

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

CM2B - Financial Engineering and Loss Reserving

Time allowed: 1 Hour 45 Minutes (14.45-16.30 Hours)

Total Marks: 100

- Q. 1)** An investor is trying to choose between the investments whose distributions of returns are described below:

Investment A: 0.35 probability that it will return 9%
 0.45 probability that it will return 12%
 0.2 probability that it will return 14%

Investment B: 0.15 probability that it will return 10%
 0.55 probability that it will return 15%
 0.25 probability that it will return 20%
 0.05 probability that it will return 40%

Investment C: A uniform distribution on the range 10% to 20%

- i) Calculate the following for each investment:

a) Expected return (3)

b) Variance of return (3)

c) Semi-variance (3)

d) Expected shortfall below 13% (3)

e) Shortfall probability below 15% (3)

- ii) An investor invested INR 10,000 in each of the three investments and a utility function $U(w) = 2w - 0.00015w^2$.

Calculate the expected utility of each investment.

(6)
[21]

- Q. 2)** The following data are taken from the records of a non-life insurance company. Claim amounts paid in the year of accident and incremental amounts paid in subsequent years are as follows:

| | Year of Payment | | | | | |
|---------------|-----------------|-------|-------|-------|-------|-------|
| | | 2019 | 2020 | 2021 | 2022 | 2023 |
| Accident Year | 2019 | 34990 | 21540 | 3105 | 488 | 88 |
| | 2020 | | 32107 | 2500 | 18625 | 9256 |
| | 2021 | | | 19995 | 10780 | 6721 |
| | 2022 | | | | 41567 | 8760 |
| | 2023 | | | | | 27890 |

Annual rate of claims inflation

| Year | Rate |
|------|------|
| 2019 | 6% |
| 2020 | 5% |
| 2021 | 7% |
| 2022 | 9% |

Expected future rate of claims inflation

| Year | Rate |
|------|------|
| 2023 | 8% |
| 2024 | 7% |
| 2025 | 7% |
| 2026 | 6% |

Reserves held in deposit on or after 31/12/2023 are expected to earn interest @7.5% per annum. Assuming that 2019 claims would have “run-off” fully by the end of 2023, you have to estimate the reserves needed in respect of claims outstanding at that time.

- i) Use the Basic Chain Ladder method without taking into account the given data regarding inflation and interest earnings. (10)
 - ii) Use inflation adjusted chain ladder method, taking into account the inflation, both past and future, and interest earned on deposits from 31/12/2023 onwards. (20)
- [30]**

Q. 3) A non-dividend-paying stock, S_t , has a current price of 300. After 1 year the price of the stock could increase to 345 or decrease to 255. After a further 1 year, the price could increase from 345 to 375, or decrease from 345 to 300. From 255 the price could increase to 300 or decrease to 225. The annually compounded risk-free rate of interest is 6% per annum and the real-world probability that the share price increases at any time step is 0.75. Adopt a binomial tree approach with semi-annual time-steps.

- i) Calculate the state-price deflator after two years. (8)
 - ii) Calculate, using the state-price deflator from (i), the price of a non-standard option which pays out $\max\{0, \log(S_2 - 270)\}$ two years from now. (8)
- [16]**

Q. 4) An employer contracts with his staff to give each of them 10,000 shares in one year's time provided the share price increased from its current level of INR 20 to at least INR 30 at the end of the year.

You may assume the following parameters:

- risk free interest rate: 8% p.a. continuously compounded.
 - stock price volatility: 25% p.a.
 - dividend yield: nil
- i) Calculate the value of the contract with each employee by considering the terms of the Black-Scholes formula. (10)

The employer now wishes to limit the gain to each employee to INR 400,000.

- ii) Calculate the value of this revised contract.

(10)

[20]

- Q. 5)** A company is planning to offer an employee health scheme suitable for their staff wherein they will be able to get reimbursement of the minor medical expenses (consultation fees and cost of medicines prescribed) of themselves and their family members. The aggregate yearly medical expenses of an individual employee's family (including the employee) has a compound Poisson distribution with Poisson parameter 0.45. The individual consultation expense has a Gamma (α, λ) distribution and the prescribed medicine cost is independent of the consultation fee with a uniformly distributed random variable between INR 500 to Rs. C ($C > 500$). Every month, the company has to set aside INR 80 for each employee to fund the scheme. Any deficit or surplus from the scheme is funded or taken back by the company at the year end to bring the net fund position at zero. The random variable S denotes the total yearly claimed consultation & medicine expenses from the scheme. You may assume S has an approximately normal distribution. You may also ignore any interest earned by the scheme.

If $\alpha = 5$, $\lambda = 0.01$ and $C = 2000$, calculate

- i) Expected value of an individual claim.

(4)

- ii) Minimum number of employees the company should have to be 95% sure it will not have to fund any deficit at the year end.

(9)

[13]
