INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

CM1 - Actuarial Mathematics (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1: Answer - C: (2)

$${}_3(aq)^d_{60} = \frac{20}{2000} + \frac{30}{2000} + \frac{40}{2000} = 0.045$$

Solution 2: Answer - D: (2)

$tp\overline{HH} = \exp[-\int_0^t (\mu + \sigma)ds]$

$tp\overline{HH} = \exp[-\int_0^t (0.020 + 0.002)ds] = e^{-0.022t}$

Force of interest =0.06

PV of Sickness benefit:

$$= 200{,}000 \int_0^{20} e^{-0.06t} e^{-0.022t} \sigma\, dt$$

$$= 200{,}000 \int_0^{20} e^{-0.082t} * 0.002\, dt$$

$$= \frac{200{,}000*0.002}{-0.082}(e^{-0.082*20} - e^{-0.082*0})$$

= 4878.04878 *(1- 0.19398)

= 3931.80

Solution 3: Answer - C: (2)

PV = 1000*(1+ 1.04/1.06 +(1.04/1.06)^2 + … + (1.04/1.06)^19

= 1000 $\ddot{a}_{\overline{20|}}$ @i= 1.9231%

= 16790.26

Solution 4: Answer - D: (2)

Premium = 10000*($a^{(12)}_{60}$ – $a^{(12)}_{65:60}$)

Premium = 10000*(16.652 – 12.682) = 39700

Effect of monthly payments cancels out.

Solution 5: Answer - D: (2)

$p_x p_y = e^{-0.035-0.025} = e^{-0.06}$

$$PV = 5000 * \{1 + \frac{1.04}{1.06}(e^{-0.06}) + \left(\frac{1.04}{1.06}\right)^2 (e^{-0.06})^2 + \left(\frac{1.04}{1.06}\right)^3 (e^{-0.06})^3 + \cdots \}$$

$$PV = 5000/(1 - \frac{1.04}{1.06}e^{-0.06}) = 65{,}785.50$$

Solution 6: Answer - B: (2)

${}_0V$ =0,

${}_1V$ = [(0 + 1231.80)*1.04 – 50000*0.02] /0.98 = 286.81

${}_2V$ = [(286.81 + 1231.80)*1.04 – 40000*0.025] /0.975 = 594.21

OR

PV of future Benefits:

s	qx	(s-1)px+t	DB	Cost	DF	PV
1	0.030000	1	30000	900	0.96154	865.38
2	0.035000	0.97	20000	679	0.92456	627.77
3	0.040000	0.93605	10000	374.42	0.88900	332.86

Total **1826.02**

PV of future Premiums = 1231.80

Hence reserve = 1826.02- 1231.80 = 594.22

Solution 7: Answer - B: (2)

npx + npy -2npx * npy = 0.9 + 0.8 - 2*0.9*0.8 = 0.26

OR

npx * (1- npy) + npy *(1- npx) = .9*.2 + .1*.8 = .18+.08 = 0.26

Solution 8: Answer - C: (2)

$${}_3V = \frac{200}{1.06} = 188.68$$

$${}_2V = \frac{188.68 * p_{52} + 250}{1.06} = 412.07$$

$${}_1V = \frac{412.07 * p_{51} + 350}{1.06} = 715.43$$

Solution 9: Answer - D: (2)

This series of payment forms an increasing annuity where k→∞.

$$(Ia)_{\overline{k|}} = \frac{\ddot{a}_{\overline{k|}} - nv^k}{i} = \frac{1}{id}$$

Therefore, PV = 1/id = 1/(.04*0.038462) = 650

Alternative method:

v+2v^2+3v^3 + 4v^4+….

=

[v+v^2+v^3 + v^4+….]

+ v[v+v^2+v^3 +…]

+v^2 [v+v^2+…]

+…

= $\frac{1}{i}$ + v $\frac{1}{i}$ +v^2 $\frac{1}{i}$ +…

= ($\frac{1}{i}$) [1+ v+v^2+v^3 + ⋯]

=($\frac{1}{i}$) (1+($\frac{1}{i}$)) = $\frac{(1+i)}{i^2}$ = $\frac{1}{i^2\,v}$ = $\frac{1}{i(i\,v)}$ =$\frac{1}{i\,d}$ (as d=iv)

Therefore, PV = 1/id = 1/(.04*0.038462) = 650

Solution 10: Answer -C: (2)

a_{11} = x and a_{21} = y

a_{21} - a_{11} = y-x

((1-v^2)/i – (1-v)/i - = y-x

v(1-v)/i = y-x

vx = y-x

v = (y-x)/x

1+i = x/(y-x)

i= (2x-y)/(y-x)

