Institute of Actuaries of India

ACET March 2023 Solutions

Mathematics

1. C. By definition of greatest integer less than or equal to x,$-1 < \lfloor x \rfloor - x \ \leq 0$, for x real. Since $|\lfloor x \rfloor - x| = -(\ \lfloor x \rfloor - x)$, it follows that

   $0 \leq |\lfloor x \rfloor - x| < 1$.

2. A. $f^\circ g(x) = f\big(g(x)\big) = f\left(\frac{x}{x-2}\right) = \left[\frac{x}{x-2}\right]^2 + 4 = \frac{x^2}{(x-2)^2} + 4.$

   Hence, $f^\circ g(3) = 9 + 4 = 13.$

3. D. Let $x = \sqrt{7^2 + x}$. Since x is positive, $x^2 - x - 49 = 0$ giving

   $x = \frac{1+\sqrt{1-4(1)(-49)}}{2} = \frac{1+\sqrt{197}}{2}$.

4. B. The r^{th} term in $(a - 3b)^{12}$ is $T_{r+1} = \binom{12}{r} a^r (-3b)^{12-r} = \binom{12}{r} a^r (-3)^{12-r} b^{12-r}$.

   Thus, the coefficient of $a^5 b^7$is obtained when $r = 5$ in T_{r+1} .

   It is given by $\binom{12}{5}(-3)^7$.

5. D. $\log_m 7 - 3\log_m 2 = 2 \ \Rightarrow\ \log_m 7 - \log_m 2^3 = 2 \Rightarrow\ \log_m \frac{7}{8}$ =2, giving $m^2 = \frac{7}{8}$.

   Thus, $m = \sqrt{\frac{7}{8}}$ since m is positive.

6. A. Since $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 + bx + 1 = 0,$ we have $\sin\theta + \cos\theta = -\frac{b}{a}$ and $\sin\theta\cos\theta = \frac{1}{a}$.

   Hence $(\sin\theta + \cos\theta)^{\ 2} = \frac{b^2}{a^2}$.

   Thus,$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2} \Rightarrow 1 + 2\frac{1}{a} = \frac{b^2}{a^2} \Rightarrow b^2 - a^2 = 2a.$

7. C. Given that n th term of an AP: $a_n = 3 + 4n$. The first term $a_1 = 7$ and $d = 4$.

   Hence, the sum of the first 15 terms $S_{15} = \frac{15}{2}[(2 \times 7) + 14 \times 4] = 525.$

8. B. $\sum_{i=1}^{n}\sum_{j=1}^{i} j = \sum_{i=1}^{n}\frac{i(i+1)}{2} = \sum_{i=1}^{n}\frac{i^2+i}{2} = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right].$

   $= \frac{n(n+1)}{4}\left[\frac{2n+1}{3} + 1\right] = \frac{2n(n+1)(n+2)}{12} = \binom{n+2}{3}.$

9. C. Given that $z = \frac{3+i}{2-i}$, $i^2 = -1$. $z = \frac{3+i}{2-i}.\frac{2+i}{2+i} = \frac{6+5i+i^2}{4-i^2} = \frac{5i+5}{5} = i + 1.$

   $z^2 = (i+1)^2 = i^2 + 1 + 2i = 2\,i;\ \ z^{16} = (2i)^8 = 256.$

10. B. $\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2(\vec{a}\,.\vec{b} + \vec{a}.\vec{c}+\vec{c}.\vec{a}) = 1+1+1 = 3$.

$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{3}.$

11. B. In order that the vectors $\vec{\imath} - x\vec{\jmath} + 3\overrightarrow{k}, -2\vec{\imath} + 3\vec{\jmath} - 4\vec{k}$ and $-\vec{\jmath} + 2\vec{k}$ are coplanar, we must have $\begin{vmatrix} 1 & -x & 3 \\ -2 & 3 & -4 \\ 0 & -1 & 2 \end{vmatrix} = 0.$

Thus, $\begin{vmatrix} 1 & -x & 3 \\ -2 & 3 & -4 \\ 0 & -1 & 2 \end{vmatrix} = 2 - 4x + 6 = 0$ implying $x = 2$.

12. D. As there are three values, y_0, y_2, y_4 the function y can be represented by a second degree polynomial. Hence, $\Delta^3 y = 0$ for all y. The difference table is given below.

