

# **INSTITUTE OF ACTUARIES OF INDIA**

## **EXAMINATIONS**

**30<sup>th</sup> November 2023**

**Subject CP2A – Actuarial Modelling (Paper A)**

**Time allowed: 3 Hour 15 Minutes (10.15 – 13.30 Hours)**

**Total Marks: 100**

### **INSTRUCTIONS TO THE CANDIDATES**

- 1. Mark allocations are shown in brackets.*
- 2. Do save your work in solution template on a regular basis.*
- 3. All the detailed guidelines are available on exam screen.*
- 4. If Any, Data set file(s) accompanying the question paper is available for download on the exam screen.*
- 5. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

#### **AT THE END OF THE EXAMINATION**

**Please return this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you. You are requested to save and submit the work before leaving the examination premises.**

**Exam requirements****Q. 1) Modelling steps and data checks**

Read the background document, which describes the scenario that need to be modelled and documented for this project.

Construct a spreadsheet model that produces the following calculations. You should ensure that your spreadsheet contains appropriate self-checks and that you have performed (and documented in the audit trail) reasonableness checks at each stage of your calculations.

i) Carry out checks on the player-wise data provided to confirm that the data is complete and fit to use. This should include analysis of the data provided to give reassurance that the data is reasonable and consistent with the instructions provided. (3)

ii) Basis the data provided, compute the following statistics for performance of each of the 110 players:

- Average runs per match.
- Average wickets per match.
- Average catches per match. (3)

iii) Calculate the same metrics as in (ii) for:

- Each player category.
- Each team. (3)

iv) Estimate the auction price for each player using the formula provided in background material.

Note: Auction price for a player needs to be rounded to nearest 25,000. One may use excel function MROUND for the same. (5)

v) Perform checks on the random numbers that have been provided, including validation that they can be treated as having been generated from a continuous uniform [0,1] distribution.

Note: You are not required to make any changes to the data.

Hints:

a) Consider using descriptive statistics of uniform distribution, such as the mean and variance. (3)

b) Perform goodness of fit chi-squared test by splitting the total data fields of random numbers into 10 groups. (To generate chi-square statistic i.e., inverse of right-tailed probability of chi-square distribution, you can use excel function CHISQ.INV.RT). (4)

vi) Simulate a series of runs for each player, using the results from steps (ii), (iii), uniform random numbers, and the following formula.

(A simulated run is required for each of the 110 players for the next 25 matches.)

Formula for simulating runs:

$$[ (1/2.5) * e^{U_{ij}} * AR_i ] + [ (1 - (U_{ij})^{0.5}) * ARP_i ], \text{ where:} \quad (3)$$

- $U_{ij}$  is the random number from uniform [0,1] distribution generated for projecting runs for player  $i$  ( $1 \leq i \leq 110$ ) in each of the  $j$  matches ( $1 \leq j \leq 25$ )
- $AR_i$  is the average run per match for player  $i$ .
- $ARP_i$  is the average run per match for player category of player  $i$ .

Note: Candidates are not expected to review or challenge the stated formula as that is beyond the scope of this exam.

- vii)** Similar to calculations performed in (vi), simulate wickets and catches for each player. The simulation formula is the same, simply replacing runs for wickets or catches (for e.g.  $AR_i$  replaced by  $AW_i$  or  $AC_i$ , and  $ARP_i$  replaced by  $AWP_i$  or  $ACP_i$ ). (4)

Note:

- Random numbers for simulating wickets and catches to be taken from respective tables.
  - In the above formula,  $AR_i$  should be replaced with  $AW_i$  for simulating wickets, where  $AW_i$  is the average wickets taken per match for player  $i$ .
- viii)** Using the outcomes of simulations carried out in (vi) and (vii), determine the team Mr. Khiladi should choose based on his original decision criterion i.e., team which is likely to score maximum points in the tournament. (5)
- ix)** Using the outcomes of simulations carried out in (vi) and (vii), determine the team Mr. Khiladi should choose based on the decision criterion proposed by your manager i.e., team which is likely to maximize Mr. Khiladi's profit by investing by MPL. (3)

[36]

## Q. 2) Modelling technique and practice

Demonstration of good modelling techniques and practice. [7]

## Q. 3) Audit trail

Produce an audit trail for your spreadsheet model that includes the following aspects:

- Purpose of the model
- Data and assumptions used
- Methodology, i.e. description of how each calculation stage in the model has been produced
- Explanation of checks performed

You should ensure that your audit trail is suitable for both a Senior Actuary, who has been asked to approve your work, and a fellow student, who has been asked to peer review and correct your model, to continue work on it or to use it again for a similar purpose in the future.

