Institute of Actuaries of India

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

November 2023 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

Shiller's methodology is about excessive volatility.

Criticism covered:

a. The choice of terminal value for the stock price
b. The use of a constant discount rate
c. Bias in estimates of the variances due to autocorrelation
d. Possible non- stationarity of the series

Outcome: Whether or not market is efficient and market participants are rational

[Max 3 Marks]

Solution 2:

i) Risk averse:

As a risk averse investor values an incremental increase in wealth less highly than an incremental decrease . $U''(w) < 0$ [2]

ii) Risk seeking :

A risk seeking investor values an incremental increase in wealth more highly than an incremental decrease. $U''(w) > 0$

[2]

[4 Marks]

Solution 3: There two investment strategies Active versus Passive

In active fund management, it attempts to exploit the mispricing since it is believed that markets are not efficient

In passive fund management, it simply aims to diversify across a whole market.

Indian Bond market :

From Covid era, the bond market has changed the trends twice .

Before COVID the yield had downward sloping trend, which modified to an upward trend, and now is slowly again progressing towards downward trend

In this context, the fund management will have to make timely calls for shifting between active and passive strategy, following any one strategy will not leads to optimal returns.

[Max 4 Marks]

Solution 4:

i) Expected cost of the gift: 1/3* 20,000 + +1/3*40,000+1/3*50,000 = 36666.67

Expected Utility: $1/3 * (20000-15^{-7} \text{ x } 20000^2) + 1/3 * (50000 – 15^{-7} \text{ x } 50000^2) +1/3 * (40000 – 15^{-7} \text{ x } 40000^2) = 36657.89$

[2]

ii) Minimum cost S is equal to the certainty equivalent of the gift

$U(x) = S – 15^{-7} \text{ X } S^2 = 36657.89$

$-S+15^{-7}S^2 + 36657.89 = 0$

using $S = \frac{-1 \pm \sqrt{(-1)^2 - 4*15^{-7}*36657.89}}{2*15^\wedge-7}$

we get (1+- 0.9996) / 2*15^-7

i.e Rs 34,172 or Rs 17,08,25,203.13

Conclusion: Minimum price of the gift should be Rs 34,172. Mr Z can buy gift of Rs 40,000 or Rs 50,000 both would please him. Gift of Rs 20,000 will not please Mr Y.

[4]

[6 Marks]

Solution 5: Given

Benchmark return = 3%

Parameters for binomial distribution: n = 3, p =0.5

Parameters for Poisson distribution: mu = 3

i)

a) Variance

Investment Y:

Mean = 3.5* np = 5.25%

Variance = 3.5^2 * npq = 9.1875%%

Investment X:

Mean = Variance = 2 * mu = 6

[2]

b) Downside semi-variance

Investment Y:

The binomial distribution is symmetric about mean. Therefore, semi variance = variance/ 2 = 4.59375

Investment X:

Downside semi variance
= exp(-3) *(3^2/2‰ (6 - 4)^2 + 3/1‰ (6 - 2)^2 + 1.(6 - 0)^2)
= 5.078%%;

[4]

ii) Maximising expected utility is equivalent to maximising semi-variance.
Therefore, Investment B would maximise utility. [2]

[8 Marks]

Solution 6:

i) The market portfolio is (0.25, 0.45, 0.30), so

RM = (0.25RA+ 0.45RB + 0.30RC)

Thus

Cov(Ri, RM) = [0.25 Cov(Ri, RA) + 0.45Cov(Ri, RB) + 0.3 Cov(Ri, RC)] .

So,
Cov(RA, RM) = 0.25 Cov(RA, RA) + 0.45Cov(RA, RB) + 0.3 Cov(RA, RC) .

= 0.25 *.09 + 0.45*.03+ 0.3 *.0375 = 0.0473 .

Similarly, Cov(RB, RM) =.03330,

Cov(RC, RM) =0.0394, .

and

Variance M= [0.25 Cov(RM, RA) + 0.45 Cov(RM, RB) + 0.3 Cov(RM, RC)] = .0385.
Or variance M = .25* Var(a) + .45* Var(b) +.3* Var(c) + 2* cov(a,b) *.25*.45+ 2* cov (b,c)* .45*.3+2* cov(c,a) *0.3*0.25

We conclude that
Beta (AM) = 1.2281 ,
Beta (BM) = 0.8577 and
Beta (CM) = 1.0234.

Finally, solving
Ei = ro +beta *(RM- ro)
EA = 13.37%, EB =11.15%, EC = 12.14%

[Max 8]

ii) Correlation co-efficient using beta:

Cor im = beta(im) * std(m)/ std(i) .

Cor AM= 0.80296, Cor BM = 0.84119, Cor CM= 0.80296

[Max 2]

iii) The corresponding single index model is
Ri = alpha + beta* RM + ei where ei= error process follows standard normal with mean 0 and variance Vei

E (Ri) = alpha + beta* E(RM) (since Ei = 0)

Under CAPM,
E (Ri) = ro + beta* E(RM) - beta* ro

Giving
alpha = ro *(1- beta)

alpha (A) = .06 *(1-1.2281) = -0.013684

[Max 3]

iv) Under the Single Index model ,

Total variance = beta^2 * V(M) + V(e)

Total variance = 0.09
Beta =1.2281
V(M) =.0385

Systematic risk = beta^2 * V(M) = 0.5803
Specific risk = V(e) = Total variance - beta^2 * V(M) =0.03197

[Max 4]

v) Correlation under CAPM:

Cor = 0.5

Correlation under Single Index between A and B:
(beta(Am)* beta(BM) * V(M)) / (std(A) * std(B)) = 0.67543

Single Index Model is not compatible with CAPM

[Max 3]
[20 Marks]

