

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

23rd November 2024

CM1 - Actuarial Mathematics (Paper A)

Time allowed: 3 Hours 15 Minutes

Total Marks: 100

- Q. 1)** Active members of a pension scheme are subject to following decrements at the given ages where r stands for retirement and d stands for death:

| Age (x) | $(al)_x$ | $(ad)_x^r$ | $(ad)_x^d$ |
|-------------|----------|------------|------------|
| 60 | 2000 | 100 | 20 |
| 61 | 1880 | 90 | 30 |
| 62 | 1760 | 80 | 40 |

Calculate ${}_3(aq)_{60}^d$.

- A. 0.023
 B. 0.054
 C. 0.045
 D. 0.135

(2)

- Q. 2)** A health insurance company uses a three-state (Healthy, Sick and Dead) continuous-time Markov model for pricing of health insurance products. The constant force of transition for all ages are as under:

- Healthy to Sick, $\sigma = 0.002$
- Healthy to Dead, $\mu = 0.020$
- Sick to Dead, $\nu = 0.050$

The company ignores all other transitions. The company issues a 20-year policy to a healthy life aged 40 years exact. Sickness benefit of Rs. 200,000 is payable immediately on becoming sick of a healthy life. Calculate the expected present value of this sickness benefit assuming force of interest of 6% p.a.

- A. Rs. 3,191
 B. Rs. 4,239
 C. Rs. 4,694
 D. Rs. 3,932

(2)

- Q. 3)** An increasing annuity is payable annually for 20 years and payments are to be made in advance i.e. at the beginning of each year. The first annuity amount is Rs. 1,000 and rate of increase is 4% p.a. compounding. Find the present value of this annuity at annual effective interest rate of 6%.

- A. Rs. 12,158
 B. Rs. 16,473
 C. Rs. 16,790
 D. Rs. 17,205

(2)

- Q. 4)** A male life aged 65 years exact wants to buy an annuity of Rs. 10,000 per annum for his wife currently aged 60 years exact. Annuity will commence on death of the male life and will be payable monthly in arrear to female life for the rest of her life. Calculate the single premium.

Basis: Mortality Male life – PMA92C20, Female life – PFA92C20, Interest - 4% per annum, Expenses- Nil.

- A. Rs. 57,120
 B. Rs. 9,840
 C. Rs. 27,260
 D. Rs. 39,700

(2)

- Q. 5)** A male life aged (x) and a female life aged (y) purchased an annuity of Rs. 5,000 per annum increasing by 4% each year payable annually in advance so long as both lives survive. Calculate the present value of this annuity.

Basis: Mortality $\mu_x = 0.035$ for all x for male lives and

$\mu_y = 0.025$ for all y for female lives
Interest 6% per annum

- A. Rs. 44,825
B. Rs. 104,460
C. Rs. 38,520
D. Rs. 65,785

(2)

Q. 6) For a 5 year decreasing term insurance on age (x) the premium is payable for 3 years. You are given:

- i. Death benefit in year $t + 1 = Rs. 10,000 * (5 - t)$ for $t = 0,1,2,3$ & 4
ii. Death benefit is payable at the end of year of death
iii. $q_{x+t} = 0.02 + 0.005t$ for $t = 0,1,2,3$ & 4
iv. $i = 0.4$
v. Annual premium = Rs. 1,231.80
vi. Pricing and reserving basis are same.

Calculate ${}_2V$, the reserve at the end of year 2. Ignore expenses.

- A. Rs. 337.81
B. Rs. 594.21
C. Rs. 573.39
D. Rs. 373.81

(2)

Q. 7) Find the probability that exactly one of the lives (x) and (y) will survive n years, given that ${}_n p_x = 0.90$ and ${}_n p_y = 0.80$

- A. 0.72
B. 0.26
C. 0.28
D. 0.02

(2)

Q. 8) A five year unit-linked policy issued by an insurance company to a life aged 50 years exact has the following profit vector:

Rs. (750, -350, -250, -200, 225)

Calculate non-unit reserves required at the end of 1st policy year in order to zeroise the negative cashflows. Assume rate of interest on non-unit fund cash flows as 6% per annum and mortality as under:

| | | | | | |
|-------------|-------|-------|-------|-------|-------|
| Age (x) | 50 | 51 | 52 | 53 | 54 |
| q_x | 0.008 | 0.009 | 0.010 | 0.011 | 0.012 |

- A. Rs. 720.61
B. Rs. 714.88
C. Rs. 715.43
D. Rs. 715.99

(2)

Q. 9) An amount of Rs. k is payable at the end k^{th} year for perpetuity where $k = 1,2,3, \dots$. Find the present value of these payment at interest rate of 4% p.a.

