INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

28th May 2024

CM2A - Financial Engineering and Loss Reserving

Time allowed: 3 Hours 15 Minutes (14.45-18.00 Hours)

Total Marks: 100

Q. 1) As we move down the indifference curve from left to right, the slope of the indifference curve tends to which of the following?

I. Unity
II. Zero
III. Decline
IV. Rise **[2]**

Q. 2) Which of the following statement(s) is(are) correct.

a. random variables are defined in respect of sample spaces.
b. probabilities measures are associated with sample spaces.
c. probabilities measures are associated with event spaces.
d. event space is a set containing the subsets of sample space.

I. a & b
II. c & d
III. Only c
IV. a, b & d **[2]**

Q. 3) If a market is inefficient and as a new information is received about a security then

I. nothing will happen.
II. the stock price will fall at first and then later rise.
III. there will be a lag in the adjustment of the stock price.
IV. there will be negative demand for the stock. **[2]**

Q. 4) Consider an American put option on a non dividend paying share. Which of the following statement(s) is/are correct.

a. Increase in the share price will decrease the value of put option.
b. Increase in the exercise price will decrease the value of put option
c. Increase in the time to expiry will decrease the value of put option
d. Increase in the risk free interest rate will decrease the value of put option

I. only a
II. Only a & b
III. Only a & d
IV. None of the above **[2]**

Q. 5) For a 3-month European call option of a non-dividend paying share the current share price is ₹ 1040 and strike price is ₹ 1000. If the risk-free interest rate is 12% per annum and volatility is 30% per annum then find the price of call option.

a. The price of this call option is 101.1.
b. If the strike price and current share price is halved then the price of call option will be half of the price calculated in (a).

c. If the strike price and current share price is halved then the price of call option will be double of the price calculated in (a).
d. Given information not sufficient to calculate the price of call option.

Which of the statement is correct?

I. Only d is correct
II. both a and b is correct
III. both a and c is correct
IV. only a is correct **[4]**

Q. 6) Weak form of efficient markets hypothesis is represented by which of following statements:

A. It should not be possible to consistently profit by selling winners and hanging onto losers.

B. It should not be possible to consistently profit by trading on any public information, such as that found on the Internet or in the financial newspaper.

C. It should not be possible to consistently profit by trading on private information, such as that obtained from a thorough analysis of the company and its industry.

D. It should not be possible to consistently profit by trading on inside information.

I. Only A
II. Both A and B
III. Only D
IV. None **[2]**

Q. 7) A company declares a cash dividend unexpectedly, what would be immediate effect on the value of call option and put option price on the company's share.

a. The value of Call option will increase.
b. The value of Put option will increase.
c. The value of Call option will remain unchanged.
d. The value of Call option will decrease.

I. both a and b
II. only c
III. both b and d
IV. only d **[2]**

Q. 8) Consider the following inequalities relating to the probability of ruin for a claims process operating in continuous time. Which of these inequalities are correct?

[Here, u, u1 and u2 represent initial surplus and t, t1 and t2 represent time.]

A. Prob(u,t1) <=Prob(u,t2) if 0 <=t1<=t2
B. Prob(u1,t) <=Prob(u2,t) if 0 <=u1<=u2
C. 0<=Prob(u,t) <= 1

I. A and B only
II. A and C only

III. B only
IV. A , B and C **[2]**

Q. 9) Which of the following(s) factors are ignored in the specification of Modern Portfolio Theory?

a. rationality of the investors
b. risk aversion of investors
c. skewness of the distribution of returns
d. kurtosis of the distribution of returns

I. a & b
II. c & d
III. Only c
IV. a, b & d **[2]**

Q. 10) Pick the correct statement what do these symbols denote: ρ , θ, λ

I. Change in derivative price w.r.t interest rate, share price, dividend rate respectively
II. Change in derivative price w.r.t interest rate, time, volatility respectively
III. Change in derivative price w.r.t volatility, share price, dividend rate respectively
IV. Change in derivative price w.r.t interest rate, time, dividend rate respectively **[2]**

Q. 11) A Gambler wins 500 when a coin comes up heads and loses 750 when the coin comes up tails.
If 'p' is the probability of the coin showing 'heads' then which of the following is a 'submartigale'.

a. Gambler's fortune over time when p = 0.45
b. Gambler's fortune over time when p = 0.5
c. Gambler's fortune over time when p = 0.6
d. Gambler's fortune over time when p = 0.65

I. a & b
II. c & d
III. a, b & c
IV. b, c & d **[4]**

Q. 12) The adjustment coefficient "R" in Lundberg's inequality is negative. Then which of the following is true:

I. The statement is incorrect as "R" is only positive.
II. The statement is incorrect as "R" is unique and positive.
III. The statement is incorrect as "R" is unique and negative.
IV. The statement is correct as "R" is always a root of Lunberg's inequality. **[2]**

Q. 13) Which of the following behaviours of insured policyholders is Adverse selection?

A. A person with co-morbid conditions availing a group term insurance through the employer

B. A vehicle owner parking the car below a tree during windy rainy season

C. A biker riding the bike carelessly after buying additional accidental cover

I. Only A
II. Both A and B
III. Both B and C
IV. All of the above **[2]**

Q. 14) An analyst uses a two state continuous time model to study the credit risk of zero coupon bonds issued by different companies.