Solution 11: Answer -A: (2)

100,000 = $k\ddot{a}_{\overline{5|}} + 2kv^5\ddot{a}_{\overline{5|}} + 4kv^{10}\ddot{a}_{\overline{10|}}$

= 4.16987k + 2*0.62092*4.16987k + 4*0.38554*6.75902k

= 19.77167k

k = 5057.74

Solution 12: Answer- C (2)

Year	Premium received	Prem allocated	Cost of allocation	Fund after allocation	Interest	Fund before mgt charges	Magt Charges	Fund at year end
1	5000	4900	4655	4655.00	372.40	5027.40	100.55	4926.85
2	5000	4900	4655	9581.85	766.55	10348.40	206.97	10141.43
3	5000	4900	4655	14796.43	1183.71	15980.15	319.60	15660.54
4	5000	4900	4655	20315.54	1625.24	21940.79	438.82	21501.97
5	5000	4900	4655	26156.97	2092.56	28249.53	564.99	27684.54

Solution 13: Answer -C: (2)

Calculation in half years so n =15*2=30

Effective rate for first 5 year = (1+ 0.08/12)^6 – 1 = 4.06726% , d= 3.9083%

Effective rate for next 10 year = 4.0%, d = 3.84615%

$$S\ due\ 10\ = \frac{1.046726^{10} - 1}{.039083} = 12.53346$$

$$S\ due\ 20\ = \frac{1.04^{20} - 1}{.0384615} = 30.96923$$

Accumulated value at the end of term = 2000*($12.53346 * 1.04^{20} + 30.96923$) = 116863

Solution 14: Answer -D: (2)

PV of Development cost = 100 + 15v@10% = 113.64
PV of income = 25 v^2 $a_{\overline{n|}}$ = 25*0.82645* $a_{\overline{n|}}$ = 20.66125 $a_{\overline{n|}}$
Discounted payback period will be n+2 years when
20.66125 $a_{\overline{n|}}$ > 113.64
$a_{\overline{n|}}$ > 5.50

OR
AV of cost as on 01.01.2027 = 100*1.10^2 +15*1.10 = 137.50
Discounted payback period will be n+2 years when

$25\, a_{\overline{n}|} > 137.50$
$a_{\overline{n}|} > 5.50$

n =9
DPP = n+2 = 11 years

Solution 15: Answer - C: (2)

111.39 = 10/1.06 + 10/(1.06*1.055) + 110 /[1.06*1.055*(1+$f_{2,1}$)]

111.39 = 9.43396 + 8.94214 + 98.36359/(1+$f_{2,1}$)

93.01390 = 98.36359/(1+$f_{2,1}$)

(1+$f_{2,1}$) = 1.0575, $f_{2,1}$ = 5.75%

Solution 16: Answer - A: (2)

$G\ddot{a}_{50} = 100{,}000\bar{A}_{50} + 3000(I\bar{A})_{50} + 0.05G\ddot{a}_{50}$

$\bar{A}_{50}$ = (1+i)^(1/2) A_{50} = 1.02956 * 0.20508 = 0.21114

$(I\bar{A})_{50}$ = (1+i)^(1/2) $(IA)_{50}$ = 1.02956 *4.84555 = 4.98880

$\ddot{a}_{50}$ = 14.044

13.3418G = 21114 + 14966.40= 36080.40

G = 2704

Solution 17: Answer -A: (2)

Profit emerging per policy in force at the start of the year is:

(15,000 + 2000)*1.06 – 20,000*0.03 – (1-0.03)*17,500 = 445

Solution 18: Answer --C: (2)

In the 1st policy year, independent rates of mortality and surrender are 2% and 10% respectively.

