INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

30th May 2024

CM1B - Actuarial Mathematics

Time allowed: 1 Hour 45 Minutes (09.30-11.15 Hours)

Total Marks: 100

Q. 1)

i) You are given a curve of one-year forward rates f_{t+r}, where t and r are given in years. Derive the n-year spot rates for the years 1 to 30 (both years included). (4)

ii) Calculate the 30-year par yield. (5)

iii) You are the financial controller at a BBU life insurance company and are looking to take over the business of one of the two health insurance companies in the market. Following are the details of projected cash flows from the two companies –

a) **XYZ Health** has an annual renewal premium income of Rs. 2.25 Crore. Their books are closed for new business and the longest policy in-force in the books currently has exactly 25 years remaining to maturity. Around 3% of their in-force policies tend to lapse each year, hence the premium income drops by 3.75% each year. The expenses are Rs. 50 lakhs per year inflating from the second year. The company currently maintains reserves worth Rs. 4 Crore which has to be bought on takeover; this can be used to pay the claims which are expected to be Rs. 80 lakh in the first year, growing by 5% every 5 years. One-time initial cost of takeover is Rs. 5 Crore. Investments yield a return of around Rs. 30 lakh per year, growing at 8% per annum.

b) **PWR Health** has an annual renewal premium income of Rs. 12 Crore, dropping at 10% each year. Premium is payable up to year 5 only. The policies have an outstanding term of 16 years. Expenses are expected to be Rs. 1.2 Crore per year. The company currently has a reserve maintained at Rs. 6.05 Crore which has to be bought on takeover. Expected claims per year are Rs. 2 Crore, growing at 2.50% every 2 years. The company invests aggressively in equity markets hence the returns are expected to be volatile. Projected investment values and expected returns on the fund are given in the workbook (expected returns are applied on an average of the fund value over the year). One-time initial cost of takeover is Rs. 3.5 Crore.

Risk discount rate considered for the cash flows**:** one-year forward rates given in the base input sheet

Inflation**:** 4.5% p.a., starting from the second year.

Taxes paid to the Government:

- 10% of the renewal premium income &
- 15% of the investment returns.

Assume:

- There is no interest earned on reserves.
- Reserves can be used to pay claims till the time reserves gets exhausted.
- No taxes on negative returns, but losses to be taken.

For each of above companies -

a) Determine the cash flows and (22)

b) Hence Net present value (NPV) of cash flows and (4)

c) Internal Rate of Returns (IRR) (4)

iv) Which of the above two companies is a better measure for the project viability? Comment on the results found in (ii) above. Which of the two should BBU company take over? (4)

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Q. 2) Shyam went to KLS Bank for a personal loan worth INR 50 lakhs. He would like to repay the loan annually in arrears over the next 15 years. The rate of interest is 5% per annum effective.

i) Calculate the annual loan repayment amount given that:

a) Level payments. (5)

b) Payments reduce by INR 20,000 every year starting from year 1.

***Note**: solve above parts without using excel function like goal seek.* (7)

c) Verify the same by creating a loan schedule for part (b). (6)

ii) What is the total amount of interest paid in each of the above arrangements? (4)

On reconsideration after a week, Shyam decides to make repayments worth Rs. 6.5 lakh each year for the next 6 years and then an amount of Rs. 2 lakh each year for the remaining years. By this time, the bank has changed their offering of the loan. The interest rate is now set to 5.5% per annum and the loan has to be repaid in 12 years.

iii) Calculate the loan amount that Shyam will now receive at his chosen repayment amount, with the new Rate of Interest 5.5% p.a. for a term of 12 years. (5)

[27]

Q. 3) A life insurance company issues a 30-year unit-linked joint life endowment assurance policy to a man aged 33 exact and a woman aged 35 exact. Should either life die during the term a benefit of INR 500,000, or the bid value of the units if higher, is payable at the end of the year of the first death. On survival of both lives to maturity, 105% of the bid value of the units is payable. Premiums, which increase at a fixed rate each year, are payable annually in advance throughout the policy or until the death of the first life. You are provided the following details:

- Initial premium: INR 60,000; premiums increasing at a compounding rate of 2% per annum, starting from the second year.
- Rate of growth on assets in the unit fund: 4% p.a.
- Rate of interest on non-unit cash flows: 2.5% p.a.

- Expenses
    - Initial – INR 5,000
    - Renewal – INR 1,000 payable second premium due date onwards

- Commission
    - Initial – 30% of premium
    - Renewal – 2% of renewal premiums

- Allocated premium
    - First year – 86%
    - Second year – 94%
    - Subsequent years – 100%

- Bid offer spread: 4%
- Management charges: 0.5% of the bid value of units, deducted at each policy year end
- Risk discount rate: 5% p.a.

Assumptions:

- Deaths are assumed to occur uniformly across each year of age
- No inflation on expenses

i) Project the unit fund for each policy year. (9)

ii) Determine the profit signature for the policy. (21)

[30]

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