	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	3	$y_1 - 3$		
x_1	y_1	$2 - y_1$	$5 - 2y_1$	$y_3 + 3y_1 - 9$
x_2	2	$y_3 - 2$	$y_3 - 4 + y_1$	$8.4 - 3y_3 - y_1$
x_3	y_3	$2.4 - y_3$	$4.4 - 2y_3$	
x_4	2.4			

This gives $y_3 + 3y_1 - 9 = 0$ and $8.4 - 3y_3 - y_1 = 0$. Solving these equations, we have $y_1 = 2.325$ and $y_3 = 2.025$.

13. A. Towards computing the value of the integral, we need the following information.

0	1	2
$y_0 = \log_e \sqrt{1+0} = 0$	$y_1 = \log_e \sqrt{1+1} = 0.3466$	$y_2 = \log_e \sqrt{1+2} = 0.5493$

$\int_0^2 \log_e \sqrt{1+x}\ \ dx = \frac{1}{3}\ [(y_0 + y_2) + 4y_1] = \frac{1}{3}\ [0 + 0.5493) + 4 \times 0.3466] = 0.6452$.

14. B. . $\lim_{x\to 2} \frac{x^6-2}{x^4-2} = \frac{2^6-2}{2^4-2} = \frac{31}{7}$. (l'Hospital's rule is not applicable, as the expression does not have the form $\frac{0}{0}$ as $x \to 2$.)

15. C. Let $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$. Then

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{\sin x}{1+\cos x}\right)^2} \times \frac{(1+\cos x)\cos x - \sin x\,(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)^2}{1+\cos^2 x + 2\cos x + \sin^2 x} \times \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{1+\ \cos x}{2 + 2\cos x} = \frac{1}{2}.$$

16. D. Let $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$. $\frac{dy}{dx} = \frac{1}{1!} + \frac{2x}{2!} + \frac{3x^2}{3!} + \cdots + \frac{nx^{n-1}}{n!}$.

$\frac{d^2y}{dx^2} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \frac{x^{n-2}}{(n-2)!}$.

Hence, $\frac{d^2y}{dx^2} + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} = y$.

17. A. Put $\sqrt{5+\log x} = t$. Then $\frac{1}{2t}.\frac{1}{x}dx = dt \Rightarrow \frac{1}{x}dx = 2tdt$. Hence,

$$\int \frac{\sqrt{5+\log x}}{x}dx = \int 2t^2dt = 2\frac{t^3}{3} + c = \frac{2}{3}\ (5+\log x)^{\frac{3}{2}} + c.$$

18. B. Let $f(x) = x^{15}\cos^6 x$. $f(-x) = (-x)^{15}\cos^6(-x) = -x^{15}\cos^6 x = -f(x)$.

Hence $f(x)$is an odd function. Thus $\int_{-1}^{1} x^{15}\cos^6 x\, dx = 0$.

19. A. Let $A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. Then, $A + B = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

The characteristic roots of $A + B$ is the solution of the equation.

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0 \Rightarrow (5-\lambda)(2-\lambda) - 4 = 0 \Rightarrow \lambda^2 - 7\lambda + 6 = 0.$$

Thus, $\lambda = 1{,}6$. The characteristic roots are 1, 6.

20. C. In a diagonal matrix, if the diagonal elements are not zero, then its inverse is also a diagonal matrix with entries on its diagonal being the reciprocal of the corresponding diagonal elements of the given matrix. Thus, the inverse of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is A^{-1} $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$.

Alternatively, $|A|$=105. $Adj\ A = \begin{vmatrix} 35 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 15 \end{vmatrix}$.

Hence $A^{-1} = \frac{1}{105}\begin{bmatrix} 35 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 15 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$.

Statistics

21. D. Twenty books can be arranged among themselves in 20! ways.
The number of ways the books can be placed so that two particular books always be together is 19! × 2!.
P(a particular pair of books is always together) = $\frac{19!\times 2!}{20!} = \frac{1}{10}$
P(a particular pair of books shall never be together)
= 1 – P(a particular pair of books is always together)
= $1 - \frac{1}{10} = \frac{9}{10}$.

22. A. Probability of drawing a red ball from the box containing 24 balls out which x are red balls is $p = \frac{x}{24}$.
If 12 more red balls are put in the box, number of red balls $= x + 12$ and total number of balls $= 24 + 12 = 36$.
The probability of drawing a red ball $= \frac{x+12}{36} = 2p$.
So $\frac{x+12}{36} = 2p = \frac{2x}{24}$. This implies $x = 6$.