- A. Rs. 25
B. Rs. 125
C. Rs. 625
D. Rs. 650

(2)

Q. 10) If $a_{\overline{1}|} = x$ and $a_{\overline{2}|} = y$, find the value of i .

- A. $\frac{y-x}{x}$
 B. $\frac{2x-y}{x}$
 C. $\frac{2x-y}{y-x}$
 D. $\frac{y-x}{2x-y}$ (2)

Q. 11) A loan of Rs. 100,000 is being repaid by 20 annual payments. The first payment is due on the date in which the loan is taken out. The first 5 payments are Rs. k each, next 5 payments are Rs. $2k$ each and last 10 payments are Rs. $4k$ each. Assume $i = 10\%$ and find the value of k .

- A. Rs. 5,058
 B. Rs. 5,563
 C. Rs. 11,746
 D. Rs. 10,678 (2)

Q. 12) A life insurance company issued a 5-year Unit Linked Endowment Assurance contract. The details are as under:

- Age at issue : 45 years
 - Annual Premium : Rs. 5,000
 - Allocation rate : 98% for all premiums
 - Bid/offer spread : 5%
 - Unit growth rate : 8% p.a.
 - Management charges : 2% p.a. (deducted at the end of year)
- No other charges are deducted from the fund.

Calculate the value of unit fund at the end of 5th year.

- A. Rs. 29,494
 B. Rs. 29,142
 C. Rs. 27,685
 D. Rs. 28,250 (2)

Q. 13) An investor pays Rs. 2,000 every half-year in advance into a 15-year savings plan. The interest rate is 8% per annum convertible monthly for the first 5 years and 8% per annum convertible half-yearly for the final 10 years. Calculate the accumulated value of the fund at the end of the 15th year.

- A. Rs. 116,656
 B. Rs. 117,843
 C. Rs. 116,863
 D. Rs. 118,052 (2)

Q. 14) A manufacturer is planning the production of robots in India. The cost of development comprises Rs. 100 lakhs payable on 1 January 2025 and Rs. 15 lakhs payable on 1 January 2026. From 1 January 2027 the robots will be ready for marketing and it is assumed that income will be received at the end of each year at a rate of Rs. 25 lakhs per annum for 20 years. Calculate the discounted payback period at an effective rate of interest of 10% per annum.

- A. 5 years
 B. 7 years
 C. 9 years
 D. 11 years (2)

Q. 15) The issue price at time $t = 0$ of a three-year bond bearing coupons of 10% per annum payable annually in arrear and redeemed at par is Rs. 111.39 per Rs. 100 nominal. One-

year spot rate per annum effective at time $t = 0$ is 6%. The one-year forward rate of interest at time $t = 1$ year is 5.5% per annum effective. Calculate the one-year forward rate per annum effective at time $t = 2$ years.

- A. 5.25%
- B. 5.50%
- C. 5.75%
- D. 6.00%

(2)

- Q. 16)** XYZ company issued a whole life policy to a life aged 50 exact. On death of the life assured, the sum assured Rs. 100,000 and annual reversionary bonuses are payable immediately on death. The annual reversionary bonuses are added at the rate Rs. 30 per thousand sum assured at the start of each year. Level premiums are payable annually in advance throughout the lifetime of the life assured. Calculate the gross annual premium.

Basis: Mortality - AM92 Ultimate
Interest - 6% per annum
Expenses - 5% of each premium.

