INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

CM2 - Financial Engineering and Loss Reserving (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1 - Answer: 1 & 2- Weak and Semi Strong form Efficient Markets. **[1.5]**

Solution 2 - Answer: 3- Transitivity **[1.5]**

Solution 3 - Answer: 3- Reduced-form models **[1.5]**

Solution 4 - Answer: 1- Investors' irrational behavior and overconfidence**.** **[1.5]**

Solution 5 - Answer: 3- Specific risk **[1.5]**

Solution 6 - Answer: 2 - The risk-free rate plus a market risk premium **[1.5]**

Solution 7 - Answer: 3 - zero; one**.** **[1.5]**

Solution 8 - Answer: 3- The marginal utility from each good consumed will be lower for X than for Y and X will have a higher total utility. **[1.5]**

Solution 9 - Answer: 3- Optimal **[1.5]**

Solution 10 - Answer: 1- Change of measure in the context of Brownian motion**.** **[1.5]**

Solution 11 - Answer: 4 - A European put option allows the holder to sell the underlying asset at the striking price on the expiration date. **[1.5]**

Solution 12 - Answer: 1- Q and P assign zero probability to the same events. **[1.5]**

Solution 13 - Answer: 1- Spot price greater than strike price **[1.5]**

Solution 14 - Answer: 3- The total utility consumers get from consumption of the good less the total expenditure on the good. **[1.5]**

Solution 15 - Answer: 4- Total utility decreases at an increasing rate as income increases**.** **[1.5]**

Solution 16 - Answer: 2- people are risk averse. **[1.5]**

Solution 17 - Answer: 4- The surplus process follows a compound Poisson process**.** **[1.5]**

Solution 18 - Answer: 2- Spot price is greater than strike price. **[1.5]**

Solution 19 - Answer: 1- Intrinsic value is the difference between the price of underlying asset and the strike price. **[1.5]**

Solution 20 - Answer: 1- The process has continuous but non-differentiable paths**.** **[1.5]**

Solution 21 - Answer: 2-

$X_t = e^{\mu t + \sigma W_t}$, where μ and σ are constants, and W_t is a Wiener process

[1.5]

Solution 22 - Answer: 1- TRUE **[1.5]**

Solution 23 - Answer: 2- Rs. 1320-1210 = 110

Time Value = 200-110 = 90 **[1.5]**

Solution 24 - Answer: 1- This model is used to calculate a theoretical price of options using the five key determinants of an option price. **[1.5]**

Solution 25 - Answer: 4- Incremental Triangle of claims development **[1.5]**

Solution 26 - Answer: 3- Ultimate Claims – Incurred Claims **[1.5]**

Solution 27 - Answer: 1- Development factors: 1.50, 1.20; Projected cumulative claims: 252,000; Estimated claims reserve: 112,000 **[2]**

Solution:

1. **Calculate Development Factors:**
   - Development Factor from Year 1 to Year 2:

$$\text{Factor}_1 = \frac{150,000}{100,000} = 1.50$$

$$\text{Factor}_1 = \frac{180,000}{120,000} = 1.50$$

   Average $\text{Factor}_1 = (1.50 + 1.50)/2 = 1.50$

   - Development Factor from Year 2 to Year 3:

$$\text{Factor}_2 = \frac{180,000}{150,000} = 1.20$$

   Average Factor_2 = 1.20 (only one data point available)

2. **Projected Cumulative Claims for Accident Year 2018 at Development Year 3:**
   - Development Year 2:

$$140,000 \times 1.50 = 210,000$$

   - Development Year 3:

$$210,000 \times 1.20 = 252,000$$

3. **Estimated Claims Reserve for Accident Year 2018:**

$$252,000 - 140,000 = 112,000$$

Solution 28 - Answer: 1- 0.875 **[2]**

Calculation:

$$d_1 = \frac{\ln(50/45)+(0.05+0.2^2/2)\cdot 1}{0.2\sqrt{1}} = \frac{\ln(1.1111)+0.07}{0.2} \approx \frac{0.105+0.07}{0.2} = \frac{0.175}{0.2} = 0.875$$

Solution 29 - Answer: 3 - Weak form Efficient Markets **[2]**

Solution 30 - Answer: 4- Security Market Line **[2]**

Solution 31 - Answer: 3- 9.0% **[2]**

Calculation:

Using the given values in the CAPM equation:

$r_i = r_f + \beta(r_m - r_f)$

$r_i = 3\% + 1.2 \times (8\% - 3\%)$

Steps:

