# Institute of Actuaries of India 

## ACET June 2023 Solutions

## Mathematics

1. A. $\frac{x}{2} \geq \frac{3 x-2}{5}-\frac{x+1}{2} \Rightarrow \frac{x}{2} \geq \frac{2(3 x-2)-5(x+1)}{10} \Rightarrow \frac{x}{2} \geq \frac{x-9}{10} \Rightarrow x \geq-\frac{9}{4}$.
2. D. Let $n(M)$ and $n(P)$ be the number of students who opt for Mathematics and Physics, respectively. Hence $n(M)=48 ; n(P)=32 ; n(M \cup P)=70$ and $n(M \cup P)=n(M)+n(P)-n(M \cap P) \Rightarrow n(M \cap P)=10$.
3. B. $f(x)=2 x+1$ is one to one and onto. Hence it is invertible. By solving the given equation, we have $x=\frac{f(x)-1}{2}$, i.e., $f^{-1}(x)=\frac{x-1}{2}$.
4. C. Let $\operatorname{cosec}^{-1} \sqrt{2}=\theta \Rightarrow \operatorname{cosec} \theta=\sqrt{2}$. The principal value branch of $\operatorname{cosec}^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$. Since $\operatorname{cosec} \theta=\sqrt{2}=\operatorname{cosec}\left(\frac{\pi}{4}\right)$ and $\frac{\pi}{4} \in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]-\{0\}$, it follows that the principal value of $\operatorname{cosc}^{-1} \sqrt{2}=\frac{\pi}{4}$.
5. B. The general term: $T_{r+1}=\binom{15}{r} x^{15-r}\left(-\frac{1}{x}\right)^{r}=\binom{15}{r} x^{15-2 r}(-1)^{r}$. To get the coefficient of $x^{5}$, we must have $15-2 r=5$. This implies $r=5$ and the coefficient is $-\binom{15}{5}$.
6. C. $\log _{3} x+\log _{9} x+\log _{27} x=\frac{11}{3}$.
$\Rightarrow \frac{1}{\log _{x} 3}+\frac{1}{2 \log _{x} 3}+\frac{1}{3 \log _{x} 3}=\frac{11}{3} \Rightarrow \frac{11}{6 \log _{x} 3}=\frac{11}{3} \Rightarrow \log _{x} 3=\frac{1}{2} \Rightarrow \sqrt{x}=3$.
Thus $x=9$.
7. D. $(x+i y)(5+4 i)=5 x+4 x i+5 i y+4 i^{2} y=5 x+4 x i+5 i y-4 y=$ $(5 x-4 y)+i(4 x+5 y)$ has the conjugate as $-4+12 i$.
Hence, $(5 x-4 y)+i(4 x+5 y)=-4-12 i$.
Equating real and imaginary parts, we have $5 x-4 y=-4$ and $4 x+5 y=-12$.
From these equations we obtain $y=-\frac{44}{41}$ and $x=-\frac{68}{41}$.
Alternatively, $(x+i y)=\frac{-4-12 i}{5+4 i}=\frac{(-4-12 i)(5-4 i)}{25+16}=\frac{-20-60 i+16 i-48}{41}=\frac{-68-44 i}{41}$.
8. B. Let the HP be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2 d}, \ldots$. Given that $\frac{1}{a}=\frac{1}{3} ; \frac{1}{a+2 d}=\frac{1}{7}$.

These give $a=3$ and $d=2$,.
The reciprocals of the HP are $a, a+d, a+2 d, \ldots$, which are AP.
Alternatively, the AP consisting of the reciprocals of the HP has 3 and 7 as its first
and third terms, respectively.
The sum to 15 terms of $3,5,7, \ldots$ is $s_{15}=\frac{15}{2}[2 \times 3+(15-1) 2]=255$.
9. A. $x^{2}-x+3 \mu=0$ and $4 x^{2}-5 x+\mu=0,(\mu \neq 0)$ have a common root $\alpha$.