- A. Rs. 2,704
- B. Rs. 2,627
- C. Rs. 2,569
- D. Rs. 2,495

(2)

- Q. 17)** An insurance company issued an Endowment Assurance policy for Sum Assured Rs. 20,000. The benefit is payable at the end of the year of death. A level premium of Rs. 2,000 is payable annually in advance, throughout the term of the policy. For a policy in force at the start of the 15th year, you are given the following information:

- Reserve at the start of the year, ${}_{14}V = Rs. 15,000$
- Reserve at the end of the year per survivor, ${}_{15}V = Rs. 17,500$
- Probability of death during the year, $q_x = 0.03$
- Rate of interest earned, $i = 6\% p. a.$

Calculate the profit expected to emerge at the end of the 15th year per policy in force at the start of that year. Ignore expenses and all decrements other than death.

- A. Rs. 445
- B. Rs. -80
- C. Rs. -1,675
- D. Rs. -575

(2)

- Q. 18)** A three-year unit-linked endowment assurance policy is sold to life aged (x) exact. The expected profit per policy in force at the start of each year for this policy, without making allowance for surrenders, is: Rs. (-400, 100, 400). It is now assumed that for the cash flows for this policy, 10% of all policies in force at the end of the first policy year are surrendered at that time. The surrender value payable at that time is the bid value of units on the date of surrender less surrender charges of Rs. 1,000. Given that independent rates of mortality are $q_x = 0.02$, $q_{x+1} = 0.03$, and $q_{x+2} = 0.04$, calculate the revised expected profit per policy in force at the start of year for first policy year.

- A. Rs. 600
- B. Rs. -300
- C. Rs. -302
- D. Rs. 198

(2)

- Q. 19)** An ordinary share pays half-yearly dividends. A dividend of Rs. 1.50 per share is due in exactly 3 months' time. Subsequent dividends are expected to grow at a compound rate

of 3% per half-year forever. The annual effective rate of return for an investor is 10%. Calculate the price per share for the investor.

- A. Rs. 76.85
- B. Rs. 81.67
- C. Rs. 79.75
- D. Rs. 44.33

(2)

Q. 20) A whole life insurance policy is issued at age (x). The gross annual premium is payable at the beginning of each year for n years. The death benefit is Rs. 100,000 in the first year and increases by Rs. 100,000 per year thereafter. Death claim is payable at the end of year of death. The expenses for the policy are as under:

- i. 30% of the first premium
- ii. 10% of the premium for renewal premiums
- iii. 5,000 at the time of issue of the policy
- iv. 500 at the beginning of each policy year

Find the gross annual premium given that:

$$(IA)x = 7, \ddot{a}_x = 24, \ddot{a}_{x+n} = 18, {}_n p_x = 0.70, i = 6\% \text{ and } n = 30.$$

- A. Rs. 44,812
- B. Rs. 37,101
- C. Rs. 38,591
- D. Rs. 36,910

(3)

Q. 21) ${}_{10}p_x = 0.867219$, $a_{x:\overline{9}|} = 6.465$, $i = 0.06$, Calculate $A_{x:\overline{10}|}^1$

- A. 0.09320
- B. 0.05774
- C. 0.04842
- D. 0.05583

(3)

Q. 22) Calculate e_{60} , the 'curtate expectation of life' of a life aged 60. Given that:

$$q_x = .02 \text{ for } 60 \leq x < 80,$$

$$q_x = .04 \text{ for } 80 \leq x < 100 \text{ and}$$

$$q_x = 1 \text{ for } x = 100.$$

- A. 25.23
- B. 29.68
- C. 27.16
- D. 30.16

(3)

Q. 23) A whole life insurance policy is issued to life aged (x) and aged (y) for sum assured of Rs. 100,000. Death benefit is payable upon second death i.e. at the end of the year of second death. No other benefits are payable under the policy. Annual premium is payable as long as (x) survives. Calculate annual premium, given that $\ddot{a}_x = 15.632$, $\ddot{a}_y = 16.652$, $\ddot{a}_{xy} = 14.090$, $i = 0.04$.

- A. Rs. 2,930
- B. Rs. 1,742
- C. Rs. 1,921
- D. Rs. 2,792

(3)

Q. 24) Present value of 15 year annuity payable at the end of each year is equal to the present value of a deferred perpetuity starting from the end of 16th year and thereafter payable at the end of each year forever. Find the effective annual interest rate.