Company A has issued 10 year zero coupon bonds. The risk neutral transition intensity function for company A is as follows:

$\lambda A(s) = 0.05s$, $0 \leq s \leq 10$

The risk-neutral probability that the bond will not default between times 5 and 10, given that it hasn't defaulted till time 5, is:

I. 0.1362
II. 0.1534
III. 0.8466
IV. 0.8825 **[4]**

Q. 15) Which of the following invalidates continuous log normal assumption in modelling investment returns?

a. Mean reverting nature of returns
b. Volatility is constant over time
c. Momentum effect of returns
d. Drift is constant over time

I. a & b
II. c & d
III. a & c
IV. a, b & d **[2]**

Q. 16) Claims occur on a portfolio of insurance policies according to a Poisson process at a Rate λ. All claims are for a fixed amount d, and premiums are received continuously. The insurer's initial surplus is U (< d) and the annual premium income is 1.5 λ d. Pick the probability that ruin occurs at the first claim is:

I. 1-e^(-1/1.5 (1-U/d))
II. 1-e^(-1/1.5 (1-U/dλ))
III. 1-e^(-1/1.5 λ (1-U/d))
IV. 1-e^(-1/1.5 d (1-U/d)) **[2]**

Q. 17) Which of the following statements is incorrect for Bornhuetter-Ferguson method:

I. That whatever claims have already developed in relation to a given origin year, the future development pattern will follow that experienced for other origin years.
II. It combines the estimated loss ratio with a projection method such as Chain ladder method.

III. it could be applied to the development of paid claims, using either an accident year or policy year cohort.

IV. It assumes that first accident year is not fully run off. **[2]**

Q. 18) Let St be a stock price process which follows a geometric Brownian motion according to the following stochastic differential equation (SDE):

dSt = (μ+ ½ σ^2) St dt+ σSt dWt

where μ and σ are constants, and Wt is a standard Brownian motion.

Let Bt be a risk-free bond whose price grows deterministically at the risk-free rate r, continuously compounded, according to the formula Bt = e^rt

i) Derive the SDE for the discounted stock price process $\boldsymbol{D_t = B_t^{-1} S_t}$. (7)

Let X = f(ST) be a path-independent claim on ST for some horizon T.

ii) Describe what is meant by a self-financing and replicating strategy for X. (3)

iii) Explain the steps needed to construct a replicating strategy for X that enables the claim to be valued. (9)

[19]

Q. 19) An investor makes decisions using the utility function U(w) = ln(w) where w > 0. The investor is going to invest INR 1000 now for a period of 1 year, and has identified the following two assets to invest in:

- Asset A is risk-free and will not change in value over the year.
- Asset B will increase in value by 50% over the year with probability 0.6 or decrease in value by 50% over the year with probability 0.4.

The investor does not make any allowance for discounting when making investment decisions. They are going to invest a proportion, x, of their wealth in Asset A and the remaining proportion, (1 – x), in Asset B.

i) Construct a formula, in terms of x, for their expected utility at the end of the year. (3)

ii) Determine, using your result from part (ii), the amount that the investor should invest in each asset to maximise their expected utility. (5)

[8]

Q. 20) An investor has Rs. 1,00,000 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest.

The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean μ = 7% p.a. and standard deviation σ = 5.5% p.a.

i) Calculate the following in respect of the investment at the end of 1 year:

a) The shortfall probability below a value of Rs. 90,000. (2)

b) The 99.5% value at risk. (3)

The second opportunity is a derivative that offers the following payoff in 1 years' time based on the performance of the index during the year.

Payoff (Rs) Scenario

92,000 when $X \leq -7.1\%$
95,000 when $-7.1\% < X \leq 7\%$
1,20,000 when $X > 7\%$

ii) Calculate the expected payoff from the derivative at the end of the year. (4)

iii) Calculate the following in respect of the payoff from the derivative:

a) The shortfall probability below a value of Rs. 90,000. (1)

b) The 99.5% value at risk. (1)

[11]

Q. 21)

i) Write down the general form of a statistical model for a claims run-off triangle, defining all terms used. (4)

The table below shows the cumulative incurred claims on a portfolio of insurance policies.

	Development Year		
Accident Year	1	2	3
2021	1300	1800	1900
2022	1400	1850	
2023	1500		

The company decides to apply the Bornhuetter-Ferguson method to calculate the reserves, with the assumption that the Ultimate Loss Ratio is 80%. Claims are assumed to be fully run off by development year 3.

The earned premium for 2023 is 3000 and the paid claims for 2023 are 1000.

ii) Calculate the reserve in respect of the accident year 2023. (6)

[10]

Q. 22) In a Black–Scholes market, the stock price is given by:

$St = S_0 \exp(0.2Bt + 0.2t)$

where Bt is a standard Brownian motion under the real-world probability measure.

A derivative security written on the stock in the same market at time t has price:

$D_t = 2 \exp(0.6(ß_t - ct) + 0.39t)$

where $ß_t$ is a standard Brownian motion under the equivalent martingale measure.

Determine the value of c such that $B_t + ct$ is a standard Brownian motion under the equivalent martingale measure. **[6]**

Q. 23) An investor has the choice of the following assets that earn rates of return as follows in each of the four possible states of the world:

State	Probability	Asset 1	Asset 2	Asset 3
1	0.2	4%	5%	6%
2	0.3	4%	12%	5%
3	0.1	4%	2%	3%
4	0.4	4%	1%	7%
Market Capitalisation		20,000	20,000	80,000

Determine the market price of risk assuming CAPM holds. Define all terms used. **[6]**

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