Hence, $\alpha^{2}-\alpha+3 \mu=0$ and $4 \alpha^{2}-5 \alpha+\mu=0$.
By eliminating $\mu$ from the two equations, we have $3\left(4 \alpha^{2}-5 \alpha+\mu\right)-$ $\left(\alpha^{2}-\alpha+3 \mu\right)=0$, i.e., $(11 \alpha-14) \alpha=0$. The solution $\alpha=0$ implies $\mu=0$, which is not allowed. Therefore, $\alpha=\frac{14}{11}$.
10. C. By interpolation, $\frac{x-1.0}{\Phi(x)-0.8413}=\frac{1.1-1.0}{0.8643-0.8413} \Rightarrow \frac{x-1.0}{\Phi(x)-0.8413}=\frac{0.1}{0.023}$.
$\Rightarrow \Phi(x)=0.8413+\frac{x-1.0}{0.1} \times 0.023$
$\Rightarrow \Phi(1.05)=0.8413+\frac{1.05-1.0}{0.1} \times 0.023=0.8413+0.5 \times 0.023=0.8413+$ $0.0115=0.8528$.
Alternatively,
$\Phi(x)=0.8413+\frac{1.05-1.0}{1.1-1.0} \times(0.8643-0.8413)=\frac{0.8643+0.8413}{2}=0.8528$.
11. D. The Trapezoidal rule is $\int_{0}^{1} f(x) d x=\frac{h}{2}\left[\left(y_{0}+y_{5}\right)+2\left(y_{1}+y_{2}+y_{3}+y_{4}\right)\right]=$ $\left.\frac{0.25}{2}[(0+2.7828)+2(0.3210+0.8244+1.5878)]\right]=1.03115$.
12. A. Let $y=x-5$. This implies as $x \rightarrow 5, y \rightarrow 0$.

$$
\begin{aligned}
\lim _{x \rightarrow 5} \frac{e^{x}-e^{5}}{x-5} & =\lim _{y \rightarrow 5} \frac{e^{y+5}-e^{5}}{y}=\lim _{y \rightarrow 5} \frac{e^{5}\left(e^{y}-1\right)}{y} \\
& =e^{5} \lim _{y \rightarrow 0} \frac{e^{y}-1}{y}=e^{5} \times 1=e^{5}(\text { using l'Hopital Rule }) .
\end{aligned}
$$

13. B. Given $y=e^{x^{3}-\frac{1}{2} \log _{e} x}$,

$$
\begin{aligned}
\frac{d y}{d x} & =e^{x^{3}-\frac{1}{2} \log _{e} x}\left(3 x^{2}-\frac{1}{2 x}\right)=e^{x^{3}} e^{-\frac{1}{2} \log _{e} x}\left(3 x^{2}-\frac{1}{2 x}\right) \\
& =e^{x^{3}} \frac{1}{e^{\frac{1}{2} \log _{e} x}}\left(3 x^{2}-\frac{1}{2 x}\right)=e^{x^{3}} \frac{1}{\sqrt{x}}\left(3 x^{2}-\frac{1}{2 x}\right) .
\end{aligned}
$$

14. C. Given $f(x)=x^{3} e^{-2 x}$,
$f^{\prime}(x)=3 x^{2} e^{-2 x}+x^{3} e^{-2 x}(-2)=e^{-2 x} x^{2}(3-2 x)>0$ if $x<\frac{3}{2}$.
The function is strictly increasing in $x$ in the interval $\left(-\infty, \frac{3}{2}\right)$.
15. A. $\int_{4}^{5} 5^{x} d x=\int_{4}^{5} e^{x \log _{e} 5} d x \quad$ Put $u=x \log _{e} 5$. Then

$$
\int_{4}^{5} 5^{x} d x=\frac{1}{\log _{e} 5} \int_{4 \log _{e} 5}^{5 \log _{e} 5} e^{u} d u=\frac{1}{\log _{e} 5}\left[e^{5 \log _{e} 5}-e^{4 \log _{e} 5}\right]=\frac{2500}{\log _{e} 5}
$$