- A. 4.00%
 B. 4.29%
 C. 4.56%
 D. 4.73%

(3)

Q. 25) An insurance company has liabilities at the end of each of the next 35 years. The total liability at the end of the first year is Rs. 1 Crore. Thereafter, the liability is expected to increase at a fixed rate of 2.83% per annum compound. Calculate the discounted mean term of the liabilities using a rate of interest of 9% per annum effective.

- A. 10.31 years
 B. 14.50 years
 C. 14.10 years
 D. 12.43 years

(3)

Q. 26) Calculate single premium for 4 year term insurance policy issued on life aged 30 years exact. Sum assured of the policy in first year is Rs. 10,000 and at the end of each year sum assured increases at the rate of 20% p.a. compounding. Assume the interest rate of 4% and mortality from the following life table:

| Age (x) | l_x |
|-------------|--------|
| 30 | 100000 |
| 31 | 99000 |
| 32 | 97500 |
| 33 | 95500 |
| 34 | 93000 |

- A. Rs. 853.41
 B. Rs. 887.88
 C. Rs. 933.34
 D. Rs. 913.52

(3)

Q. 27) An insurance company issued 3 year Endowment Assurance policy on aged (x) for Sum assured Rs. 100,000. Annual Premium payable for 3 years in advance is Rs. 35,000. Assuming $q_x = 0.10$, $q_{x+1} = 0.11$, $q_{x+2} = 0.12$, risk discount rate = 6%, and the expected present value of profits is Rs. 3,142. Calculate the profit margin.

- A. 3.50%
 B. 6.65%
 C. 3.71%
 D. 3.89%

(3)

Q. 28) On 1st January 2023, an insurance company sold 1000 Pure Endowment policies to lives aged 50 exact. Sum Assured of each policy was Rs. 100,000 and term of all policies was 10 years. Reserve at the end of 1st year is Rs. 8,106.80. During the first year of policies, there were 4 deaths from the policies written. Calculate the total mortality profit or loss to the insurer in the year 2023.

Pricing and reserving Basis:

- Interest: 4% per annum,
- Mortality: AM92 Ultimate,

- A. Rs. 9,695
 B. Rs. 12,095
 C. Rs. -137,105
 D. Rs. 137,105

(3)

Q. 29) You are given:

$$A_x = 0.20, A_{x+n} = 0.24, A_{x:\overline{n}|} = 0.62 \text{ and } i = 0.06$$

Assume deaths are uniformly distributed over each year of age. Calculate $\bar{A}_{x:\overline{n}|}$.

- A. 0.63833
- B. 0.62199
- C. 0.62404
- D. 0.63507

(3)

Q. 30) A Select Mortality Table has select period of 2 years. The following table shows the extract from the Select Mortality Table:

| Age (x) | $q_{[x]}$ | $q_{[x-1]+1}$ | q_x |
|---------|-----------|---------------|-------|
| 75 | 0.07 | 0.09 | 0.11 |
| 76 | 0.08 | 0.10 | 0.12 |
| 77 | 0.09 | 0.11 | 0.13 |

You are given that $A_{77} = 0.8357$ and $i = 4\%$, find the value of $A_{[75]}$

- A. 0.800
- B. 0.797
- C. 0.775
- D. 0.815

(3)

Q. 31) An IIT graduate is planning to start an AI project. At the beginning of each of the first two years Rs. 1000 lakhs will be invested in the project. From the beginning of the first year until the end of the 15th year, net revenue will be received continuously. The net revenue for next 15 years will be Rs. 200 lakhs per annum and growing continuously at a rate of 6% per annum effective. Calculate the net present value of the project at an effective rate of interest of 10% per annum.

- A. Rs. 392.57 lakhs
- B. Rs. 1301.67 lakhs
- C. Rs. 119.34 lakhs
- D. Rs. 613.77 lakhs

(3)

Q. 32) A two-year index linked bond was issued on 1st January 2022. The coupons on the bond were of nominal amount 8% pa payable half-yearly in arrears i.e. first coupon was payable on 1st July 2022. The nominal redemption rate was 100%. Coupon and redemption payments were indexed by reference to the value of an inflation index with a time lag of six months. A tax-exempt investor purchased Rs. 100 nominal at issue and held the bond till redemption.

