Institute of Actuaries of India

Subject SP6 – Financial Derivatives Principles

December 2022 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

Solution 1: The amount of expected credit loss is [Rs. (20000000 * 0.02 * 0.4) + Rs. (30000000 * 0.04 * 0.6)] = Rs. 880,000 **[3 Marks]**

Solution 2:

i) Ito's formula (or lemma) forms the basic extension of differential calculus to variables which are stochastic in nature. It is used to derive stochastic differential equations (SDEs) for valuing derivatives whose payoffs depend on the evolution of a stochastic process. Using suitable boundary conditions, these SDEs can then be solved analytically or numerically to give derivative prices and sensitivities. [1 Marks]

ii) A martingale Xt is a stochastic process under a probability measure P, for which all expected future values of X, conditional on a known history up to time t, are equal to its current value at time t, i.e. EP[Xu | Ft] = Xt for all u > t.The martingale must also satisfy the boundedness condition EP[|Xt|] < ∞ for all t. [2 Marks]

iii)

$$\frac{\partial Y_t}{\partial t} = -1, \frac{\partial Y_t}{\partial W_t} = 2W_t \ and \ \frac{\partial^2 Y_t}{\partial W_t^2} = 2.$$

Using Ito's Lemma, we get

$$dY_t = \left[-1 + \frac{1}{2}.2\right] dt + 2W_t dW_t = 2W_t dW_t.$$

This is driftless and hence a martingale. [3 Marks]

iv)

$$\frac{\partial Z_t}{\partial t} = -6W_t^2 + 2at, \frac{\partial Z_t}{\partial W_t} = 4W_t^3 - 12W_t t \ and \ \frac{\partial^2 Z_t}{\partial W_t^2} = 12W_t^2 - 12t.$$

Using Ito's Lemma, we get

$$dZ_t = \left[-6W_t^2 + 2at + \frac{1}{2}(12W_t^2 - 12t)\right] dt + [4W_t^3 - 12W_t t] dW_t$$
$$= (2at - 6t)dt + (4W_t^3 - 12W_t t) dW_t.$$

This is driftless if an only if a = 3. [3 Marks]

[9 Marks]

Solution 3:

i) The price tree takes the values 320 at time 0, 480 or 160 at time 1, 720 or 240 or 80 at time 2 and 1080 or 360 or 120 or 40 at time 3. [3 Marks]

ii) The value of p is 0.5779 and that of (1 - p) = 0.4221 [2 Marks]

iii) $140.4606 [4 Marks]

iv) $60.01522 [4 Marks]

v) $69.6209 [4 Marks]

[17 Marks]

Solution 4:

i) Credit – Default Swap is defined as:

- The holder of a credit default swap (CDS) retains the right to sell the reference entity (bond) for its face value (to the seller of the CDS) when a credit event occurs (i.e. the reference entity defaults).

- The CDS therefore provides insurance for the holder against the risk of a default by a particular company.
- The holder makes periodic payments to the seller until the end of the life of the CDS or until a credit event occurs. These payments are typically made in arrears but sometimes payments can be made in advance.
- The settlement in the event of a default involves either physical delivery of the bonds or a cash payment. [2 Marks]

ii) A basket default swap is a product with which the investor gains either long or short exposure to a relatively small basket of credits.

Baskets typically consist of up to a dozen credit names.

The holder of a third-to-default basket swap is protected against the third default of the basket, and the seller is exposed to it. No protection is provided for the first or second defaults

The holder has the right to sell the reference entity (bond) for face value when the third credit event occurs. At this point, the swap is terminated.

The holder makes periodic payments to the seller until the end of the life of the third–to-default basket swap or until a credit event occurs.

Third-to-default baskets can be traded in funded or unfunded form. [2 Marks]

iii) If the CDS bond basis is positive and the bond is trading at par, one can then theoretically have arbitrage. The strategy would be as follows:

- Sell CDS protection and short the corporate (risky) bond.
- Use the proceeds from the sale of a CDS as well as the bond to invest in a risk – free instrument.

Now, in the event of a default of the bond, the loss on the CDS should be exactly offset by the profit on the short corporate bond.

Thus, this portfolio would always return greater than the risk-free rate, if the CDS – bond basis is positive. [4 Marks]

iv) In practice, such arbitrage may not exist due to the following:

- The CDS may provide greater protection in that it could pay out on credit events that are technical defaults which would not fully impact a cash bondholder.
- There may be other minor contractual differences in the CDS contract such as provisions allowing delivery of a range of bonds or how the accrued coupons are dealt with.
- The CDS spread may include a premium (or a discount) for counterparty risk. This is because, if the protection buyer defaults, the CDS will terminate, and the protection seller will no longer receive the premia and the protection seller may default if the credit event occurs.
- The CDS spread may incorporate a premium to reflect the greater liquidity of the CDS compared to the bond, and this liquidity premium can only be harvested if the position is held to term.
- It may be either impractical or costly to short the underlying bond.
- The transaction costs may also remove the arbitrage. [4 Marks]

v) The effectiveness of these transactions in meeting the objective can be examined based on the "Impact on Cost" and the Impact on Credit Risk".