16. D. $\int\left(3 x^{2}-4 \tan x+3 \sqrt{x}\right) d x=3 \frac{x^{3}}{3}+4 \log (\cos x)+3 x^{\frac{3}{2}} \times \frac{2}{3}+C=x^{3}+$ $4 \log (\cos x)+2 x \sqrt{x}+C$.
17. B. $\left|\begin{array}{cc}\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta\end{array}\right|=\sin ^{2} \theta+\cos ^{2} \theta=1$. $\operatorname{Adj}\left[\begin{array}{cc}\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta\end{array}\right]=$ $\left[\begin{array}{cc}\sin \theta & -\cos \theta \\ \cos \theta & \sin \theta\end{array}\right]$.
Hence inverse is $\left[\begin{array}{cc}\sin \theta & -\cos \theta \\ \cos \theta & \sin \theta\end{array}\right]$.
18. C. Given that $\left|\vec{a}{ }^{\circ} \vec{b}\right|=|\vec{a} \times \vec{b}| \Rightarrow|\vec{a}||\vec{b}|[\cos \theta|=|\vec{a}|| \vec{b} \mid\lceil\sin \theta \mid \Rightarrow \tan \theta=1$. Hence $\theta=\frac{\pi}{4}$.
19. A. Given $|\vec{a}|=1,|\vec{b}|=2$ and $|\vec{c}|=3$ and $\vec{a}+\vec{b}+\vec{c}=0$,

Now, $|\vec{a}+\vec{b}+\vec{c}|^{2}=|\vec{a}|^{2}+|\vec{b}|^{2}+|\vec{c}|^{2}+2\left(\vec{a}^{\circ} \vec{b}+\vec{b}^{\circ} \vec{c}+\vec{c}^{\circ} \vec{a}\right)$.
Hence, $0=1+4+9+2\left(\vec{a}^{\circ} \vec{b}+\vec{b}^{\circ} \vec{c}+\vec{c}^{\circ} \vec{a}\right)$ and $\vec{a}^{\circ} \vec{b}+\vec{b}^{\circ} \vec{c}+\vec{c}^{\circ} \vec{a}=-7$.
20.
D. $\quad M=\left[\begin{array}{l}2 \\ 3 \\ 4\end{array}\right] ; M^{T}=\left[\begin{array}{lll}2 & 3 & 4\end{array}\right] . M M^{T}=\left[\begin{array}{l}2 \\ 3 \\ 4\end{array}\right]\left[\begin{array}{lll}2 & 3 & 4\end{array}\right]=\left[\begin{array}{ccc}4 & 6 & 8 \\ 6 & 9 & 12 \\ 8 & 12 & 16\end{array}\right]$.

The rank of $M M^{T}$ is 1 since all minors of order above 2 are not zero.

## Statistics

21. B. If all the weights are multiplied by a constant, then the A.M. does not change. So the A.M. remains same, that is, 16.
22. C. $M=\frac{1+3+4+4+5+7}{6}=\frac{24}{6}=4 . \quad \frac{2+2+2+3+3+4+p}{7}=M-1=3 \Rightarrow p=5$.

Then median of the numbers $2,2,2,3,3,4,5$ is 3 . $p$ and $q$ are 5 and 3 .
23. A. Mean $=\frac{7+7+8+X+10+12+Y}{7}=11 \Rightarrow X+Y=33$.
$\operatorname{Corr}(X, Y)=\frac{\operatorname{Cov}(X, Y)}{\sqrt{\operatorname{Var}(X) \operatorname{Var}(Y)}} \cdot \operatorname{Cov}(X, Y)=\operatorname{Cov}(X, 33-X)=-\operatorname{Var}(X)$.
$\operatorname{Var}(X)=\operatorname{Var}(Y)$. So $\operatorname{Corr}(X, Y)=-\frac{\operatorname{Var}(X)}{\operatorname{Var}(X)}=-1$.
24. D.

| Marks | Frequency | Cumulative <br> frequency |
| :---: | :---: | :---: |
| 20 | 8 | 8 |
| 40 | 12 | 20 |
| 50 | 18 | 38 |
| 60 | 6 | 44 |
| 70 | 12 | 56 |
| 75 | 9 | 65 |
| 90 | 5 | 70 |