23. C. $P(E \cap F) = p, P(E \cap \bar{F}) = p, P(\bar{E} \cap F) = p.$
$P(E \cup F) = P(E \cap \bar{F}) + P(\bar{E} \cap F) + P(E \cap F) = 3p.$

24. D. E and F are independent, so $P(E \cap F) = P(E)P(F)$.
A. $P(E \cap \bar{F}) = P(E) - P(E \cap F) = P(E) - P(E)P(F) = P(E)\big(1 - P(F)\big) = P(E)P(\bar{F})$, so E and $\bar{F}$ are independent.
B. $P(\bar{E} \cap \bar{F}) = 1 - P(E \cup F) = 1 - [P(E) + P(F) - P(E \cap F)] = 1 - \big[P(E)\big(1 - P(F)\big) + P(F)\big] = 1 - P(F) - P(E)P(\bar{F}) = P(\bar{F}) - P(E)P(\bar{F}) = \big(1 - P(E)\big)P(\bar{F}) = P(\bar{E})P(\bar{F}).$
C. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = P(E) + P(F) - P(E)P(F) = P(E) + P(F)\big(1 - P(E)\big) = P(E) + P(F)P(\bar{E}).$
D. $P(E|F)P(F|E) = P(E)P(F).$

25. B. Let E be the event that number 5 appears at least once.
F be the event that the sum of the numbers appearing is 8.
$E = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(1,5),(2,5),(3,5),(4,5),(6,5)\}$
$F = \{(2,6),(3,5),(4,4),(5,3),(2,6)\}$
$P(E) = \frac{11}{36},\ P(F) = \frac{5}{36}.$
$E \cap F = \{(3,5),(5,3)\}$. So $P(E \cap F) = \frac{2}{36}$. $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}.$

26. D. The average number of accidents per day $= \frac{0+1\times 30+2\times 25+3\times 12+4\times 11+5\times 2}{100} = 1.7.$
Median $=$ (50th obs. + 51st obs.)$/2 = (1 + 2)/2 = 1.5.$
Mode = 1.

27. B. Mean $= \frac{-1+1+(n-2)\times 0}{n} = 0.$
Mean absolute deviation about the mean
$= \frac{1}{n}(|-1-0| + |1-0| + (n-2)\times 0) = \frac{1+1}{n} = \frac{2}{n}.$

28. B. Coefficient of variation $= \frac{s}{\bar{x}} \times 100$. Standard deviation $s = 3.8$.
Coefficient of variation cv $= \frac{s}{\bar{x}} \times 100 = 7.6. \Rightarrow \bar{x} = 50$
If each observation is multiplied by 2, then new $\bar{x} = 2 \times 50$ and $s = 2 \times 3.8$.
New CV $= \frac{2\times 3.8}{2\times 50} \times 100 = 7.6$.

29. A. The probability distribution of X is
$P(X = x) = \frac{1}{10}, x = 21, 22, 23, 25, 27, 29, 30, 32, 35, 40$.
$P(X > 25|X \geq 23) = \frac{P(X>25, X\geq 23)}{P(X\geq 23)} = \frac{P(X>25)}{P(X\geq 23)}$.
$P(X > 25) = \frac{6}{10}$, $P(X \geq 23) = \frac{8}{10}$. $P(X > 25|X \geq 23) = \frac{6}{8} = \frac{3}{4}$.

30. B. Mean of X is $\int_0^\infty \frac{x}{\theta} e^{-\frac{x}{\theta}} dx = \theta \int_0^\infty u e^{-u} du = \theta.1 = \theta$.
The median of $X = 25$. So $P(X \leq 25) = 0.50$. This implies $\int_0^{25} \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = 0.50$
$\Rightarrow 1 - \exp\left(-\frac{25}{\theta}\right) = 0.5 \Rightarrow \exp\left(-\frac{25}{\theta}\right) = 0.5 \Rightarrow \theta = \frac{25}{\log_e 2}$.

31. D. $X \sim Binomial\ (n, p)$. $E(X) = np$ and $\mathrm{Var}(X) = npq$.
So $np = \frac{4}{3}$ and $npq = \frac{8}{9}$. $\frac{npq}{np} = \frac{2}{3} \Rightarrow q = \frac{2}{3} \Rightarrow p = \frac{1}{3}$. $np = \frac{4}{3} \Rightarrow n = 4$.
$P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - \left(\frac{2}{3}\right)^4 - 4\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 = 1 - \frac{16}{81} - \frac{32}{81} = \frac{33}{81}$.