Therefore, dependent rate of surrender at the end of 1st policy year $(aq)^s_x$= (1-.02)*0.10 = 0.098

Revised profit signature at the end of 1st year, after including surrender charges = -400 + 1000 *0.098 = -400 + 98 = -302

Solution 19: Answer -B: (2)

The price is equal to the present value of dividends it pays:

$P = dv^{\frac{1}{4}} + d(1+g)v^{\frac{3}{4}} + d(1+g)^2V^{\frac{5}{4}} + \cdots$

$P = dv^{\frac{1}{4}}\,[\,1 + (1+g)v^{\frac{1}{2}} + (1+g)^2 v + \cdots$

$$P = \frac{dv^{\frac{1}{4}}}{1-(1+g)v^{\frac{1}{2}}}$$

Given $d = 1.5, g = 0.03, i = 0.10, v = 0.90909, v^{\frac{1}{2}} = 0.95346, v^{\frac{1}{4}} = 0.97645$
$P = 81.67$

Solution 20: Answer -D: (3)

$100{,}000 * (IA)x$ = 0.9G $\ddot{a}_{x:\overline{n}|}$ - 0.2G - 5,000 - 500$\ddot{a}_x$

$\ddot{a}_{x:\overline{n|}} = \ddot{a}_x - v^n \, {}_np_x \ddot{a}_{x+n} = 24 - 0.7 * 0.17411 * 18 = 21.80621$

$100{,}000 * 7$ = 0.9G $* 21.80621$ - 0.2G - 5,000 - 500$* 24$

19.42559G = 717000 and G = Rs.36,910

Solution 21: Answer - A: (3)

d = 0.056604

$v^{10} = 0.55839$

$\ddot{a}_{x:10\rceil} = 1 + a_{x:9\rceil} = 1+6.465 = 7.465$

$A_{x:10\rceil} = 1 - d\,\ddot{a}_{x:10\rceil} = 1 - 0.056604 * 7.465 = 0.57745$

$A^1_{x:10\rceil} = A_{x:10\rceil} - {}_{10}p_x v^{10} = 0.57745 – 0.867219*0.55839 = 0.09320$

Solution 22: Answer - A: (3)

$e^x = \sum_{k=0}^{\infty} {}_kp_x$

= 0.98 + 0.98^2 + …..+ 0.98^20 + 0.98^20*(0.96+ 0.96^2 +…+0.96^20) + 0

$= \frac{0.98*(1-0.98^{20})}{1-0.98} + (0.98\text{^}20)\frac{0.96*(1-0.96^{20})}{1-0.96} + 0$

= 16.28721 + 0.66761*13.39194 = 25.23

Solution 23: Answer - C: (3)

d = 0.038462
Ax = 1 - d $\ddot{a}_x$ = 1- 0.038462*15.632 = 0.39876
Ay = 1 - d $\ddot{a}_y$ = 1- 0.038462*16.652 = 0.35953
Axy = 1 - d $\ddot{a}_{xy}$ = 1- 0.038462*14.090 = 0.45807
Axy_bar = Ax + Ay – Axy = 0.30022

OR
$A_{\overline{xy}} = 1 - d\ddot{a}_{\overline{xy}}$, and $\ddot{a}_{\overline{xy}} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}$
Therefore
P $\ddot{a}_x$ = $100000\, A_{\overline{xy}}$
15.632P = 30022, P = 1920.54

Solution 24: Answer - D: (3)

$a_{15\rceil} = v^{15} a_{\infty\rceil}$

$1 - v15 = v^{15}$

$v^{15} = 0.50$

v = 0.95484

i= 4.729%

Solution 25: Answer -D: (3)

DMT= $\frac{v+2g \quad v^2+3g^2v^3+\cdots\ldots\ldots+35g^{34}v^{35}}{v+g \quad v^2+g^2v^3+\cdots\ldots\ldots+g^{34}v^{35}}$

Where g = 1.0283, v = 1/1.09 and gv = 0.94339
gv = v’ @ i'=6%

DMT= $\frac{g(v'+2v'^2+3v'^3+\cdots\ldots\ldots+35v'^{35})}{g(v'+v'^2+v'^3+\cdots\ldots\ldots+v'^{35})}$

DMT = $(Ia)_{\overline{35}|}$ / $a_{\overline{35}|}$ @ 6%

= 180.2410/14.4982 = 12.43 years

Solution 26: Answer : B (3)

Year	Age	lx	dx	Benefit	Expected outgo for Death Benefit per policy	DF	PV
1	30	100000	1000	10000	100	0.96154	96.15
2	31	99000	1500	12000	180	0.92456	166.42
3	32	97500	2000	14400	288	0.88900	256.03
4	33	95500	2500	17280	432	0.85480	369.28
5	34	93000					
							887.88

Solution 27: Answer - A: (3)

Year	t-1px	Prem.		DF	PV
1	1	35000	35000	1	35000
2	0.9	35000	31500	0.943396	29716.98
3	0.801	35000	28035	0.889996	24951.05
					89668.03
	Profit	3142			
	PM	3.50%			