Median $=\left(35^{\text {th }}\right.$ obs. $+36^{\text {th }}$ obs. $) / 2=(50+50) / 2=50$.
Mode $=50$.
25. C. $P(\bar{E} \mid \bar{F}) P(\bar{F} \mid \bar{E})=\frac{P(\bar{E} \bar{F})}{P(\bar{F})} \times \frac{P(\bar{E} \bar{F})}{P(\bar{E})}$.
$P(\bar{E} \bar{F})=1-P(E \cup F)=1-[P(E)+P(F)-P(E \cap F)]=1-\left(\frac{2}{5}+\frac{3}{10}-\frac{1}{5}\right)=$ $\frac{1}{2}$.
$(P(\bar{E} \bar{F}))^{2}=\frac{1}{4} . \quad P(\bar{E}) P(\bar{F})=\frac{3}{5} \times \frac{7}{10}=\frac{21}{50} . \quad P(\bar{E} \mid \bar{F}) P(\bar{F} \mid \bar{E})=\frac{1}{4} \times \frac{50}{21}=\frac{25}{42}$.
26. D. $P(E)=p, P(F)=2 p$.
$P($ exactly one of $E$ and $F$ occurs $)=\frac{5}{9} \Rightarrow P(E \bar{F})+P(\bar{E} F)=\frac{5}{9}$

$$
\begin{gathered}
\Rightarrow P(E) P(\bar{F})+P(\bar{E}) P(F)=\frac{5}{9} \Rightarrow p(1-2 p)+(1-p) 2 p=\frac{5}{9} \\
\Rightarrow 36 p^{2}-27 p+5=0 \Rightarrow p=\frac{5}{12}, \frac{1}{3}
\end{gathered}
$$

27. B. The probability density function can be expressed as

$$
f(x)=\frac{1}{2 \sqrt{2 \pi}} e^{-\frac{1}{2.2^{2}}(x-10)^{2}}
$$

So $X \sim N\left(10,2^{2}\right)$. Mean $=10$. For normal distribution, mean $=$ median $=$ mode .

So mean, median and mode are 10,10 and 10.
28. A. $f(x)=\frac{1}{\lambda^{2}} x e^{-\frac{x}{\lambda}}$. Mode is that value of $x$, for which $f(x)$ is maximum.
$f^{\prime}(x)=\frac{1}{\lambda^{2}}\left(e^{-\frac{x}{\lambda}}-\frac{x}{\lambda} e^{-\frac{x}{\lambda}}\right)=0 \Rightarrow x=\lambda$. It can be checked that $f^{\prime \prime}(\lambda)<0$. So $\lambda$ is the mode of the distribution.
Hence $\lambda=2$.
29. D. Required probability $=P(X>6)=\int_{6}^{\infty} \frac{1}{8} x e^{-\frac{x^{2}}{16}} d x=\int_{36 / 16}^{\infty} e^{-y} d y=e^{-2.25}$.
30. C. Let $X$ be the number of chocolate chips in a cookie. $X \sim \operatorname{Poisson}(\mu)$
$P(X \geq 1)=1-e^{-4.2} \Rightarrow 1-P(X=0)=1-e^{-4.2} \Rightarrow P(X=0)=e^{-4.2}$.
$e^{-\mu}=e^{-4.2} \Rightarrow \mu=4.2$. Therefore, $P(X=1)=4.2 e^{-4.2}$.
31.
B. $E(X)=1 \times \frac{1}{4}+2 \times \frac{1}{4}+3\left(\frac{1}{4}+\frac{p}{4}\right)+4\left(\frac{1}{4}-\frac{p}{4}\right)=\frac{10}{4}-\frac{p}{4}$.
$E(X)=2.4 \Rightarrow \frac{10}{4}-\frac{p}{4}=2.4 \Rightarrow p=0.4$.
32. A. $\quad \phi(x)=\frac{1}{\sqrt{2 \pi}} e^{-\frac{x^{2}}{2}} . \quad E(X)=\frac{1}{\Phi(b)-\Phi(a)} \int_{a}^{b} x \frac{1}{\sqrt{2 \pi}} e^{-\frac{x^{2}}{2}} d x$

$$
=\frac{1}{\Phi(b)-\Phi(a)}\left(\frac{1}{\sqrt{2 \pi}}\right)\left[e^{-\frac{a^{2}}{2}}-e^{-\frac{b^{2}}{2}}\right]=\frac{\phi(a)-\phi(b)}{\Phi(b)-\Phi(a)} .
$$