1. Calculate the market risk premium:
   $r_m - r_f = 8\% - 3\% = 5\%$
2. Multiply the market risk premium by the stock's beta:
   $1.2 \times 5\% = 6\%$
3. Add the risk-free rate:
   $r_i = 3\% + 6\% = 9\%$

Solution 32 – Answer: 2- Kurtosis **[2]**

Solution 33 - Answer: 4 – D only **[2]**

Solution 34 - Answer: 4- People who know they are particularly bad risks are more inclined to take out insurance than those who know they are good risks **[2]**

Solution 35 - Answer: 1- 20 **[2]**

Calculation:

$$U(100) = \frac{100^{1-0.5}}{1-0.5} = \frac{100^{0.5}}{0.5} = \frac{10}{0.5} = 20$$

Solution 36 - Answer: 2- Basic coverage: Rs.7,500; Premium coverage: Rs.12,500; Effect of moral hazard: Rs.5,000 **[2]**

1. **Basic Coverage Expected Annual Cost**:
   - Average number of claims per year: 1.5
   - Average cost per claim: $5,000
     $\text{Expected cost} = 1.5 \times \$5,000 = \$7,500$
2. **Premium Coverage Expected Annual Cost**:
   - Average number of claims per year: 2.5
   - Average cost per claim: $5,000
     $\text{Expected cost} = 2.5 \times \$5,000 = \$12,500$
3. **Effect of Moral Hazard**:
   - Increase in the number of claims due to higher coverage: 2.5 (Premium) - 1.5 (Basic) = 1 additional claim
   - Cost due to moral hazard: 1 additional claim × $5,000 per claim = $5,000

Solution 37 - Answer: 1- the marginal utility of an extra pound of income decreases as more income is received. **[2]**

Solution 38 - Answer: 3- Equities **[2]**

Solution 39 - Answer: 1- Ceded premium: Rs.4,800,000; Ceding commission: Rs.240,000; Net premium retained: Rs.7,440,000 **[2]**

1. **Ceded Premium**:
   - Total premium: $12,000,000
   - Proportional reinsurance cession: 40%

   $$\text{Ceded premium} = \$12,000,000 \times 0.40 = \$4,800,000$$

2. **Ceding Commission**:
   - Ceding commission rate: 5%

   $$\text{Ceding commission} = \$4,800,000 \times 0.05 = \$240,000$$

3. **Net Premium Retained**:

   $$\text{Net premium retained} = \text{Total premium} - \text{Ceded premium} + \text{Ceding commission}$$
   $$\text{Net premium retained} = \$12,000,000 - \$4,800,000 + \$240,000 = \$7,440,000$$

Solution 40 - Answer: 1- The Vasicek model **[2]**

Solution 41 – Answer: 3- **[3]**

$\Sigma x = 50(-2) + 0 + 60(2) = 20$ $\Sigma x^2 = 50(4) + 0 + 60(4) = 440$

$\Sigma y = 50(2) + 0 + 60(-1) = 40$ $\Sigma y^2 = 50(4) + 0 + 60(1) = 260$

$\Sigma xy = 50(-4) + 0 + 60(-2) = -320$

so $r = [-320 - (20 \times 40)/200]/[(440 - 20^2/200)(260 - 40^2/200)]^{1/2} = -0.975$

Solution 42 – Answer: 2- Mean = 92780/50 = Rs. 1855.60

25.5th value in order = (17.3 + 17.4)/2 = 17.35 so median amount = Rs. 1735

[3]

Solution 43 – Answer: 3- **[3]**

$X \sim$ exponential with $\lambda = 1/1000$ and density $\lambda e^{-\lambda x}$.

$P(X > t) = e^{-\lambda t}$ (stated or by integration)

$$\therefore P(X > 5000) = e^{-\frac{1}{1000}5000} = e^{-5} = 0.0067$$

Solution 44 (a) – Answer: None of the above

- Calculate Development Factors without Inflation = (2796+2440)/ (2515+2216) = 1.1067

Apply Development Factor to Accident Year 2010 without Inflation = 2880*1.1067 = 3187.42

Adjust for Inflation = (3187.42-2880) * 1.05 = 322.79

total amount of claims paid in 2013 in respect of accidents that occurred in 2010 is: 322.79 **[3]**

Solution 44 (b) – Answer: 2-

AY 2012

$$1{,}182+(2{,}142-1{,}220)+(2{,}880-1{,}814)+(2{,}796-2{,}515)+(2{,}519-2{,}440)$$

$$=3{,}530$$

AY 2011
1220 + (1814 – 995) + (2515 – 1575) + (2440 – 2216)
= 3203

[3]