The inflation index was as follows:

| Date | Inflation Index |
|-----------------|-----------------|
| 01 July 2021 | 105.00 |
| 01 January 2022 | 108.50 |
| 01 July 2022 | 111.40 |
| 01 January 2023 | 117.20 |
| 01 July 2023 | 120.25 |
| 01 January 2024 | 130.00 |

The annual effective money yield obtained by the investor was 8% per annum. What was the issue price per Rs. 100 nominal?

- A. Rs. 100
- B. Rs. 114

- C. Rs. 119
D. Rs. 122

(3)

- Q. 33)** Calculate variance of the present value of 15-year Endowment assurance issued on life aged 45 years exact for sum assured of 1. The death benefit is payable at the end of the year of death.

Basis:

Mortality - AM92 Ultimate

Rate of interest - 6% per annum

- A. 0.00234
B. 0.02620
C. 0.24446
D. 0.42556

(3)

- Q. 34)** A loan is being repaid with annual installments for 15 years at interest rate 10% p.a. First installment will be due one year after the loan is taken. The amount of principal repaid in 5th installment is Rs. 529.93. Find the amount of original loan.

- A. Rs. 10,000
B. Rs. 10,500
C. Rs. 11,000
D. Rs. 11,500

(4)

- Q. 35)** A bond pays coupons of 7% per annum half yearly on 1 January and 1 July. The bond will be redeemed at par on any date between 1 January 2026 and 1 January 2031 inclusive, at the option of the borrower.

An investor pays tax at 30% on interest income and 10% on capital gains. He purchased the bond on 1 January 2021, immediately after the payment of the coupon then due, at a price which gave him a net yield of at least 6% per annum effective. Calculate the price per Rs. 100 nominal at which the investor bought the bond.

- A. Rs. 92.44
B. Rs. 95.32
C. Rs. 95.67
D. Rs. 91.99

(4)

- Q. 36)** A whole life insurance policy is issued at age 80 years on male life for Sum Assured of Rs. 100,000. Sum Assured is payable at the end of the year of death, whenever that may be. Assuming male mortality for $x \geq 80$ is constant and $q_x = 0.25$ for all ages, the Net Single Premium for this policy is Rs. 82,608.70.

Based on above information, calculate the Net Single Premium for a whole life insurance policy on female life aged 80 years for Sum Assured of Rs. 100,000. Assume that female mortality for $y \geq 80$ is constant and $q_y = 0.20$ for all ages and interest rate is same as used for male life.

- A. Rs. 66,086.96
B. Rs. 72,626.82
C. Rs. 79,166.67
D. Rs. 80,887.69

(4)

- Q. 37)** The present value of an n year annuity of Rs. 500, deferred for n years and payable annually in arrear (i.e the annuity payments will be made at the end of year $n + 1, n + 2, \dots, 2n - 1, 2n$ year) is Rs. 1,788.19.

Further, the sum of the present value of following two immediate annuities is Rs. 37,986.14:

- (1) Rs. 1,000 payable at the end of each year for $2n$ years; and

(2) Rs. 2,000 payable at the end of each year for first n years.

Find the annual effective interest rate.

- A. 5.0%
- B. 6.0%
- C. 7.0%
- D. 8.0%

(4)

Q. 38) A single premium special term insurance policy was issued at age 40 for 15 years. Death benefit is payable at the end of year of death. During first 5 years death benefit shall be equal to accumulated value of single premium upto the date of payment with interest at rate used to calculate the premium. Death benefit during policy term after 5 years shall be Rs. 100,000. You are given-

- i. $i = 0.05$
- ii. $A_{45} = 0.20814$
- iii. $A_{55} = 0.31700$
- iv. ${}_5p_{40} = 0.994422$
- v. ${}_{10}p_{45} = 0.975157$

Ignore expenses. Calculate the single premium.

- A. Rs. 1,298
- B. Rs. 1,681
- C. Rs. 2,526
- D. Rs. 1,439

(4)