**Marks available for audit trail:****Audit approach**

- i) Communication skills (the audit trail provides enough detail to be read as a stand-alone document). (4)
- ii) Fellow student can review and check methods used in the model. (7)
- iii) Senior Actuary can scrutinise and understand what has been done. (7)
- iv) Written in clear English. (4)
- v) Written in a logical order. (3)

**Audit content**

- vi) All steps clearly explained. (8)
  - vii) Clear signposting included throughout. (4)
  - viii) Statement of assumptions made. (5)
  - ix) All model steps accurately covered. (15)
- [57]**

**Background:**

A men's Thirty30 (T30) cricket league MPL (Mody Premier League) is expected to start next year and shall be contested by ten franchise teams.

Each team is expected to have 11 players, with following composition in each team:

- 1) A minimum of 3 Bowlers
- 2) A minimum of 3 Batsmen
- 3) A minimum of 1 All-Rounder
- 4) A minimum of 1 Wicketkeeper

During the tournament, a total of 25 matches shall be played by each team (in various combinations of competitors) and winner shall be decided based on a pre-set point-based criterion as follows:

- 50 match-points for each run scored.
- 2000 match-points per each wicket taken.
- 750 match-points per each catch taken.

The team scoring maximum match-points across 25 matches wins the tournament and the franchise owner gets awarded a sum of INR 100 per point scored during the tournament.

Your client, Mr. Khiladi, has been approached by the MPL directors to be owner of one of the franchises. Mr. Khiladi is a sports enthusiast and is keen to participate in the auction. However, he is not sure which team to bet on and hence has reached out to your company, NKM Actuarial Consultants, to provide him data-driven insights to decide the team he should own.

He has provided following information for assessment:

1. Historic Data of Individual Players' Performance:

For each of the 110 players (11 players for 10 teams), following data has been provided:

- Name of the team for which each player plays.
- Player Category i.e., whether the player is a bowler, batsman, wicketkeeper or an all-rounder.
- Number of matches played till date by the player.
- Total runs scored by the player till date.
- Total wickets taken by the player till date.
- Total catches taken by the player till date.

2. Decision Criterion:

Since the winning amount of the franchise is solely a function of match-points scored through runs, wickets and catches by its players, Mr. Khiladi wants to bet on the team which is likely to score maximum match-points.

Your manager has advised Mr. Khiladi that making decision only basis expected match-points may not be the best strategy. Since there is an auction and Mr. Khiladi would need to invest money, your manager wants him to decide his team basis expected Net Profit from each team, which shall be equal to Expected Winnings for each team (based on match-points) less auction price paid.

Mr. Khiladi has agreed to evaluate your manager's suggestion in addition to his original decision criterion. For the same, he as provided the following additional information.

Auction Pricing Philosophy:

The auction price which franchise owner needs to pay to own a MPL team shall be equal to the total of auction price of each of the players in that team.

Auction Price for a player = Auction Points for a player \* Price per Auction Point, where

Price per auction point = INR 1,000 and

Auction Points for a player =  $\sum (a_i * b_{xi})$ ;  $1 \leq i \leq 4$ ,

where:

I	Description of $a_i$
1	Number of matches
2	Average runs per match
3	Average wickets per match
4	Average catches per match

and  $b_{xi}$  is the auction points for player category x, based on criterion i.

The  $b_x$  factor, or auction points to be applied, are provided in the following table:

Player Category/Criterion	$b_x$ for No. of Matches	$b_x$ for Average Runs	$b_x$ for Average Wickets	$b_x$ for Average Catches
<b>Bowler</b>	2.5	100	1,000	750
<b>Batsman</b>	2.5	50	3,000	750
<b>All-Rounder</b>	2.5	75	2,000	750
<b>Wicketkeeper</b>	5.0	90	0	500

Note: Auction points are not the same as match points discussed earlier to calculate the team scoring the highest points. Auction points, calculated based on the table above, varies by player category and historic performance averages of the player. **This is only to be used to calculate the auction price of a player at onset of tournament.**

### Scenario:

Your manager has asked you to analyse the data provided and then produce a set of simulations of each of the 110 players' performance over the 25 matches in the tournament.

To simulate runs scored by each of the 110 players over 25 matches, your manager has provided you with 2750 random numbers that were generated from a continuous Uniform Distribution on [0,1].

He has provided a similar set of 2750 random numbers each for simulating the wickets and catches taken, respectively, by each of the 110 players over 25 matches.

After completing the simulations, your manager has asked you to use these simulations to determine the team that Mr. Khiladi should pick under each of the criterion agreed.

\*\*\*\*\*