, with price S_t at time t , has a European call option written on it with value c_t at time t . The call matures at time T and has a strike price of K . The continuously compounded risk-free rate is r . The upper bound for the value of the call option, c_t is

- I. $c_t \leq S_t$
- II. $c_{t+1} \leq S_{t+1}$
- III. $c_{t-1} \leq S_{t-1}$
- IV. $S_t \leq c_t$

[2 marks]

- Q.5)** Which of the following are the assumptions underlying the Black-Scholes model:
- The price of the underlying share follows a geometric Brownian motion.
 - There are no risk-free arbitrage opportunities.
 - The risk-free rate of interest is constant, the same for all maturities and the same for borrowing or lending.
 - Limited short selling (that is, negative holdings) is allowed.

- a & b only
- a & c only
- a, b and c
- a, b, c & d

[2 marks]

- Q.6)** Which of the following are true for the price of an European put option on a dividend-paying share
- The price would increase as the underlying share price increased
 - The price would increase as the strike price increased.
 - The price would increase as the dividend rate increased.
 - The price would increase as the volatility of the underlying share decreased.

- a, b and c
- only b and c
- only c and d
- all a, b, c and d

[2 marks]

- Q.7)** A bank sells 10,000 European put options on a non-dividend paying share St. The initial share price is $S_0 = \$17$ and the strike price of the option is \$19 at expiry in 2 years' time. The share has volatility 15% p.a. and the risk-free force of interest is 3% p.a.

The option price is \$1.963 and the Delta of the option is -0.446 . What is the initial income for the bank from selling the options

- 17000
- 19630
- 19000
- 20180

[2 Marks]

- Q.8)** A bank has received a loan application from a customer to borrow £8,000 for 3 years. The loan will be repaid by a single lump sum at the end of the 3 years with compounded interest. The bank estimates that there is a 5% chance the customer will be declared insolvent in each future year. If the customer is declared insolvent before the loan is repaid, the bank will receive nothing.

What is the probability that the customer will still be solvent when the loan Matures?

- 0.8574
- 0.8674
- 0.95
- 0.33

[2 marks]

Q.9) A company specialises in long-term insurance contracts. It holds reserves in respect of these contracts and it wants to build a model of its future investment returns. Why the company may prefer to use a stochastic model.

- A. A stochastic model avoids this issue by assuming a range of possible outcomes could apply
- B. This is particularly useful for the insurer because they hold long-term contracts
- C. For a deterministic model, deciding which set of input variables to use may be a challenge

- I) A and B only
- II) A and C only
- III) B only
- IV) A , B and C

[2 marks]

Q.10) The term ‘loss ratio’ as used in the Bornhuetter–Ferguson method for estimating outstanding claim amounts means

- a. the ratio of incurred claims to earned premiums over a defined period
- b. the ratio of to be incurred claims to earned premiums over a defined period
- c. the ratio of outstanding claims to earned premiums over a defined period
- d. the ratio of incurred claims to future premiums over a defined period

- I. a
- II. b
- III. c
- IV. d

[2 marks]

Q.11) Which of the following are true for Variance as a Measure of Investment Risk

- a. It is mathematically tractable.
- b. leads to elegant solutions for optimal portfolios, within the context of mean-variance portfolio theory.
- c. It is a measure of downside risk of low investment returns

- I. Only A
- II. Only b
- III. Both A and B
- IV. All of the above

[2 marks]

Q.12) An investor is contemplating an investment with a return of R , where:
 $R = 250,000 - 100,000N$

and N is a Normal $[1,1]$ random variable.

Calculate variance of return

- I. 10^{10}
- II. 100^{10}
- III. 1000^{10}
- IV. 10000^2

[2 marks]

Q.13) An investor is contemplating an investment with a return of R , where:

$$R = 250,000 - 100,000N$$

and N is a Normal $[1,1]$ random variable.

Calculate Downside Semi-variance of return

- I. 5×10^8
- II. 5×10^9
- III. 5×10^7
- IV. 5×10^{10}

[2 marks]

- Q.14)** An investor is contemplating an investment with a return of R , where:
 $R = 250,000 - 100,000N$

and N is a Normal $[1,1]$ random variable.

