INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

28th May 2024

SP6 - Financial Derivatives

Time allowed: 3 Hours 15 Minutes (09.30 – 12.45 Hours)

Total Marks: 100

Q. 1)

i) What is Martingale representation theorem? (1)

ii) If V_t and W_t are two standard Brownian motions, then show that X_t is a martingale where X_t is given by:

$$X_t = V_t^2 W_t - \int_o^t W_u du$$ (5)

[6]

Q. 2) The Cox-Ingersoll-Ross (CIR) model is a one-factor interest-rate model of the short rate r (t) of the following form:

$$dr(t) = a\big(b - r(t)\big)dt + \sigma\sqrt{r(t)}dz(t)$$

where a, b, σ are constants and z (t) is a Brownian motion.

i) Describe the economic arguments in favor of mean reversion of interest rates. (2)

Under the CIR model, the price $P(t, T)$ of a zero coupon bond of maturity T valued at time t is given by:

$$P(t,T) = A(t,T)exp[-B(t,T)r(t)]$$

Where: A(t,T) is the function representing the discount factor and B(t,T) is a function that represents the instantaneous forward rate.

ii) Show that the continuously compounded interest rate at time t for a term of (T – t), R(t,T), is given by:

$$R(t,T) = \frac{B(t,T)r(t)}{T-t} - \frac{\ln\ A(t,T)}{T-t}$$ (2)

iii) Comment on the relationship between r (t) and the term structure of the continuously compounded rate of interest R(t, T). (2)

iv) Justify, in the context of derivative pricing, why it would **NOT** be appropriate to model bond prices as mean reverting in a risk neutral world. (2)

[8]

Q. 3) XXX Industries decided to invest in a 3-year at par bond of nominal Rs 1000 with coupon rate of 8%. The bond is given credit rating of A. Annual default rate of the bond is contributed by two factors one of which is historical behavior, and the other is current equity market level.

i) Explain why the current equity market level might impact the default rate of bonds. What are the various approaches by which this can be allowed for when estimating the default rate. (3)

An investment actuary has suggested Capital Asset Pricing Model (CAPM) to calculate the default rate for each year.

ii) Describe CAPM in its usual context of use for asset returns and describe the terms in the model. (4)

iii) How can CAPM be adapted to estimate the default probability. Suggest an approach to calibrate the model to derive the parameters and state possible limitations of the approach. (3)

iv) Using the CAPM model Investment actuary has calculated the default rate for 2 scenarios:

Equity market movement	Default rate
Up by 10%	4%
Down by 10%	6%

Assuming a recovery rate of 50% and interest rate of 5% per annum estimate the expected price of the bond under the two scenarios.

Please state any assumptions about the cashflow timing used. (7)
[17]

Q. 4) Consider the following:

- Assume a principal of $1 and term of 10 years.
- Conditional on no earlier default, a reference entity has a (risk-neutral) probability of default of 3% in each of the next 5 years.
- Assume payments are made annually in arrears, while defaults happen halfway through a year.
- The expected recovery rate is 30%.
- The interest rate in the market is 8% per annum.
- The breakeven CDS rate is '*s*' per dollar of notional principal.

Answer the following questions:

i) What is the probability of default in year 4? (2)

ii) What is the probability of survival at the end of the 5 years? (2)

iii) As a function of "s", what is the present value of the payments to be made in the case of no default and in the accrual payment in the event of default? (4)

iv) What is the present value of the expected payoff in the event of a default? (2)

v) What is the breakeven CDS spread in this case? (3)

vi) Assume that the value of a swap is negotiated at an earlier point in time with a CDS spread of 323.98. What would be the value as a function of the principal? Comment on the result. (4)
[17]

Q. 5) The price of a non-dividend paying equity worth 500 today has four possible values in a year's time: 300, 450, 550 or 700. The risk neutral probabilities with which the future prices would be reached are p_1, p_2, p_3 and p_4 respectively. The risk-free interest rate is 5% continuously compounded.

i) Calculate the price of a one year Call option with strike 600 and a one year Put option with strike 400 using the Black Scholes formula, and hence calculate the four risk neutral probabilities. You may assume the implied volatility of both the options as 22.5%. (10)

Now assume that risk-free interest rates remain at 5% continuously compounded but there is a volatility skew observed in the implied volatilities due to which the implied volatility is 20% for options with a strike of 600 and 25% for options with a strike of 400.

ii) Explain (without calculating any of the values once again) how the Black-Scholes values of the Call and Put, and hence the four risk-neutral probabilities, will differ relatively from those found in (i). (4)

iii) Assume that you have prepared a plot of risk neutral probability distribution versus corresponding equity prices for the two scenarios: (a) flat implied volatility and (b) there is a volatility skew. Comment on the tail behavior of the two curves at the low equity price end and the high equity price end. Also, comment on the mean of the distribution under volatility skew with respect to the distribution without the skew. (3)

iv) An investment actuary has proposed that the issue of volatility skew can be addressed to an extent by using a skew adjusted volatility which is a function of the underlying stock price. Suppose we model the dependency of volatility on stock price as $\sigma = \alpha S+\beta$. Show that the skew adjusted delta of a call option of the stock will be the sum of the unadjusted delta and α times its vega. (8)

[25]

Q. 6) Clearing Corporation of country X (CCX) is considering setting the initial margin requirements for interest rate derivatives by modelling the behavior of interest rates in the future. It wishes to carry out scenario analysis. In this context, CCX is considering the use of Vasicek model for interest rates.

i) Describe why CCX should carry out both real-world and risk-neutral projections of interest rates. (2)

ii) What will be the stochastic differential equation (SDE) of the Vasicek model in the real world? Define all terms used. Also, describe how CCX could calibrate the parameters of the real world SDE. (4)

iii) What is the market price of risk for an interest rate derivative f, including equations and defining all terms where appropriate. (2)

iv) How can the risk-neutral SDE for the Vasicek model be derived from the real-world SDE for the Vasicek model and the market price of risk. (3)

v) Explain why the Hull-White two-factor model may be more appropriate than the Vasicek model for CCX's purpose. (4)

The CCIX has derived the following levels of initial margin for both payer and receiver swaps for all counterparties. All initial margins will be cash only.

Term outstanding on swap	*Level of initial margin required for payer and receiver swaps*
Less than 5 years	1% of notional
More than 5 years	2% of notional

vi) Assess how effective these initial margin requirements will be for mitigating counterparty risk to the CCX from a counterparty with a range of payer and receiver swaps. (6)

The regulator of X has requested CCX for settlement of OTC derivatives also.

vii) What are the advantages and disadvantages of this suggestion? (6)

[27]

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