INSTITUTE OF ACTUARIES OF INDIA

Subject CM1A - Actuarial Mathematics

May 2024 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1: (A) **[2]**

Solution 2: (B) **[2]**

Amount needed= $(10,00,000 / 1.065^3)$ = Rs.827849.10

Solution 3: (C) **[4]**

First amount = $2500*(1-18\%/12)^{-3}$= Rs. 2616

Second Amount =$2616*(1+20\%/4)^3$ = Rs. 3028

Solution 4:(D) **[2]**

Solution 5:(C) **[4]**

PV of Income = Int(0,5)(v(t)p(t)dt

=Int(0,5)(35,00,000 * (1-0.01t)dt

=Rs. 170,62,500

PV of outgo= 60,00,000 * v (1) +40,00,000 v (3) = 60,00,000*0.99 + 40,00,000*0.97

= Rs. 98,20,000

So NPV =170,62,500-98,20,000= Rs.72,42,500

Solution 6: (C) **[2]**

Solution 7: (C) **[2]**

${}_4q_1=1-{}_4p_1=1-l_5/l_1$

=1-99032/99186=0.00155

Solution 8: (D) **[4]**

EPV=500,000 X $v^5 X {}_5P_{55} X A_{60}$

$=500,000 \text{ X } (1+4\%)^{-5} X l_{60}/l_{55} X A_{60}$

=Rs. 182253.45

Solution 9: (A) **[4]**

EPV of the maturity benefit = $300,000*D_{65}/D_{[40]}$ =Rs. 100738.11

EPV of the death benefit = 6700*(*IA*)(1, [40]:25|) =Rs. 5870.36

Total EPV = 10073.81+587.02 =Rs. 106608.47

Solution 10: (B) **[2]**

The accumulated fund value=1,000,000*1.05^20=Rs. 2653297.71

The Expected number of survivors =$10,000*l_{60}/l_{40}$

= 10,000*9287.216/9856.2863

= 9422.63 rounded to nearest integer 9423

Expected Payout=2653297.71/9423= Rs. 281.59

Solution 11: (B) **[2]**

$S_{55:10|} = (1+i)^{10} (l_{55}/l_{65})\, a_{55:10|}$

$=1.04^{10} * (l_{55}/l_{65}) * a_{55}-a_{45}$ putting values gives

=12.166

Solution 12: (D) **[4]**

a(due)(m,65)-a(due)(65:65)- v^{10} ${}_{10}p_{65:65}$(a(due)(m,75)- a(due)(75:75))

=13.666-11.958-1.04^{-10} *(8405.160/9647.797) * (8784.955/9703.708) * (9.456-7.679)

=0.76117

So the EPV to husband is =20,000*0.76117= Rs. 15223

Solution 13: (D) **[4]**

For policyholders who die during the year, no funds are required at the end of the year
The reserve required at the end of the year for each surviving policy plus the annuity payment due at that time is:
$10{,}000a_{61} + 10{,}000 = 10{,}000adue_{61} = 10{,}000 * 16.311 = 163{,}110$
Death strain at risk is 0-163110=-163110

From the tables $q_{60}=0.002058$
Expected number of deaths during the year is $5000q_{60} = 10.29$
EDS is -163110*10.29=-1678402
ADS is -163110*9=-1467990
Mortality profit is given by EDS-ADS=-1678402-(-1467990) =-210412
Hence EDS is a loss of Rs. 210412

Solution 14: (B) **[2]**

Solution 15:

i)

a) A multiple decrement table is a computational tool to deal with decrements in a population which is subject to more than one decrement like Healthy, sick, terminally sick and dead. (2)

b) Independence of force of mortality means that the presence of multiple causes of decrement in a population does not affect the forces of decrement of each cause i.e. the force of decrement is independent of each other

(2)

(4)

ii) The required force of mortality is

Mu(d,50)=0.8*[-ln(1-q(d,50)]

Where q(d,50) is the relevant probability of death in the ELT15(Males) table so,

Mu(d,50)=0.003721

Calculating the dependent probabilities

$aq(d,50)=[0.003721/(0.003721+0.075)]x[1-e^{-(0.003721+0.075)}]=0.003578$

$aq(s,50)=[0.075/(0.003721+0.075)]x[1-e^{0.078721}]=0.072124$

So the first line of required decrement table is

$Age_{x.}$	$(al)_x$	(ad)(d,x)	(ad)(s,x)
50	100,000	357.80	7212.39
51	92429.81		

(6)

[10]

Solution 16:

i) Market segmentation theory

Market segmentation theory says that the shape of yield curve is determined by supply and demand at different terms. So, for example the yield at the short end of the curve will be determined by the demand from the investors who prefer short dated stocks i.e. those with short tail liabilities, as well as the supply of the short-dated stocks. Similarly, the yields at the long end will be a function of demand from investors interested in buying long bonds to match long dated liabilities, and supply of long dated stocks.