- *Impact on cost:* The basket swap will be much cheaper since there is no protection given to the first or second defaults, whereas the CDS would be expected to cover defaults on half the portfolio.

  The CDS transaction would be expected to reduce the credit spread on the portfolio by around half, whereas the basket swap will reduce it by much less.

- *Impact on credit* risk: Both transactions will partially reduce credit risk in the portfolio, as required.

  In particular, the CDS will substantially reduce the credit risk by covering defaults on five of the reference entities; on the other hand, the basket swap covers only the third default event.

  The basket swap will provide some protection from credit risk across the whole portfolio, whereas the CDS will not cover any credit risk for five of the holdings.

  Both provide protection against increased default correlation.

  The basket swap is particularly designed to protect against highly correlated default events.

  As both the CDS and basket swap would be unwound on a credit event, the manager would need to purchase new derivatives to ensure the protection is maintained. This will be even more important for the basket swap, as immediately following a relevant credit event there would be no protection in place against the remainder of the portfolio. [6 Marks]

vi) In this scenario, the recommendation would be that given the large cost of the CDS and consequent reduction in exposure to credit yield, it is likely that the second transaction would be most suitable (given the requirement for "limited cost") and so the basket swap is seems the best option. [3 Marks]

[21 Marks]

Solution 5:

i) A futures contract is an **exchange traded contract** that involves two parties having agreed to buy or sell an **asset at a certain future time** for a certain price. As such if someone has the right (but not the obligation) to buy or sell this futures contract for a specified price then it is referred to as a futures option.

Call Futures Option

When the holder of a call futures option exercises it the writer of the option delivers:

1. A long position in the underlying futures contract and
2. An amount of cash equal to the excess of the of the futures price over the strike price of the option. [3 Marks]

ii) The payoff from a call futures option is the same as the payoff from a call option on a stock with the stock price replaced by the futures price.

As explain above the payoff for call S-K which is 110 – 100 = 10 and payoff from the option on future would be 112- 100 = 12 [3 Marks]

iii) Forward

		INR
1000000	79	7,90,00,000
1000000	81	8,10,00,000
		20,00,000

[2 Marks]

iv) Spot

		USD	INR
60 days	74	1000000	74000000
90 Days	83	1000000	83000000
			90,00,000

Loss of INR 70 lk

Company would continue to use forwards to remove the volatility of payments [3 Marks]

v) Suggested forward contract has payoff of

$$P(\text{ Day} = 90\text{ }) = 1\text{ million x} \begin{cases} -S + X2 \; if \; S < X2 \\ 0 \; if \; X1 < S < X2 \\ -S + X1 \; if \; S > X2 \end{cases}$$

Hence for spot rate of 70, 78, 85 pay offs are

40lk , 0 , -20 lk INR

This caps and floor both profit and loss hence reduces the volatility but is not as prohibitive on liquidity if company to not have exact match of actual money payoff compared with amount in forward contract. [4 Marks]

vi) The Black-Scholes formula adapted for currencies

$C = S\exp(-r_1 T)N(d_1) - K\,\mathrm{Exp}(-r_2 T)N(d_2)$

$P = K\exp(-r_2 T)N(-d_2) - S\exp(-r_1 T)N(-d_1)$

where S = currency value, K = strike, T = expiry time, r_1 = risk-free rate in foreign currency, r_2 = risk-free rate in local currency. N(..) is the cumulative Normal distribution, and the values in the brackets (..) are the adapted Black-Scholes parameters d_1 and d_2.

Hence

$P - C = K \exp(-r_2 T) - S \exp(-r_2 T)$

$\Rightarrow S = K \exp((r_1 - r_2)T) - (P - C)\mathrm{e\,xp}(r_1 T)$

since $N(-d_1) = 1 - N(d_1)$ and $N(-d_2) = 1 - N(d_2)$. [3 Marks]

vii) The implied volatilities are estimated by using market price of option and revised black-sholes formula for currency. These are different from actual price volatility observed in the market.

The implied volatilities are different because of a skew (or smile) effect. There are several reasons why this can occur.

- The implied volatility of options with strikes further out-of-the-money is often higher, reflecting the gearing effect to the seller of dealing in small premiums (small upside = premium, unlimited downside).
- Where the market has a tendency to move rapidly or jump in a particular direction, the implied volatility of options in that direction is often higher, reflecting the demand for protection against those moves and the potential jump diffusion effects that are not priced into the Black-Scholes model.
- The price process is not exactly log normal (e.g. has fatter tail).
- There can be supply/demand effects for particular option strikes. [3 Marks]

[21 Marks]

Solution 6:

i) Ito lemma for function G(x,t) is given by

$$dG = \left(\frac{\partial G}{\partial t} + \mu x \frac{\partial G}{\partial x} + \tfrac{1}{2}\sigma^2 x^2 \frac{\partial^2 G}{\partial x^2} \right) dt + \sigma x \frac{\partial G}{\partial x} dW_t$$