Solution 7:

i) Value of the liability using the method specified by the Regulator:
Summation of probability * value of liability = Summation of probability*(1 – v^ 10) /i *30000

Where = v = 1/(1+i), i = interest rate applicable under scenario

= 0.3* 231,652 + 0.2* 210,707 + 0.4 * 201,302 + 0.1 * 192,530

= 2,11,411

[Max 3]

ii) Value of the liability using the method used by the Bank:

value of liability at the mean interest rate :

mean rate = .07

Liability
= (1 – v^ 10) /.07 * 30000

= 2,10,707

Yes, It is underestimating its liability

[Max 3]

iii) **Solvency value of the liability using the method specified by the Regulator:**

STD of the liability :

√(E (liability^2) – mean liability ^2)

= E (liability^ 2) = 0.3* 231,652^2 + 0.2*210,707^2 + 0.4 * 201,302^2 + 0.1*192,530^2

= 44,894,165,955

STD :
= √ (44,894,165,955 - 2,11,411 ^ 2)

= 14125.6

Solvency value of the liability using the method used by the Bank:

STD of i = √ (0.0051- .07^2)
= 0.014142

Liability using .07 - .014142 = 5.586%
= (1 – v^ 10) /.07 * 30000

= 225201

Solvency:
= 225201- 210707 = 14494

It is not underestimating its Solvency

[Max 8]

iv)

- Bank would have felt that the difference between Values derived from its method and regulator's method is very small
- Time taken to estimate average liability and standard deviation of liability using Regulators method would have been high
- Regulator would have allowed the bank to use an approximate methodology

[Max 1]

[15 Marks]

Solution 8:

Consider a single FD, If W is the aggregrate claim amount from each FD

$W = Y_1 + ...Y_n$

Here, E(N) = k(1-p)/p =0.4 and Var (N) = K(1-p)/p^2 =0.6^2

Moments of Y

E(Y) = m = $e^{\mu+1/2\sigma\wedge 2}$ = $e^{5.65125}$ = 284.6470

And Var (Y) = $s^2 = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$ = 1651340.819

So, the mean and variance of W are :

E(W) = 0.4 * 284.6470 = 113.8588

Var (W) = 0.6 (284.6470^2 + 0.4 * 1651340.819)

Var (W) = 0.6(284.6470^2+660536.3276)

=444936.1453

Aggregate claim amount E(S) = 1500 * 113.8588 = 170788.2

Var(S) = 1500 * 444936.1453 = 667404218

Let interest rate be i

P[1500 * 150 *(1+i) – S*(1+i)^0.5 > 50000)=0.99

P [S < 225000(1+i)^0.5 – 50000 (1+i)^-0.5] = 0.99

P [$\frac{S-E(S)}{\sqrt{Var\ (S)}} < \frac{225000\ (1+i)^{0.5}-50000\ (1+i)^{-0.5}-170788.2}{\sqrt{667404218}}$]

$\frac{225000\ (1+i)^{0.5}-50000\ (1+i)^{-0.5}-170788.2}{\sqrt{667404218}}$ = 2.3263

$225000\ (1+i)^{0.5} - 50000\ (1+i)^{-0.5} - 170788.2$ = 60098.0242

$225000\,(1+i)^{0.5} - 50000\,(1+i)^{-0.5} - 230886.2242 = 0$

$225000\,(1+i) - 50000 - 230886.2242(1+i)^{0.5} = 0$

$(1+i)^{\wedge}0.5 = \frac{230886.2242 \pm \sqrt{230886.2242^2 - 4 \times 225000 \times -50000}}{2 \times 225000}$

= 544428.00 / 2*225000 or (a negative number)

= 1.209

1+i = 1.4637

i = 46.37%

[Max 15 Marks]

Solution 9:

i)

Greek	American call	European put
Delta	Positive	Negative
Vega	Positive	Positive
Theta	Negative	Negative
Rho	Positive	Negative
Lambda	Negative	Positive

[5]

ii) Theta in terms of gamma and delta

$$\Theta = \frac{\partial f}{\partial t}$$

Black-Scholes pdf:

$$\frac{\partial f}{\partial t} + rs\frac{\partial f}{\partial s} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 f}{\partial s^2} = rf$$

Substituting,

$$\Theta + rs\Delta + \frac{1}{2}\sigma^2 s^2 \Gamma = rf$$

[4]

iii) For a deep out of the money option, delta and gamma close to 0.
Substituting in the above expression, Theta will be close to rf.

[3]

[12 Marks]

Solution 10:

i) Given: Derivative price = 0.1448,

payoff = 1 at time 2 if S2 < So

A = $\begin{cases} 0.761, S_t = uS_{t-1} \\ 1.522, S_t = dS_{t-1} \end{cases}$

Calculating value of p:

derivative pays after 2 downward moves

0.1448 = A1^2 *(1-p)^2

0.1448/ (1.522^2) = (1-p)^2

P = 0.749983 [5]

ii) **Calculating value of q:**

A1 = 0.716 for an upward move

A1 = exp(-r) * (q/p)

0.716 = exp (-0.05) * q/ 0.749983

q = 0.56452 [5]

iii) **Derivative price at time 0, if payoff = 1 for S2>So, using q**

If candidates are assuming ud =1, which is generally accepted assumption,

price = o.6^2 * e^(-0.05x2) *1

price = 0.3257 [3]

Or

If the candidates assume specifically for recombining binomial tree and use the condition ud = du. This gives payoffs for upper and middle nodes.

price = 0.6^2 * exp(-0.05x2) + 2 x 0.6 x 0.4exp(-0.05x2)

price = 0.7601 [3]

[13 Marks]

****