Market segmentation theory provides a framework for analyzing the complexities of financial markets and understanding how segmentation affects asset prices, investor behavior, and the efficiency of capital allocation. By studying market segmentation, economists and policymakers seek to identify opportunities for enhancing market integration, reducing inefficiencies, and promoting broader access to diversified investment opportunities

(2)

ii) Liquidity preference theory

Liquidity preference theory states that the investors will be in general prefer more liquid (i.e. shorter) stocks to less liquid ones as the short dated stocks are less sensitive to changes in interest rates. Hence investors purchasing long dated bonds would require higher yields in order to compensate them for the greater volatility of the stocks they are purchasing.

Liquidity preference theory provides insights into how interest rates are determined by the demand and supply of money, driven by individuals' motives for holding money versus investing in interest-earning assets. This theory underscores the importance of monetary policy in influencing economic activity and interest rate levels.

(2)

[4]

Solution 17:

i) Features of unit linked policies

a. The benefits are directly linked to the value of the underlying investments
b. The benefit payable in respect of each policy depends on the units allocated to the policy
c. Every time the policyholder pays premium, part of it is invested on policyholder's behalf in a fund chosen by her
d. The remainder goes to non-unit fund
e. The investment fund is divided into units which are priced continuously
f. There will be different prices for buying /selling the units, the difference is called bid/offer spread
g. Bid price is usually 5% lower than the offer price
h. At the time of claim/maturity/surrender, the cumulative value of the units available will be paid to the policyholder at the current rates of units (NAV)
i. Insurance company regularly (mostly monthly) deducts the charges for expenses and charge for insurance (mortality)
j. The charges deducted by the company will depend on the company's experience and are variable

k. There may be a minimum guaranteed sum assured to protect the policyholder against poor investment performance or to provide some benefit in the event of early claim (death)
l. The most common type of unit linked assurances are whole life and endowment assurances

(Max 5)

ii) Unit Fund—The unit fund is the amount held in units on behalf of the policyholder
Non-Unit Fund—The Non-unit fund is the amount deducted from premium and held by the insurance company to meet its expenses, cost of cover and profits.

Policyholder is entitled to receive money from the Unit fund. However, at times Policyholder may get higher or lower than the amount available in the unit fund.
E.g. in case of surrender insurance company would levy some penalty and will pay lower than the amount in Unit fund.

In case of early death when the unit fund is less than the sum assured agreed (or any guarantee) the balance is paid from non-unit fund.
Also, in case of maturity, if the unit fund value is less than agreed sum assured or guaranteed benefit, the balance is paid from the non- unit fund.

Items in both funds are

i. Premium less cost of allocation
ii. Expenses incurred by the insurance company
iii. Interest earned/charged on the non-unit fund
iv. Management charges taken from the unit fund
v. Extra death or maturity charges (if the benefits payable on death or maturity is greater than the value of Units held at the time of death or maturity)
vi. Profits on surrender, may arise when
vii. Surrender value paid is less than the unit fund value.

(Max 5)
[10]

<u>Solution 18:</u>

i) Future Loss Random Variable
The Future Loss Random Variable at time zero is the present value of the future benefits and expenses paid out less the present value of the premium received.

If the life dies we need to use K_{50} to determine the value of the sum assured (since this decreases by discrete amounts), but T_{50} for the discount factor (since the benefit is paid immediately on death). We add INR 400 to the amount paid on death as the claims expense.

The future loss random variable at the outset is
For K_{50} <10
$(150{,}400-10{,}000K_{50})\, v^{(T_{50})} + 300 + 43\, a_{\overline{K50}|} - P\ddot{a}_{\overline{K50+1}|}$

For K_{50}>=10

$300+43\, a_{\overline{9}|} - P\ddot{a}_{\overline{10}|}$

(2)

Just before the payment of the fifth premium, the life is aged 54 and the balance of the term is 6 years. So the future loss random variable is now

For K_{54} <6

$(110{,}400-10{,}000K_{54})\, v^{(T_{54})} + 43\, a_{\overline{K55+1}|} - P\ddot{a}_{\overline{K54+1}|}$

For K_{54}>=6

$43\, a_{\overline{6}|} - P\ddot{a}_{\overline{6}|}$

(2)

(4)

ii) Calculation of Premium

The premium equation using equivalence principle is

$P\ddot{a}_{50:10|} = 160{,}400\, A^{1}_{50:\overline{10}|} - 10{,}000\, IA^{1}_{50:\overline{10}|} + 300 + 43(\ddot{a}_{50:10|} - 1)$

from tables the level annuity factor is 8.314

For the other factors we shall need

D_{60}/D_{50}= 882.85/1366.61= 0.64601

The level term assurance factor is

$A^{1}_{50:\overline{10}|} \cong (1+i)^{0.5}\, A^{1}_{50:\overline{10}|} = (1.04)^{0.5}[\, A_{50:\overline{10}|} - (D_{60}/D_{50})\,]$
= 0.03490

The increasing annuity factor is

$IA^{1}_{50:\overline{10}|} \cong (1+i)^{0.5} IA^{1}_{50:\overline{10}|} = (1.04)^{0.5}[(IA)_{50} - (D_{60}/D_{50})((IA)_{60} + 10A_{60}))]$
= 0.21282

So we have

8.314P=160,400*0.03490-10,000*0.21282+300+43*7.314

This gives P = 491.31 (4)

iii) Gross Premium Prospective reserve

The gross premium prospective reserve for the policy at time 4 is the expected present value of the future benefits and expenses less the expected present value of the future premiums:

${}_4V = 120{,}400\, A^{1}_{54:\overline{6}|} - 10{,}000\, IA^{1}_{54:\overline{6}|} - (491.31-43)\, (\ddot{a}_{54:6|})$

We shall need

D_{60}/D_{54}=882.85/1154.22=0.76489

Term assurance factor is calculated as earlier

$A^1_{54:\overline{6}|}$=0.02830

$IA^1_{54:\overline{6}|}$=0.10502.