Estimate

$$\frac{\partial G}{\partial t}, \frac{\partial G}{\partial x}, \frac{\partial^2 G}{\partial x^2}$$

For the function to get

For ln x

$$dG = \left(\mu - \tfrac{1}{2}\sigma^2 \right) dt + \sigma dW_t$$

and x^2

$$dG = \left(2\mu + \sigma^2 \right) G dt + 2\sigma G dW_t$$

[5 Marks]

ii) N(x) is the cumulative probability that a variable with a standardised normal distribution will be less than x. So

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

[2 Marks]

iii) N ′(x) is the probability density function for a standardised normal variable distribution so:

$d_1 - d_2 = \sigma\sqrt{T-t}$ which is independent of S hence

$$\frac{\partial d_1}{\partial S} - \frac{\partial d_2}{\partial S} = 0$$

[3 Marks]

iv)

$$c = SN(d_1) - Xe^{-r(T-t)} N(d_2)$$

$$\frac{\partial c}{\partial S} = N(d_1) - Xe^{-r(T-t)} N'(d_2) \frac{\partial d_2}{\partial S} + SN'(d_1)\frac{\partial d_1}{\partial S}$$

As

$$SN'(d_1) - Xe^{-r(T-t)} N'(d_2) = 0 \quad \frac{\partial c}{\partial S} = N(d_1)$$

$c = SN(d_1) - Xe^{-r(T-t)} N(d_2)$

doing partial differential with t

$$\frac{\partial c}{\partial t} = SN'(d_1) \frac{\partial d_1}{\partial t} - rXe^{-r(T-t)}N(d_2) - Xe^{-r(T-t)} N(d_2) \frac{\partial d_2}{\partial t}$$

assuming that $SN'(d_1) = X\, e^{-r(T-t)}N'(d_2)$

$$\frac{\partial c}{\partial t} = -\, rXe^{-r(T-t)} N(d_2) + SN'(d_1) \left(\frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial t}\right)$$

$$d_1 - d_2 = \sigma\sqrt{T-t} \quad \text{so} \quad \frac{\partial d_1}{\partial t} - \frac{\partial d_2}{\partial t} = -\frac{\sigma}{2\sqrt{T-t}}$$

$$\frac{\partial c}{\partial t} = -\, rXe^{-r(T-t)} N(d_2) - \frac{SN'(d_1)\sigma}{2\sqrt{T-t}}$$

[4 Marks]

[14 Marks]

Solution 7:

i)

- Duration is the average maturity time of the present value of the bond s cashflows.
- Forward price volatility is standard deviation of percentage changes in forward prices. It is used in the formula for an option valuation based on prices, the assumption being (in Black s forward price option model) that bond prices follow a Brownian motion. The appropriate forward date will be the option expiry date.
- As for 2 but for yields, so the assumption is that bond yields follow a Brownian motion.

Duration is approximately equal to sensitivity of price

$$D \approx -\left(\frac{1}{P} \cdot \frac{dP}{dy}\right)$$

So $\frac{dP}{P} \sim -Ddy$ or $\frac{dP}{P} \sim -D\, y \frac{dy}{y}$

taking variance on both side gives the approximation of

$$\sigma_P = Dy\ \sigma_Y$$

[5 Marks]

ii) A callable bond, also known as a redeemable bond, is a bond that the issuer may redeem before it reaches the stated maturity date. Call option in callable bond provides flexibility to the issuer hence value of callable bond for owner is lower than vanilla bond of similar term and conditions.

[2 Marks]

iii) Using GRY we estimate the value of vanilla bond

	interest	6%
	GRY	8%
bond		95.63
1	7	6.46
2	7	5.97
3	7	5.51
4	107	77.70

For estimating the value of the option in 1 years' time we need to calculate the forward price of the bond which is (Spot bond price – Discounted value of coupon) x exp (risk free rate)

Discounted value of coupon

discounted value of coupon	
time	1
value	7
discounted	6.59

Future value =

Spot	PV coupon	(Spot - PV coupon)	(Spot - PV coupon) * exp (rfr x 1)
95.63	6.59	89.04	94.54

Forward price volatility

estimating the price volatility		
Price volatility = Duration x GRY x fwd yld volatility		
Duration	2.8	
GRY	8%	
fwd Volatility	15%	
3.36%		

$$C = \exp(-rt)[F\, \mathbf{N}(d_1) - X\, \mathbf{N}(d_2)]$$

$$d_1 = \frac{\ln\left(F/X\right) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln\left(F/X\right) - \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}}$$

F	94.54
X σ	100
	3.36%
$\frac{1}{2}\sigma^2 t$	0.00056448
$\sigma\sqrt{t}$	3.36%
ln(F/X)	-0.05609783
d1	-1.652778284
N(D1)	0.049187998
d2	-1.686378284
N(D2)	0.045861481
Call	0.060569236
Price of bond	Non callable - call option value
Bond Price	95.57

[8 Marks]

[15 Marks]

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