INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd November 2024

CS2 - Risk Modelling and Survival Analysis

(Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Solution 1: **B** Transition matrix = **[3]**

$$0.5\begin{bmatrix}0 & 1 & 1\\1 & 0 & 1\\1 & 1 & 0\end{bmatrix}$$

Roots of characteristic polynomial are 0, 1, -0.5

$P_{11}^{(p)} = T_1x_1^p + T_2x_2^p + T_3x_3^p = T_2x_2^p + T_3x_3^p$

$T_1 = 1/3$

$T_2 = 2/3$

$P_{11}^{(p)} = \frac{1}{3} + \frac{2}{3}(-0.5)^p$

Solution 2: **None** **[4]**

Solution 3: **C** **[4]**

$\frac{d}{dt}P_{11}(t) = -0.4\, P_{11}(t)$

$P_{11}(t) = e^{-0.4t} > 0.5$

➔ $-0.4T > \ln(0.5)$

➔ $T > 1.73286$

The cost which may be paid upto: 1.73286*591 = 1024.12

Solution 4: **D** The transition matrix is: T1 **[4]**

$$\begin{bmatrix}0.5 & 0.5 & 0\\0.25 & 0.5 & 0.25\\0 & 0.5 & 0.5\end{bmatrix}$$

The transition matrix is: T4

$$2^{-4}\begin{bmatrix}4.5 & 8 & 3.5\\4 & 8 & 4\\3.5 & 8 & 4.5\end{bmatrix}$$

Using iterative progressively, the nth generation would of residual type with a probability of $\{(2^{n-2}-1)+1.5\}(2)^{-n}$

Solution 5: **B** At 5% significance, (+-)1.96/sqrt(9504) = (+-) 0.0201 **[3]**

Co-efficients at lags are outside this range, therefore, statistically significant.

Solution 6: **C** **[3]**

$\mu_{49} = 304/303330 = 0.00100$

$\mu_{51} = 307/295880 = 0.00104$

Solution 7: **B** **[3]**

0	5744
1	5477
2	4500
3	3254
4	874

Sum product = 27735

Sum = 19849

λ = 1.3972
var = 0.0000704

92% confidence interval = 1.3972 +- 1.75 *(0.0000704)
= (1.383, 1.412)

Solution 8: **A** [3]

Let 'X' denote charge life
Using Weibull distribution
P (X > 410) = 0.80
exp(- c* 410 ^ gamma) = 0.80
c*410 ^gamma = Ln(1/0.80…….(1)
P (X > 820) = 0.60
exp(- c*820 ^ gamma) = 0.60
c*820 ^gamma = Ln(1/0.60) …….(2)
From (1) and (2):
2.289224 = 820 ^gamma / 410 ^ gamma
ln(2.289224) = gamma * ln (820/410)
Gamma = 1.194859
From (1) and (2):
c = 0.00017
So,
P (X > 1100) = Exp(-0.00017* 1100 ^ 1.194859)
= 0.48097

Solution 9: **C** [3]

y_t will have mean equal to μ.
Only the past 3 lags seem to affect the process, and autocorrelation at lag zero is the correlation of y_t with itself.

Solution 10: **B** [2]

Solution 11: **D** [3]

$F_0(x) = \int_0^x f_0(t)dt$,
As force of mortality is constant
$S_x(t) = e^{-\mu t}$
${}_tp_x = S_0(x+t) / S_0(x)$

$= \left(1 - \frac{t}{117-x}\right)^{\frac{1}{5}}$

$\mathring{e}_x = -(117-x) \int_1^0 u^{1/5} . du$

= 29.1667

Solution 12: **D** [4]

mean of y_t = 1/(1-0.79) = 1.33
variance of of y_t = $1/(1-0.79^2)$ = 2.6602

value of autocovariance function at lag 3 = $0.79^3 / (1-0.79^2)$ = 1.31162
value of autocorrelation function at lag 3 = 0.79^3 = 0.4930

Solution 13: **E** [2]

Solution 14: **B** [3]

$F_0(x) = \int_0^x f_0(t)dt$,
As force of mortality is constant
$S_x(t) = e^{-\mu t}$
${}_tp_x = S_0(x+t) / S_0(x)$

$= \left(1 - \frac{t}{120-x}\right)^{\frac{1}{6}}$

$\dot{e}_x$ = -(120-x) $\int_1^0 u^{1/6}.du$

= 77.1428

Solution 15: **C** $P_n = \phi 1 + \ldots\ldots + \phi n$. By independence, **[3]**
$E[e^{\alpha P_n}] = E[e^{\alpha\phi_1} \ldots\ldots\ldots\ldots e^{\alpha\phi_n}] = E[e^{\alpha\phi_1}]^n = (pe^{\alpha} + qe^{-\alpha})^n$

Solution 16: **E** **[4]**

- var(X_0) + tσ^2
- X_t is is not stationary.
- Z_t is equivalent to random walk with drift but it I(1) process.
- Conditional distribution of X_{n-1} is dependent on X_n and X_{n-1}, so it is not Markov.