32. D. $Var(X - Y) = Var(X) - 2\,cov(X, Y) + Var(Y) = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$.
$Var(X) + Var(Y) - Var(X - Y) = 2\rho\sigma_1\sigma_2 \Rightarrow \rho = \dfrac{Var(X) + Var(Y) - Var(X - Y)}{2\sigma_1\sigma_2}$.

33. C.
A. $X \sim Poisson(3)$. $P(X \leq 1) = P(X = 0) + P(X = 1) = e^{-3} + 3e^{-3} = 4e^{-3}$
B. Suppose $X \sim Poisson\ (\lambda)$. $P(X \geq 1) = 1 - e^{-\lambda}$
$P(X \geq 1) = 1 - e^{-\lambda} = 1 - e^{-3} \Rightarrow \lambda = 3$. So variance = 3.

C. Suppose $X \sim Poisson\ (\lambda)$.

$$P(X = 2) = 2\,P(X = 1) \Rightarrow \frac{\lambda^2}{2!} e^{-\lambda} = 2 \times \frac{\lambda}{1!} e^{-\lambda} \Rightarrow \lambda = 4$$

Mean = 4, Variance =4. So standard deviation = 2.
D. $X \sim Poisson(1)$. $E\left(\frac{1}{1+X}\right) = \sum_{x=0}^{\infty} \frac{1}{1+x} . \frac{e^{-1}}{x!} = \sum_{x=0}^{\infty} \frac{e^{-1}}{(x+1)!} = 1 - e^{-1}$.

34. C. $E(X^3) = E(3 + Z)^3 = E\left(3^3 + 3 \times 3^2 Z + \binom{3}{2} 3 \times Z^2 + Z^3\right)$
$= E(27 + 27Z + 9Z^2 + Z^3)$.
$Z \sim N(0, 1)$. $E(Z) = 0, E(Z^3) = 0$. $E(Z^2) = Var(Z) + \left(E(Z)\right)^2 = 1 + 0 = 1$.
$E(X^3) = 27 + 27E(Z) + 9E(Z^2) + E(Z^3) = 27 + 27 \times 0 + 9 \times 1 + 0 = 36$.

35. A. $P(X = Y) = \sum_{x=1}^{n} p(x, x) = \sum_{x=1}^{n} \frac{2x}{n^2(n+1)} = \frac{2}{n^2(n+1)} \sum_{x=1}^{n} x = \frac{2}{n^2(n+1)} \times \frac{n(n+1)}{2} = \frac{1}{n}$.

36. B. A_1: the event that a student has 100% attendance
A_2: the event that a student is irregular
E: the event that a student attains A grade
Given that $P(A_1) = 0.40,\ P(A_2) = 0.60,\ P(E|A_1) = 0.70, P(E|A_2) = 0.10$

$$P(A_1|E) = \frac{P(A_1E)}{P(E)} = \frac{P(E|A_1)P(A_1)}{P(E|A_1)P(A_1) + P(E|A_2)P(A_2)} = \frac{0.7 \times 0.4}{0.7 \times 0.4 + 0.6 \times 0.1} = \frac{14}{17}.$$

37. C. $\bar{x} = 1.2, \bar{y} = 12$
Regression coefficient of y on x is $b_{yx} = 3.2$
Regression coefficient of x on y is $b_{xy} = 0.2$

A. $r^2 = b_{yx}b_{xy} = 3.2 \times 0.2 = 0.64 \Rightarrow r = \pm 0.8$. So $r = 0.8$, since b_{yx} and b_{xy} are positive.
B. $b_{yx} = r\frac{s_y}{s_x} \Rightarrow 3.2 = 0.8 . \frac{s_y}{s_x} \Rightarrow s_y = 4s_x$.
C. The regression of y on x is $-\bar{y} = b_{yx}(x - \bar{x}) \Rightarrow y - 12 = 3.2(x - 1.2) \Rightarrow y - 3.2x - 8.16 = 0$.
D. The regression of x on y is $x - \bar{x} = b_{xy}(y - \bar{y}) \Rightarrow x - 1.2 = 0.2\,(y - 12) \Rightarrow x - 0.2y + 1.2 = 0$.