33. B. The number of arrangements of the letters of the word 'ANACONDA' is $\frac{8!}{3!2!}$.

Now, other than 3 A's, there 5 letters which can be arranged in $\frac{5!}{2!}$ ways. In each arrangement of these five letters there are 6 places, 4 between the 5 letters and one on extreme left and other on extreme right. No two A's will occur together, if A's are arranged in these 6 places. This can be done in $\binom{6}{3}$ ways.
The number of ways in which no two A's are together $=\frac{5!}{2!} \times\binom{ 6}{3}=\frac{5!}{2!} \times \frac{6!}{3!3!}$
Required probability $=\frac{\frac{5!}{2!} \times\left(\frac{6!}{3!3!}\right)}{\frac{8!}{3!2!}}=\frac{5!}{2!} \times \frac{6!}{3!\times 3!} \times \frac{3!\times 2!}{8!}=\frac{5}{14}$.
34. A. $\quad E=A$ hits the target, $F=B$ hits the target and $G=C$ hits the target
$P(E)=\frac{5}{6}, P(F)=\frac{4}{5}$ and $P(G)=\frac{3}{4}$.
P (exactly two of $A, B$ and $C$ will hit the target)

$$
\begin{aligned}
&=P(E \cap F \cap \bar{G})+P(\bar{E} \cap F \cap G)+P(E \cap \bar{F} \cap G) \\
&=P(E) P(F) P(\bar{G})+P(\bar{E}) P(F) P(G)+P(E) P(\bar{F}) P(G) \\
&=\frac{5}{6} \times \frac{4}{5} \times\left(1-\frac{3}{4}\right)+\left(1-\frac{5}{6}\right) \times \frac{4}{5} \times \frac{3}{4}+\frac{5}{6} \times\left(1-\frac{4}{5}\right) \times \frac{3}{4} \\
&=\frac{1}{6}+\frac{1}{10}+\frac{1}{8}=\frac{47}{120} .
\end{aligned}
$$

35. D. Let $A$ be the event that an insured vehicle meets with an accident. Then
$P(A)=P(A \mid S) P(S)+P(A \mid C) P(C)+P(A \mid T) P(T)$.
Given that $P(S)=\frac{1000}{6000}=\frac{1}{6}, P(C)=\frac{2000}{6000}=\frac{2}{6}, P(T)=\frac{3000}{6000}=\frac{3}{6}$.
$P(A \mid S)=0.02, P(A \mid C)=0.03, P(A \mid T)=0.04$.
$P(A)=0.02 \times \frac{1}{6}+0.03 \times \frac{2}{6}+0.04 \times \frac{3}{6}=\frac{1}{30}$.
$P(T \mid A)=\frac{P(A \mid T) P(T)}{P(A)}, P(A \mid T) \times P(T)=0.04 \times \frac{3}{6}=0.02, P(T \mid A)=0.02 \times 30=$ 0.6.
36. A. $P(X=3 \mid X \geq 3)=\frac{P(X=3, X \geq 3)}{P(X \geq 3)}=\frac{P(X=3)}{P(X \geq 3)} . \quad P(X=3)=\frac{1}{4}\left(\frac{3}{4}\right)^{3}$.
$P(X \geq 3)=1-P(X=0)-P(X=1)-P(X=2)=1-\frac{1}{4}-\frac{3}{16}-\frac{9}{64}=\frac{27}{64}$.
$P(X=3 \mid X \geq 3)=\frac{1}{4}\left(\frac{3}{4}\right)^{3} \times \frac{64}{27}=\frac{1}{4}$.
37. 