Solution 44 (c) – Answer: 2- 3807.102 **[3]**

	Development Years				
AY	**1**	**2**	**3**	**4**	**5**
2008	786	1410	2216	2440	2519
2009	904	1575	2515	2796	
2010	995	1814	2880		
2011	1220	2142			
2012	1182	**2100.963**	**3332.034**	**3687.705**	**3807.102**
Development Factors:		1.777465	1.585955	1.106743	1.032377

Solution 45 – Answer: 4-

Let n denote the sample size which is determined by the limits of the 99% confidence interval, i.e.

$$2.58\times\frac{6}{\sqrt{n}}\le 2$$

$\Rightarrow n\ge(3\times 2.58)^2=59.9$. Therefore n should be at least 60.

[3]

Solution 46 – Answer: None of the above - 2100.3. **[3]**

Calculations:

1. **Initial Expected Utility Without Insurance:**

$$\text{Expected Utility} = 0.9 \times \sqrt{100,000} + 0.1 \times \sqrt{100,000 - 20,000}$$

$$\text{Expected Utility} = 0.9 \times \sqrt{100,000} + 0.1 \times \sqrt{80,000}$$

$$\text{Expected Utility} = 0.9 \times 316.23 + 0.1 \times 282.84 = 313.68$$

2. **Utility with Insurance (No Loss) at Wealth Minus Premium (P):**

$$U(W - P) = \sqrt{100,000 - P}$$

The consumer is indifferent when the expected utility without insurance equals the utility with insurance:

$$\sqrt{100,000 - P} = 313.68$$

Solving for P:

$$100,000 - P = 313.68^2$$

$$100,000 - P = 98,408.27$$

$$P = 100,000 - 98,408.27 = 1,591.73$$

Expected Utility =0.9*316.23+0.1*282.84= 312.89

(100,000-P)^(0.5)= 312.89

P= 2100.3

Solution 47 – Answer: 1- **[3]**

$$\text{Mean} = \int_0^{100} x(0.0002)(100-x)dx = 0.0002\left[50x^2 - \frac{x^3}{3}\right]_0^{100}$$

$$= 0.0002\{500000 - \frac{1000000}{3}\} = 33.33$$

Solution 48 – Answer: 2 –

a. Primary Insurer: Rs.150,000
b. Layer 1 Reinsurer: Rs.370,000
c. Layer 2 Reinsurer: Rs.350,000
d. Layer 3 Reinsurer: Rs.200,000

Calculations:

Claim 1:
Primary Insurer = 50,000
Layer 1 Reinsurer = 70,000
Layer 2 Reinsurer = 0
Layer 3 Reinsurer = 0

Claim 2:

Primary Insurer = 50,000
Layer 1 Reinsurer = 150,000
Layer 2 Reinsurer = 150,000
Layer 3 Reinsurer = 0

Claim 3:
Primary Insurer = 50,000
Layer 1 Reinsurer = 150,000
Layer 2 Reinsurer = 200,000
Layer 3 Reinsurer = 200,000

Total Paid:
Primary Insurer = 150,000
Layer 1 Reinsurer = 370,000
Layer 2 Reinsurer = 350,000
Layer 3 Reinsurer = 200,000

Solution 49 – Answer: 4 **[3]**

	Expected Return	Variance
Portfolio A	6.80%	0.60%
Portfolio B	6.80%	0.74%

Expected Return:

Portfolio A = Expected Return of Security 1 * Weight given to Security 1 + Expected Return of Security 2 * Weight given to Security 2 + Expected Return of Security 3 * Weight given to Security 3
= 6.2% * 33.3% + 4.9% * 0% + 7.1% * 66.7%
= 6.8%

Portfolio B = Expected Return of Security 1 * Weight given to Security 1 + Expected Return of Security 2 * Weight given to Security 2 + Expected Return of Security 3 * Weight given to Security 3
= 6.2% * 14.9% + 4.9% * 7.5% + 7.1% * 77.6%
= 6.8%

Variance:

Portfolio A
= 33.3% * 33.3% * 0.38% + 0% * 0% * 0.2% + 66.7% * 66.7% * 1.1% + 2 * 33.3% * 0% * 0.25% + 2 * 33.3% * 66.7% * 0.15% + 2 * 0% * 66.7% * 0.27%
=0.60%

Portfolio B
= 14.9% * 14.9% * 0.38% + 7.5% * 7.5% * 0.2% + 77.6% * 77.6% * 1.1% + 2 * 14.9% * 7.5% * 0.25% + 2 * 14.9% * 77.6% * 0.15% + 2 * 7.5% * 77.6% * 0.27%
=0.74%

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