38. A.

$$P\left(0 < X < \frac{2}{3} \middle| 0 < Y < \frac{1}{3}\right) = \frac{P\left(0 < X < \frac{2}{3}, 0 < Y < \frac{1}{3}\right)}{P\left(0 < Y < \frac{1}{3}\right)}$$

$P\left(0 < X < \frac{2}{3}, 0 < Y < \frac{1}{3}\right) = \int_0^{\frac{2}{3}} \int_0^{\frac{1}{3}} 4x(1-y)dydx = \int_0^{\frac{2}{3}} 4x\left[y - \frac{y^2}{2}\right]_0^{\frac{1}{3}} dx = \frac{10}{9}\int_0^{\frac{2}{3}} xdx = \frac{20}{81}$.

$$P\left(0 < Y < \frac{1}{3}\right) = \int_0^1 \int_0^{\frac{1}{3}} 4x(1-y)dy\,dx = \frac{10}{9}\int_0^1 xdx = \frac{5}{9}.$$

$$P\left(0 < X < \frac{2}{3} \middle| 0 < Y < \frac{1}{3}\right) = \frac{20}{81} \times \frac{9}{5} = \frac{4}{9}.$$

39. D. $E(X) = \sum_{x=1}^{\infty} x.\frac{\lambda^x}{x!(e^\lambda - 1)} = \frac{\lambda}{e^\lambda - 1}\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!} = \frac{\lambda}{e^\lambda - 1}e^\lambda = \frac{\lambda e^\lambda}{e^\lambda - 1}$.

$E(X^2) = E(X(X-1) + X) = \sum_{x=1}^{\infty} x(x-1).\frac{\lambda^x}{x!(e^\lambda - 1)} + E(X) = \sum_{x=2}^{\infty}\frac{\lambda^x}{(x-2)!(e^\lambda - 1)} + \frac{\lambda e^\lambda}{e^\lambda - 1} = \frac{\lambda^2 e^\lambda}{e^\lambda - 1} + \frac{\lambda e^\lambda}{e^\lambda - 1} = \frac{\lambda(1+\lambda)e^\lambda}{e^\lambda - 1}$.

$Var(X) = E(X^2) - \left(E(X)\right)^2 = \frac{\lambda(1+\lambda)e^\lambda}{e^\lambda - 1} - \left(\frac{\lambda e^\lambda}{e^\lambda - 1}\right)^2 = \frac{\lambda e^\lambda}{e^\lambda - 1}\left(1 + \lambda - \frac{\lambda e^\lambda}{e^\lambda - 1}\right) = \frac{\lambda e^\lambda}{(e^\lambda - 1)^2}(e^\lambda - \lambda - 1)$.

40. A. $E(X) = \sum_{x=0}^{2}\sum_{y=0}^{2} x\,p(x,y) = 1\left(\frac{3}{12} + \frac{4}{12}\right) + 2 \times \frac{1}{12} = \frac{9}{12}$.
$E(X^2) = \sum_{x=0}^{2}\sum_{y=0}^{2} x^2\,p(x,y) = 1^2\left(\frac{3}{12} + \frac{4}{12}\right) + 2^2 \times \frac{1}{12} = \frac{11}{12}$.
$\mathrm{Var}(X) = \frac{11}{12} - \frac{81}{144} = \frac{51}{144}$.

$E(Y) = \sum_{x=0}^{2}\sum_{y=0}^{2} y\, p(x,y) = 1\left(\frac{2}{12}+\frac{3}{12}\right)+2\left(\frac{1}{12}+\frac{4}{12}+\frac{1}{12}\right) = \frac{17}{12}.$

$E(Y^2) = \sum_{x=0}^{2}\sum_{y=0}^{2} y^2\, p(x,y) = 1^2\left(\frac{2}{12}+\frac{3}{12}\right)+2^2\left(\frac{1}{12}+\frac{4}{12}+\frac{1}{12}\right) = \frac{29}{12}.$

$\mathrm{Var}(Y) = \frac{29}{12} - \frac{289}{144} = \frac{59}{144}.$

$E(XY) = \sum_{x=0}^{2}\sum_{y=0}^{2} xy\, p(x,y) = 1\times 1\times\frac{3}{12} + 1\times 2\times\frac{4}{12} + 2\times 2\times\frac{1}{12} = \frac{15}{12}.$

$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = \frac{15}{12} - \frac{9}{12}\times\frac{17}{12} = \frac{27}{12^2} = \frac{27}{144}.$

$\mathrm{Corr}(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{27}{144}\Big/\sqrt{\frac{51}{144}\times\frac{59}{144}} = \frac{27}{\sqrt{3009}}.$

Data Interpretation

41. C. Number of families in 125000 - 150000 group owning at least 2 vehicles $= 30 + 2 = 32$.

Number of families in 150000 - 175000 group owning at least 2 vehicles $= 62 + 5 = 67$.