C. $P(X \geq 1)=\frac{5}{9} \Rightarrow 1-P(X=0)=\frac{5}{9} \Rightarrow P(X=0)=\frac{4}{9}$.
$P(X=0)=q^{2} \Rightarrow q^{2}=\frac{4}{9} \Rightarrow q=\frac{2}{3}$.
$P(Y \geq 1)=1-P(Y=0)=1-\left(\frac{2}{3}\right)^{4}=1-\frac{16}{81}=\frac{65}{81}$.
38. B. $\operatorname{Cov}\left(X, X^{4}\right)=E\left(X^{5}\right)-E(X) E\left(X^{4}\right)$.

$$
\begin{aligned}
& E(X)=\int_{-1}^{1} \frac{x}{2} d x=0, E\left(X^{5}\right)=\int_{-1}^{1} \frac{x^{5}}{2} d x=0 . \operatorname{Cov}\left(X, X^{4}\right)=0 . \\
& \operatorname{Corr}\left(X, X^{4}\right)=\frac{\operatorname{Cov}\left(X, X^{4}\right)}{\sqrt{\operatorname{Var}(X) \operatorname{Var}\left(X^{4}\right)}}=\frac{0}{\sqrt{\operatorname{Var}(X) \operatorname{Var}\left(X^{4}\right)}}=0 .
\end{aligned}
$$

39. B. Regression coefficient of $x$ on $u$ is $b_{x u}=\frac{\operatorname{Cov}(x, u)}{\operatorname{Var}(u)}=2.4$.

Hence $\operatorname{Cov}(x, u)=2.4 \operatorname{Var}(u)$.
Regression coefficient of $y$ on $v$ is $b_{y v}=\frac{\operatorname{Cov}(y, v)}{\operatorname{Var}(v)}=\frac{\operatorname{Cov}(-2 x, 6+3 u)}{\operatorname{Var}(v)}=$ $-6 \frac{\operatorname{Cov}(x, u)}{\operatorname{Var}(v)}=-6 \times 2.4 \frac{\operatorname{Var}(u)}{3^{2} \times \operatorname{Var}(u)}=-1.6$.
40.
C. $\quad E(Z)=1 \times P\left(X_{1}+X_{2} \leq 1\right)+0 \times P\left(X_{1}+X_{2}>1\right)=P\left(X_{1}+X_{2} \leq 1\right)$.

$$
\begin{gathered}
\int_{0}^{1} \int_{0}^{1} f\left(x_{1}, x_{2}\right) d x_{2} d x_{1}=1 \Rightarrow \int_{0}^{1} \int_{0}^{1} k\left(x_{1}+x_{2}\right) d x_{2} d x_{1}=1 \\
\Rightarrow k \int_{0}^{1} \int_{0}^{1} x_{1} d x_{2} d x_{1}+k \int_{0}^{1} \int_{0}^{1} x_{2} d x_{2} d x_{1}=1 \Rightarrow k\left(\frac{1}{2}+\frac{1}{2}\right)=1 \Rightarrow k=1 \\
P\left(X_{1}+X_{2} \leq 1\right) \\
=\int_{0}^{1} \int_{0}^{1-x_{1}}\left(x_{1}+x_{2}\right) d x_{2} d x_{1} \\
=\int_{0}^{1}\left[x_{1}\left(1-x_{1}\right)+\frac{\left(1-x_{1}\right)^{2}}{2}\right] d x_{1}=\int_{0}^{1}\left(\frac{1}{2}-\frac{1}{2} x_{1}^{2}\right) d x_{1}=\frac{1}{3}
\end{gathered}
$$

## Data Interpretation

41. D. In order that percentage of male students is less than twice that of female students, percentage of female students must be more than $33 \%$. It is evident from the bar chart that this happened in the last 3 years.
42. B. Number of male students admitted in

2017: $520 \times 0.75=390$.
2018: $550 \times 0.72=396$ (highest).
2019: $500 \times 0.70=350$.
2020: $580 \times 0.65=377$.
43. C. Number of female students admitted in

2014: 140.
2015: 144.
2017: 130 (minimum).
2019: 150.
2020: 203.
2021: 228.
2022: 248 (maximum).
44. D. Calculate percentage male students/percentage female students.