Solution 17: **A** **[2]**

Solution 18: **E** $(1-D)(1-D^m)y_t = (1-D-D^m+D^{m+1})y_t$ **[2]**

$= y_t - y_{t-1} - y_{t-m} + y_{t-m-1}$

Solution 19: **A** $Var[T_x] = E[T_x^2] - \{E[T_x]\}^2$ **[4]**

$S_0(x) = \left(1 - \frac{x}{120}\right)^{\frac{1}{6}}$
$f_0(x) = -\frac{d}{dx} S_0(x)$
$= (1/120)^* \left(1 - \frac{x}{120}\right)^{\frac{-5}{6}} *(1/6)$
$x = 30$

$f_{30}(t) = (1/540)(1\text{-}t/90)^{-5/6}$

$E[T_{30}^2] = \int_0^{90} \frac{t^2}{540} (1 - \frac{t}{90})^{-5/6} dt$
$= 90^2 x 72/91$
= 6408.79

Std. dev = sqrt(6408.79 - 77.1428^2)

= 21.395

Solution 20: **C** $\mu_{48} = 291/353367 = 0.00082$ **[4]**
$\mu_{50} = 313/303616 = 0.00103$

$q_x = 1 - \exp(-\mu_{48})$

Therefore,
$q_{48} = 0.000823$
$q_{50} = 0.001030$

Solution 21: **A** $\frac{d}{dt}P_{21}(t)$ = A. $0.6P_{22}(t) – 0.3P_{21}(t)$ **[2]**

Solution 22: **D** $Var[T_x] = E[T_x^2] – \{E[T_x]\}^2$ **[4]**

$S_0(x) = \left(1 - \frac{x}{117}\right)^{\frac{1}{5}}$

$f_0(x) = -\frac{d}{dx} S_0(x)$

$= (1/117)* \left(1 - \frac{x}{117}\right)^{\frac{-4}{5}} *(1/5)$

$x = 82$

$f_{82}(t) = 0.1x\ (35-t)^{-4/5}$

$E[T_{82}^2] == \int_0^{35} \frac{t^2}{10} (35 - t)^{-4/5} dt$

= 944.8128

Std. dev = sqrt(944.8128- 29.1667^2)

= 9.701

Solution 23: **E** X ~ Pareto (4, 1200) **[3]**

E[S] = 2.4 X 1200/3 = 960

With deductible,

λ E[(X-d)] = 960

$(1440/(1440+d))^4$ = 400X3/(1440+d)

➔ d = 90.23

Solution 24: **D** E[N] = 2.4 X $(1440/(1440+90.23))^4$ **[2]**

= 1.882

Solution 25: **D** 1440/3 (1-$(1440/(1440+M))^3$) - 1440/3 (1-$(1440/(1440+30))^3$) = 400 **[3]**

➔ M = 1596.2

Solution 26: **A** E[S] = 10 **[2]**

Var[S] = 60 + 90 = 150

Std. dev. = 12.25

Solution 27: **B** Based on the comparisons of mean and variance of S, the distribution is binomial with m = 10, q = 0.1 **[2]**

Solution 28: **C** $M_x(t) = (1-850)^{-1}$ **[2]**

$M_y(t) = M_x(0.73t) = (1-620.5)^{-1}$

Solution 29: **D** α = 2 **[4]**

$\Psi(x) = (-\ln x)^{\alpha}$

$C(u,v,w) = \Psi^{[-1]}(\Psi(u)+ \Psi(v) + \Psi(w))$

$= \exp[-\{(-\ln u)^{\alpha}+(-\ln v)^{\alpha}+(-\ln w)^{\alpha}\}^{1/\alpha}]$

$= \exp[-((-\ln 0.04)^{2})+(-\ln 0.07)^{2}+(-\ln 0.18)^{2}\}^{1/2}]$

= 0.010958

Solution 30: **B** Probability = (0.8X0.01)/(0.8x0.01+0.10x0.99) = 0.075 **[4]**

Solution 31: **C** β_0 is expected value of Y when X = 0. **[2]**

Solution 32: **B** Prediction interval may not become zero. With law of large numbers, the error may be zero in I. Prediction may be higher uncertainty, so III is also true. **[3]**

Solution 33: **D** D is incorrect. **[2]**

Solution 34: **A** Total sum of squares = 874.45/(1-0.87) = 6727.23 **[2]**

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