Number of families in 175000 - 200000 group owning at least 2 vehicles $= 75 + 11 = 86$.

Number of families in 200000 or more group owning at least 2 vehicles $= 62 + 20 = 82$.

42. D. Number of families owning exactly 1 vehicle $= 22 + 90 + 165 + 290 + 258 + 307 + 159 + 65 = 1356$.
Number of families owning exactly 2 vehicles $= 0 + 3 + 7 + 14 + 30 + 62 + 75 + 62 = 253$.
Number of families owning more than 2 vehicles $= 0 + 0 + 0 + 0 + 2 + 5 + 11 + 20 = 38$.
Number of families owning at least 1 vehicle $= 1356 + 253 + 38 = 1647$.

43. A. Number of families whose earning is Rs. 100000 or more per month owning exactly 1 vehicle $= 290 + 258 + 307 + 159 + 65 = 1079$.

44. A. S4: From about 2% in 2019 to about 95% in 2022.
S5: From about 6% in 2019 to about 75% in 2022.
S9: From about 3% in 2019 to about 55% in 2022.
S10: From about 4% in 2019 to about 54% in 2022.

45. A. It is evident from the graph that S1 has 100% households with tap water connections in 2022.

46. C. It is evident from the graph that S4 has minimum and S2 has maximum share in 2019.

47. B. The states having more than 65% of the households with tap water connections are S1, S2, S3, S4, S5, S6, S7 and S8.

48. D.
A. The minimum number of frauds occurred in FY13.
B. The maximum number of frauds occurred in FY22.
C. This is not true. The number of frauds decreased in FY 21 over FY 20.
D. The number of frauds decreased over the previous year is in FY21.

49. C. Budget allocation on Centrally sponsored Schemes and Central Sector Schemes are 9% and 17%.

Total budget allocation on these sectors is 40×0.26 lakh crore $= 10.4$ lakh crore.

50. B. The sectors with less than 10% allocation are
Centrally Sponsored Schemes – 9%.
Subsidies – 7%.

Defence – 8%.
Finance Commission and other transfers – 9%.
Pensions – 4%.
Other expenditure – 8%.
Percentage allocation in these sectors – 45%.

Total allocation in these sectors $= 40 \times 0.45$ lakh crore $= 18$ lakh crore.

51. B. Maximum allocation – 20%.
Minimum allocation – 4%.

The difference between maximum and minimum allocation is $40 \times 0.16 = 6.4$ lakh crore.

English

52. A.
53. C.
54. B.
55. A.
56. D.
57. D.
58. C.
59. B.
60. D.
61. D.
62. A.

Logical reasoning

63. A. Assuming A is the number of students who play exactly 1 game, B who play exactly 2 games and C be those who play exactly 3 games.
$A + B + C = 100$.
$A + 2B + 3C = 60 + 84 + 72 = 216$.
$\text{Second Equation} - 2 \times \text{First Equation} = C - A = 16$.
Thus, $C = A + 16$. Since the minimum value of A is 0, the minimum value of C is 16.

64. B. Barfis, Tikkis, Jalebis and Samosas are successive subsets.

65. D. Before Alexa starts walking towards her house, she is $50 - 30 = 20$ meters to the South and 20 meters to the East of her house.

66. D. $(360/12) \times 6 = 180$.

67. C. From Friday to Wednesday we need 5 odd days. There will be 1 odd day in 2017, 2018 and 2019 and 2 odd days in 2020 which is a leap year. So 5 odd days will be completed in 2020.

68. D. He shot balloons in the order of 2,4,6,8,10,12,14,1,5,9,13,3,11 and 7. At the tenth shot it was balloon numbered 9.

69. A. No cube is painted black on more than three faces.

70. C. There is no requirement for drawing a family tree to solve this question. Since Nimrit is the grandmother of Shiv and Shiv and Ankit are cousins (from the first statement). Ankit should be a grandson to Nimrit.

Alternative logic: Since Kishore is the grandfather of Ankit and Nimrit is the wife of Kishore, Ankit should be a grandson to Nimrit.

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