2013: 2.33.
2014: 2.57 (> 2.5).
2015: 2.33.
2017: 3 ( $>$ 2.5).
2018: 2.57 (> 2.5).
2019: 2.33.
45. A. Share of Brand I in Market C is smaller than its share in Market A. It is also smaller than the share of Brand IV in Market A and the share of Brand II in Market B.
46. C. For Brand IV, business in Market A / business in Market B $=\frac{20}{15} \times \frac{1}{2}<1$.

For Brand I, business in Market C / business in Market $\mathrm{A}=\frac{15}{25} \times \frac{1}{2}<1$.
For Brand II, business in Market B / business in Market $\mathrm{A}=\frac{20}{35} \times \frac{2}{1}>1$.
For Brand III, business in Market C / business in Market $\mathrm{A}=\frac{15}{20} \times \frac{1}{2}<1$.
47. D. The share of Market B in the combined market is $\frac{4}{2+4+1}=57.1 \%$.
48. C. The shares of various combinations of brands and markets in the combined market are summarized below.

|  | Market A | Market B | Market C | All three <br> markets |
| :--- | :--- | :--- | :--- | :---: |
| Brand I | $\frac{0.25 \times 2}{7}=7.1 \%$ | $\frac{0.3 \times 4}{7}=17.1 \%$ | $\frac{0.15}{7}=2.1 \%$ | $26.3 \%$ |
| Brand II | $\frac{0.35 \times 2}{7}=10.0 \%$ | $\frac{0.2 \times 4}{7}=11.4 \%$ | $\frac{0.25}{7}=3.6 \%$ | $25.0 \%$ |


| Brand III | $\frac{0.2 \times 2}{7}=5.7 \%$ | $\frac{0.35 \times 4}{7}=20.0 \%$ | $\frac{0.15}{7}=2.1 \%$ | $27.8 \%$ |
| :--- | :---: | :---: | :---: | :---: |
| Brand IV | $\frac{0.2 \times 2}{7}=5.7 \%$ | $\frac{0.15 \times 4}{7}=8.6 \%$ | $\frac{0.45}{7}=6.4 \%$ | $20.7 \%$ |
| All three <br> brands | $29 \%$ | $57 \%$ | $14 \%$ | $100 \%$ |

49. D. In all the countries, girls perform better in reading than boys.
50. C. The number of countries where proficiency in reading for both boys and girls is at least 75 is 14 .
51. A. It is evident from the table that boys are better in mathematics with proficiency level greater than 75 for both boys and girls in the countries UK, Belgium, New Zealand and Australia.

## English

52. B.
53. B.
54. A.
55. B.
56. C.
57. A.
58. B.
59. C.
60. D.
61. B.
62. C.

## Logical reasoning

63. A.
64. D. At 12:00 noon, both the hands of the clock are together. Relative speed between the two hands is $\frac{360-30}{60}=5.5$ degrees per minute. So time taken by the hands to form the angle of 275 degrees is $275 / 5.5=50$ minutes.
65. A. Each day of the week is repeated after 7 days. So 49 days before any Sunday is another Sunday. Additional 3 days back would make it a Thursday.
66. C. HCF of 45,75 and 90 is 15 .
67. A. Option A is the only one where the successive items do not include the preceding ones. It only describes a food chain.
68. B. $X$ and $P$ are brothers and $Z$ and $S$ are their children. $S$ and $M$ are the only nephews of P , and D has no kids. Therefore, P must be a parent of Z and Y , the remaining two grandsons of C . So Z and S are cousins.
69. D. If all Turns are Good and some Turns are Cans then some Cans are Good.
70. B. C is standing exactly between $G$ and $D$, and $A$ is standing to the immediate left of D. Therefore, irrespective of the positions of the others, A, D, C and G must be standing consecutively from left to right. Hence C is standing to the immediate left of G.

The remaining statements imply that the order of standing is $\mathrm{B}, \mathrm{F}, \mathrm{A}, \mathrm{D}, \mathrm{C}, \mathrm{G}$ and E .