The annuity function is available in the Tables as 5.391

Therefore , the reserve is calculated as

$_4V$=-59.60

(4)

iv)

a. The gross premium reserve at time 4 is negative. This is typical of a decreasing term assurance where the cost of claims and expenses are relatively high in the early years of the policy and low in the later years

b. As the premiums are level, the future premiums have greater expected present value than the future claims and expenses, producing the negative reserve value.

c. The insurer would incur an overall loss from a group of policies of this type, should some of them lapse when their reserves are negative.

d. The insurer can avoid this problem if the premium paying term is reduced, for example to 6 years for a 10-year policy, so for reserves at later policy durations the EPV of the future premiums is reduced, causing the reserve value to rise.

(2)

v) The gross prospective and retrospective reserves will be same if both are calculated on the same basis which is same as premium calculation basis. (1)

[15]

Solution 19:

i) Net Present Value of the project. Working in crores

PV of outgo=85*(1+v+v^2) +125v^{10} @7%= Rs. 302.225 Crores (1)

PV of the income

= (30v^2 +32v^3 +34 v^4)* $\bar{a}_{\overline{1}|}$ +36v^5 *(1+1.02v+1.02^2 Xv^2 +………..+1.02^{14}XV^{14}) $\bar{a}_{\overline{1}|}$ (2)

The first part is calculated directly and second part is the geometric progression of 15 terms with first term 1 and common ratio 1.02v

Solving this

PV of the income = Rs. 347.709 Crores (2)

So the NPV is (347.709-302.225)*10000000=Rs. 45.48 Crores (1)

(6)

ii) To show that discounted pay back period does not fall within 10 years

The NPV of the income received up to 10 years can be calculated in part (i)

PV of the income (10 years) =78.263*0.96692+36*1.07^{-5}*[(1-1.02^5v^5)/ (1-1.02v)]*0.96692

= Rs. 188.698 Crores (3)

The PV of the outgo excluding the refurbishment at time 10 years is

PV of the outgo (10 Years) $=85*(1+v+v^2)$ =Rs. 238.682 Crores (1)

As the PV of the income is less than the PV of outgo up to the end of the 10^{th} year and the initial outgo precedes the start of the income, the accumulated value of the project's cash flow is negative throughout first 10 years. So the DPP does not fall within first 10 years. (1)

(5)

iii) Equating the PV of the income for the first n years (n>10) of the project to the PV of the outgo

$78.263*0.96692+36*1.07^{-5}*[(1-1.02^{n-5}v^{n-5})/ (1-1.02v)]*0.96692=302.225$ (1)

➔ $75.674+24.818**[(1-1.02^{n-5}v^{n-5})/(1-1.02v)]*0.96692 \quad =302.225$

This equation will only be exact if n is an integer

Solving this we get

$(1.02v)^{n-5}=0.57344$

Solving n=16.6 (1)

So the DPP ends between time 16 and time 17

To find exact value of DPP, we need to value the payments in the final part year (of length t say) exactly which gives the equation

$75.674+24.818* *[(1-1.02^{n11}v^{11})/ (1-1.02v)] +36X1.02^{11} v^{16} a_{\bar{t}|} \quad =302.225.$ (1)

Solving this, we find

$a_{\bar{t}|} \quad =(1-1.07^{-t})/\ln 1.07=0.60514$

$1.07^{-t}=0.95906$

$t=0.618$ (1)

So the discounted payback period is 16.618 years (1)

(5)

iv) Discounted Payback period is not a good criterion due to following reasons

a. It ignores cash flows after payback period
b. It does not consider time value of money
c. It is biased towards long term projects
d. Project decisions based on DPP may be contradicting the decision based on NPV

(2)

[18]

<u>Solution 20:</u>

i) EPV is given by

$5000*\bar{A}_{35} =50,000*1.06^{0.5} A_{[35]} =50,000*1.06^{0.5}*0.09475=4877.5.$ (1)

ii) The variance is given by

$50{,}000*[{}^2\bar{A}_{35}-(\bar{A}_{35})^2]$

Where ${}^2\bar{A}_{35}$ is evaluated using an interest rate of $1.06^2-1=12.36\%$. (1)

So the variance is
$=50{,}000^2\{1.1236^{0.5}X\,{}^2\bar{A}_{35}-(1.06^{0.5}-\bar{A}_{35})^2\}$

$=50{,}000^2X\{1.1236^{0.05}X0.01765-(1.06^{0.5}X0.09475)^2\}$

$=(4793.75)^2$ (1)

[3]

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