Calculate the shortfall probability, where the shortfall level is £50,000

- I. 0.15466
- II. 0.15266
- III. 0.15866
- IV. 0.16866

[2 marks]

- Q.15)** An investor is contemplating an investment with a return of R , where:
 $R = 250,000 - 100,000N$

and N is a Normal $[1,1]$ random variable.

Calculate the Value at Risk at the 95% confidence level

- I. 14,490
- II. 15,675
- III. 15,890
- IV. 14,580

[2 marks]

- Q.16)** Equation of Security Market Line is

- I. $E_P = r + (E_M - r)\beta_P$
- II. $E_M = r + (E_P - r)\beta_P$
- III. $E_P = r + (E_M - r)\beta_M$
- IV. $E_P = (E_M - r)\beta_P$

[2 marks]

- Q.17)** Which of the following is NOT an assumption of The Binomial Model
- a. Assets may be bought and sold at integer times.
 - b. Assets can be held only in limited amounts.
 - c. There are limited taxes or transaction costs
 - d. There are no arbitrage opportunities.

- I. only a
- II. only b
- III. both b and c
- IV. both a and b

[2 marks]

Q.18) Choose the method(s) in which inflation of claims is allowed for explicitly:

- a. Basic chain-ladder method
- b. Bornhuetter-Ferguson method
- c. Inflation adjusted chain-ladder method

- I. c only
- II. a and b only
- III. a only
- IV. b and c only

[2 marks]

Q.19) Claims occur on a portfolio of insurance policies according to a Poisson process at a Rate λ . All claims are for a fixed amount d , and premiums are received continuously. The insurer's initial surplus is U ($< d$) and the annual premium income is $1.4 \lambda d$. Pick the probability that ruin occurs at the first claim is:

- I. $1 - e^{-1/1.4 (1-U/d)}$
- II. $1 - e^{-1/1.4 (1-U/d\lambda)}$
- III. $1 - e^{-1/1.4 \lambda (1-U/d)}$
- IV. $1 - e^{-1/1.4 d (1-U/d)}$

[2 marks]

Q.20) Which of the following statements is incorrect for Bornhuetter-Ferguson method:

- I: That whatever claims have already developed in relation to a given origin year, the future development pattern will follow that experienced for other origin years.
- II: It combines the estimated loss ratio with a projection method such as Chain ladder method.
- III: it could be applied to the development of paid claims, using either an accident year or policy year cohort
- IV: It assumes that first accident year is not fully run off.

[2 marks]

Q.21) Consider two independent assets, Asset A and Asset B, with expected returns of 6% pa and 11% pa and standard deviations of returns of 5% pa and 10% pa, respectively. Let x_i denote the proportion of the portfolio invested in Asset i .

If only Assets A and B are available, determine the equation of the efficient frontier in expected return-standard deviation space.

- I: $\sigma = \sqrt{5E^2 - 70E + 265}$, $E \geq 6\%$
- II: $\sigma = \sqrt{5E^2 - 70E + 265}$, $E \geq 7\%$
- III: $\sigma = \sqrt{10E^2 - 70E + 265}$, $E \geq 6\%$
- IV: $\sigma = \sqrt{5E^2 - 70E + 255}$, $E \geq 7\%$

[3 marks]

Q.22) Consider the following assets in a world where the Capital Asset Pricing Model (CAPM) holds. There are three risky assets and one risk-free asset. No other assets exist in the market.

Asset	Expected return (% p.a.)	Total value of assets in market (m)	Beta
Risky asset A	5	20	β
Risky asset B	10	40	1
Risky asset C	x	20	2
Risk-free asset	3	40	n/a

Calculate the expected return on the market portfolio.

- I. 5%
- II. 7.5%
- III. 10.25%
- IV. 11%

[3 marks]

Q.23) Consider the following assets in a world where the capital asset pricing model holds. These are the only risky assets in the market.