Solution 28: Answer -B: (3)

q_{50} = 0.002508

DSAR for 1st year = S – $_1V$ = 0- 8106.80 = -8106.80

EDS = 1000*q_{50}(S-$_1V$) = 1000*0.002508* (-8106.80) = - 20331.85

ADS = Actual Death * DSAR = 4* (-8106.80) = -32427.20

Mortality profit = EDS - ADS = -20331.85 – (-32427.20) = 12095.35

Solution 29: Answer -B: (3)

$A^1_{X:\overline{n}|} = A_x - v^n\,{}_np_x\,A_{x+n}$ = 0.20 - 0.24$v^n\,{}_np_x$

$A_{x:\overline{n}|} = A^1_{X:\overline{n}|} + v^n\,{}_np_x$ = 0.20 - 0.24$v^n\,{}_np_x + v^n\,{}_np_x$ = 0.20+ 0.76 $v^n\,{}_np_x$

0.62 = 0.20+ 0.76 $v^n\,{}_np_x$

$v^n\,{}_np_x = 0.55263$

$A^1_{X:\overline{n}|}$ = 0.20 -0.24*0.55263 = 0.06737

$\bar{A}_{x:\overline{n}|} = \bar{A}^1_{X:\overline{n}|} + v^n\,{}_np_x$

$\bar{A}^1_{X:\overline{n}|} = (1+i)^{\frac{1}{2}}\,A^1_{X:\overline{n}|}$ = 1.02956* 0.06737 = 0.06936

$\bar{A}_{x:\overline{n}|}$ = = 0.06936+0.55263 = 0.62199

Solution 30: Answer - A (3)

Since select period is two years

$A_{[x]} = vq_{[x]} + v^2 p_{[x]} q_{[x]+1} + \ v^2 \ {}_2p_{[x]} A_{x+2}$

$A_{[75]} = vq_{[75]} + v^2 p_{[75]} q_{[75]+1} + \ v^2 \ {}_2p_{[75]} A_{x+2}$

$p_{\{75\}} = 1 - q_{\{75\}} = 1 - .07 = 0.93$

${}_2p_{[75]} = p_{[75]} * \left(1 - q_{[76-1]+1}\right) = 0.93 * (1 - .01) = 0.93 * .90 = 0.837$

$= 0.961538 * .07 + \ 0.961538^2 * 0.93 * 0.10 + 0.961538^2 * 0.837 * 0.8357$

= 0.06731+0.8598+0.64671 = 0.80

Solution 31: Answer - A: (3)

Working in millions

PV of outgoing = 1000 $\ddot{a}_{\overline{2}|}$@10%

= 1000*1.90909 = 1909.09

Annuity is increasing by 6% and discounted by 10%, which is equivalent to PV at $v^t = \left(\frac{1.06}{1.10}\right)^t = \ 0.96363^t$

Hence $i = 3.7736\% \ and \ \delta = 3.7041\%$

PV of Income = $200 \ \bar{a}_{\overline{15}|}$ = 200*11.5083 = 2301.66

OR

Force of interest = ln(1.10) = 0.095310179

Force of growth = ln(1.06) = 0.058268908

PV of income =$200 \int e^{0.058268908\text{t}} * e^{-0.095310179\text{t}} dt$ = $200 \int e^{-0.037041271\text{t}} dt$

$= \frac{200}{0.037041271} \left[1 - e^{-0.037041271*15}\right]$ = 2301.66

NPV = 2301.66 – 1909.09 = 392.57 million

Solution 32: (3)

Answer -B:							
Date	Index	Coupon	Mty	CF	t	vt	PV
01 July 2021	**105.00**						
01 January 2022	108.50				0	1.0000	
01 July 2022	111.40	4		4.133	0.5	0.9623	3.977
01 January 2023	117.20	4		4.244	1	0.9259	3.929
01 July 2023	120.25	4		4.465	1.5	0.8910	3.978
01 January 2024		4	100	119.105	2	0.8573	102.113
							113.998

Solution 33: Answer -A: (3)

Variance of PV of endowment assurance = ${}^2A_{45:\overline{15}|}$ - $(A_{45:\overline{15}|})^2$

$A_{45:\overline{15}|}$) = 0.42556

$^2A_{45:\overline{15|}} = {}^2A^1_{45:\overline{15|}} + {}^2A_{45:\overline{15|}}{}^1$

$= {}^2A_{45} - (v^2)^{15}\ {}_{15}p_{45}\ {}^2A_{60} + (v^2)^{15}\ {}_{15}p_{45}$