Asset	Expected return (% p.a.)	Total value of assets in market (\$m)	Beta
Risky asset A	3.5	200	1.5
Risky asset B	2.2	300	0.2
Risky asset C	4.4	100	2.4

The risk-free rate of interest is:

- I. 2.1%
- II. 2.2%
- III. 3%
- IV. 2%

[3 marks]

Q.24) A pension fund has the following investment opportunity.

Asset A gives an annual return of $2P\%$, where P is a Poisson random variable with parameter $\mu = 3$

The semi-variance of the asset is:

- I. 14.815%%
- II. 6.780%%
- III. 5.08%%
- IV. 14.815%

[3 marks]

Q.25) A pension fund has the following investment opportunity.

Asset A gives an annual return of $5P\%$, where P is a Poisson random variable with parameter $\mu = 2$

Shortfall probability versus a benchmark return of 5% is:

- I. 0.14543
- II. 0.13534
- III. 0.13544
- IV. 0.13543

[3 marks]

- Q.26)** A non-dividend paying stock has a price at time $t = 0$ of \$8. In any unit of time $(t, t + 1)$, the price of the stock either increases by 25% or decreases by 20%, and \$1 held in cash at time t receives interest to become \$1.04 at time $t + 1$. The stock price after t time units is denoted by S_t .

A derivative contract is written on the stock with expiry date $t = 2$, which pays \$10 if and only if S_2 is not \$8 (and otherwise pays \$0).

What is the price (at $t = 0$) of the derivative contract.

- I. 4.64
- II. 10.11
- III. 8
- IV. 3.51

[3 marks]

- Q.27)** The run-off triangle below shows the incremental claims incurred on a portfolio of car insurance policies.

	0	1	2	3
2021	3620	2720	5060	3500
2022	4440	1160	1650	
2023	4870	1950		
2024	5180			

The outstanding claim amount calculated using basic chain ladder method for the year 2022 is:

- I. 2300
- II. 2226
- III. 7103
- IV. 1583

[3 marks]

- Q.28)** The run-off triangle below shows the incremental claims incurred on a portfolio of car insurance policies.

	0	1	2	3
2021	3620	2720	5060	350
2022	4440	1160	16500	
2023	4870	1950		
2024	5180			

The outstanding claim amount calculated using basic chain ladder method for the year 2023 is:

- I. 8103
- II. 2226
- III. 7103
- IV. 1583

[3 marks]

- Q.29)** Consider a market that satisfies the assumptions of the capital asset pricing model. The market contains only three assets, with the following attributes:

	Market capitalisation (\$m)	Expected return (p.a.) (%)
Asset 1	60	5.0
Asset 2	40	7.0
Asset 3	100	8.5

The variance/covariance matrix is as follows:

	Asset 1	Asset 2	Asset 3
Asset 1	0.005	0.006	0.005
Asset 2	0.006	0.0015	0.0026
Asset 3	0.005	0.0026	0.0042

The beta of market portfolio is:

- I. 2
- II. 3
- III. 1
- IV. 4

[3 marks]

- Q.30)** Let B_t ($t \geq 0$) be a standard Brownian motion process starting with $B_0 = 0$. $E(B_s B_t) = s$ when:

- I. $s < t$
- II. $t < s$
- III. Neither of the above
- IV. $s = 0$

[3 marks]

- Q.31)** If W_t is a Wiener process, then which of the following statement(s) is/are correct.

- a. W_t is a martingale
- b. Covariance of W_{100} & W_{150} , is a martingale, and is 150
- c. Covariance of W_{100} & W_{150} , is a martingale, and is 100
- d. $X_t^2 - t$ is a martingale

- I. a & b
- II. c & d
- III. a, b & d
- IV. a, c & d

[3 marks]

- Q.32)** In a Black Scholes market, a European call option has a strike price Rs. 1200 and current stock price is Rs. 1100. The expiry is in 1 year, continuously compounded risk-free rate is 2% p.a. The price of the option is Rs. 10.9.