= 0.04172 – 0.17411*9287.2164/9801.3123*0.14098 + 0.17411*9287.2164/9801.3123

= 0.04172 – 0.16498*.014098 -0.16498

= 0.18344

Variance = 0.18344 – 0.42556^2 = 0.00234

Solution 34: Answer -D: (4)

n =15, $a_{\overline{n|}}$ @10% = 7.6061, v =0.90901

L = 7.6061X or X = L / 7.6061

Principal O/s before payment of 5th Instalment = 6.4951X

Principal O/s before payment of 6th Instalment = 6.1446X

Principal in 5th instalment = 6.4951X - 6.1446X = 0.3505 X

Given 0.3505L/7.6061 = 529.93, L = 11500

OR

Principal in 5th instalment = Xv^11 = 0.35049 X = 0.35061L / 7.6061

Given 0.35061L/7.6061 = 529.93, L = 11500

Solution 35: Answer-D: (4)

Capital gain test: $i^{(2)} > (1-t) * \frac{D}{R}$

$i^{(2)} = 2 * (1.06^{.5} - 1)$=.05913

$(1-t) * \frac{D}{R} = 0.70 * \frac{7}{100} =$ .049

Since $i^{(2)} > (1-t) * \frac{D}{R}$, there is a capital gain on redemption. The worst case scenario is the latest possible redemption date (i.e. time 10 years). This will give the minimum yield therefore:

Calculate the price at the final date of redemption, Hence $n = 10$

P = $(1 - 30\%) * 7 * a^{(2)}_{\overline{10|}} + 100v^{10} - 10\% * (100 - P)v^{10}$

= 0.70*7*7.4689 +100*0.55839 – 10*0.55839 + 0.055839P

P = 91.99

Solution 36: Answer -C: (4)

P = S [$vq_x + v^2p_xq_{x+1} + v^3\,{}_2p_xq_{x+2}$ + ……….]
Since $q_x = q_{x+1}$ and ${}_2p_x = (p_x)^2$
P = S [$vq_x + v^2p_xq_x + v^3(p_x)^2q_x$ + ……….]
P = S vq_x [$1 + vp_x + v^2(p_x)^2$ + ……….]
P = S vq_x [$1/(1- vp_x)$]
And v = P / [$Sq_x + P(1-q_x)$]

v = 82,608.70 /[100,000*0.25 + 82,608.70*(1-0.25)] = 0.95

P = 100,000*0.95*0.20*[1/(1-0.95*0.80)] = 79,166.67

Solution 37: Answer- B (4)

Since $_{m|}a_{\overline{n|}} = a_{\overline{m+n|}} - a_{\overline{n|}} = v^n a_{\overline{n|}}$
Therefore,
500 $v^n a_{\overline{n|}}$ = 1788.19
$v^n a_{\overline{n|}}$ = 1788.19/500 = 3.576376
And
3000 $a_{\overline{n|}}$ + 1000 $v^n a_{\overline{n|}}$ = 37,986.14
3000 $a_{\overline{n|}}$ + 1000* 3.576376 = 37986.14
3000 $a_{\overline{n|}}$ = 37986.14 – 3576.376
$a_{\overline{n|}}$ = 11.469921
Since $v^n a_{\overline{n|}}$ = 3.576376, Therefore v^n = 0.311805

➔ $v^n a_{\overline{n|}}$ = 3.576376

➔ $v^n (1- v^n)/ I$ = 3.576376

0.311805*(1-0.311805)/i = 3.576376

→ 0.214583/i=3.576376

→ i = 0.214583/3.576376

➔ i = 0.06

and hence i = 6.0%

Solution 38: Answer - D (4)

d= 0.047619,

Let single premium = S

PV of death benefit of first 5 years=

$S[(1+i)\ v\ q_{40} + (1+i)^2 v^2\ p_{40}\ q_{41} + \ldots. + (1+i)^5 v^5\ {}_4p_{40}\ q_{44}]$

$= S\ [q_{40} + p_{40}\ q_{41} + \ldots. + {}_4p_{40}\ q_{44}]$

$= S\ {}_5q_{40}$ = S (1- 0.994422) = 0.005578*S

PV of death benefit of next 10 years =

$100{,}000^*\ v^5\ {}_5p_{40}\ [\ A_{45} - v^{10}\ {}_{10}p_{45}\ A_{55}]$

=100,000*0.78353*0.994422*[0.20814–0.61391*0.975157*0.31700]

= 1430.94

S= 1430.94 + 0.005578*S

S= 1438.97

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