The implied volatility is:

- I. 7.96%
- II. 6%
- III. 8.2%

IV. 10%

[3 marks]

- Q.33)** An investment bank writes European call options on a stock with an expiry of six months. The price of this stock on 1-Dec-2023 is Rs. 1000 and on 1-Dec-2023 is Rs. 750. But this is expected to increase in the next six months to Rs. 1500. The option price on 1-Dec-2023 is Rs. 60.

The call option price using Black Scholes method on 2-Dec-2023 is expected to be:

- I. Greater than 60
- II. Less than 60
- III. Equal to 60
- IV. Equal to 100

[3 marks]

- Q.34)** Assume a long position in the following options written on the same stock

Option	Type	Strike	Time to expiry
A	American	100	3 years
B	American	100	2 years
C	American	120	3 years
D	European	100	3 years
E	European	100	2 years

Which of the following statements are true in terms of value of the options:

- a. $A > C$
- b. $A > B$
- c. $D > A$
- d. $B > E$

- I. a and d
- II. a, b, c and d
- III. a, b and d
- IV. All of the above

[3 marks]

- Q.35)** Which of the following statements are true:

- a. Value of American call option increases when share price increases
- b. Value of American call option increases when time to expiry increases
- c. Value of American put option increases when volatility of share price increases
- d. Value of American put option increases when risk free rate of interest increases

- I. a, b and c
- II. only a and c
- III. only a and b
- IV. All of the above

[3 marks]

- Q.36)** Consider a two-period recombining binomial model for S_t the price of a non-dividend paying security at times $t = 0, 1$ and 2 , with real world dynamics:
 $S_{t+1} = S_t u$ with probability p

= St d with probability $1-p$
 $u > d > 0$

There also exists a risk-free instrument that offers a continuously compounded rate of return of 8% per period. The state price deflator in this model after one period is:

$A_1 = 0.7610$ when $S_1 = S_0 u$
 $= 1.5220$ when $S_1 = S_0 d$

The value of 'p' is:

- I. 0.9980
- II. 0.7748
- III. 0.1234
- IV. 0.6654

[3 marks]

Q.37) The current value of a company's share price is Rs. 800. Yearly movements of a Company's share price follows log normal distribution.

The following details are given with respect to Mean Variance portfolio theory:

- beta between company's share price and market = 0.5
- correlation between company's share price and market = 0.7
- Expected market return = 6% p.a.
- Market $\sigma = 20\%$ p.a.
- market price of risk = 4%

The risk free return and expected return on the share prices are:

- I. 5.2% and 5.6%
- II. 5% and 5.6%
- III. 5.2% and 6%
- IV. 5% and 6%

[3 marks]

Q.38) The current value of a company's share price is Rs. 420. Yearly movements of a Company's share price follows log normal distribution.

The following details are given with respect to Mean Variance portfolio theory:

- beta between company's share price and market = 0.5
- correlation between company's share price and market = 0.7
- Expected market return = 6% p.a.
- Market $\sigma = 20\%$ p.a.
- market price of risk = 4%

Standard deviation on the return on the share prices is:

- I. 12%
- II. 10%
- III. 16%
- IV. 14%

[3 marks]

Q.39) An investor, assesses his utility of wealth using the utility function
 $U(w) = 2(w^{0.5} - 1)$.

Which of the following are true in respect of the Investor:

- a. Constant relative risk aversion
- b. Not-satiated and risk seeking
- c. Satiated and risk averse
- d. Not-satiated and risk averse

- I. a and d
- II. a and b
- III. a and c
- IV. only d

[3 marks]

Q.40) A Poisson claims process has security loading $\theta = 2/5$ and claim size density function:

$$f(x) = \frac{4}{2} e^{-4x} + \frac{7}{2} e^{-7x}, x > 0$$

The Moment Generating Function (MGF) for the claim size distribution is:

- I. $\frac{4}{(4-t)} + \frac{7}{(7-t)}$
- II. $\frac{4}{2(4-t)} + \frac{7}{2(7-t)}$
- III. $\frac{4}{2(7-t)} + \frac{7}{2(4-t)}$
- IV. $\frac{4}{2(7-t)}$

[3 marks]
