Institute of Actuaries of India

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

December 2022 Examination

INDICATIVE SOLUTION

Introduction
The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i)

a. It makes information cheaper and more accessible thus making markets more efficient.
b. It is subject to new regulation thus marking markets less efficient.
c. It increases the volatility of security prices thus making markets less efficient.
d. It increases competition among brokers thus making markets more efficient.

[1 mark for each, 4 Marks]

ii) An excessive volatile market is one in which the changes in the market values of stocks (the observed volatility) are greater than can be justified by the news arriving. This is claimed to be evidence of market over-reaction, which is not compatible with efficiency. [2]

[6 Marks]

Solution 2:

i)

a) $U'(w) = w^{-0.5} > 0$ for $w > 0$ [1]
b) $U''(w) = -0.5w^{-1.5} < 0$ for $w > 0$ [1]

ii) $R(w) = w*(-U''(w)/U'(w)) = 0.5$ [1]
$==-w*(-0.5w^{-1.5})/ w^{-0.5} =0.5$ [1]

[2]

iii) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0) [½]
= 0.6*2*((1.69a)^0.5-1) + 0.1*2*((6.25b)^0.5-1) – 0.6 [½]
= 1.2*(1.3a^0.5-1) + 0.2*(2.5b^0.5-1) – 0.6
= 1.56a^0.5 + 0.5b^0.5 – 1.4 – 0.6 [½]
= 1.56a^0.5 + 0.5(1000-a)^0.5 –2 [½]
dE[U]/da = 0.78a^-0.5 – 0.25(1000-a)^-0.5 [1]
Setting dE[U]/da = 0 gives 0.78a^-0.5 = 0.25(1000-a)^-0.5 [½]
=> 0.78 = 0.25a^0.5(1000-a)^-0.5 => 3.12
= (a/(1000-a))^0.5 [½]
Squaring both sides => 9.7344 = a/(1000-a) [½]
=> 9,734.4 = 10.7344a or 8.7344a => a = 906.8 or 1,114.50 [½]
Rejecting the figure >£1,000 gives a = 906.80 and b=93.20 [1]
Checking the second derivative d^2E[U]/da^2
= -0.39a^-1.5 – 0.25(1000-a)^-1.5 < 0 [½]
hence this is a maximum [½]

[Max 7]

iv) E[U] = 0.6U(1.69a) + 0.1U(6.25b) +0.3U(0) [½]
=1.56a^0.5 + 0.5(1000-a)^0.5 –2 [½]
Putting the value of a=906.80 in above equation we get
E[U}= 49.801 [1]

[Max 2]

[13 Marks]

Solution 3:

i) Anti-selection – It describes the fact that people who know that they are particularly bad risks are more inclined to take out insurance than those who know that they are good risks. [1]

ii) Moral hazard – It describes the fact that a policyholder because they have insurance, act in a way which makes the insured event more likely. It makes the insurance more expensive. [1]

iii) VAR – It represents the maximum potential loss on a portfolio over a given future time period with a given degree of confidence interval. It can be measured either in absolute terms or relative to a benchmark. [1]

iv) TailVAR – It is a measure of expectation in the tail of the distribution. It measures the expected shortfall below a certain level. [1]

[4 Marks]

Solution 4:

i)

a) X = (720000/800000)-1 = -10% [1]
P(X<-10%) = P(Z<-3.09)=0.1% [1]
[2]

b) P(Z<(t-7%)/5.5%) =0.005 [0.5]
(t-7%)/5.5% =-2.5758 [0.5]
t= -7.1669% [1]
800000*(1 – 7.1669%) = 742664.8 [1]
[3]

ii)

P(X≤-7.1%) = P(Z≤-2.56) =0.00518 [1]
P(X>7%) = 0.5 (as 7% is the mean) [1]
P(-7.1%<X≤7%) = 1-0.5-0.00518 = 0.49482 [1]

Expected pay out = 730000*0.00518 + 750000*0.49482 + 962000*0.5 = 855896.40 [1]
[4]

iii)

a) 0% . Payout is discrete and has 3 payouts all greater than 720000 and hence shortfall probability is 0 [1]

b) Probability payout is ≤ 730000 is 0.52% therefore the 99.5% VaR is 730000 [1]

iv) The expected return from investing in the index is 800000*1.07 =Rs. 856000 [½]
So the expected returns are very similar for each investment. [½]
Based on the expected shortfall below Rs. 720000, the derivative is less risky as there is no possibility of this event occurring. [½]
If the investor has a utility function with a discontinuity at the minimum required return then he may base his decision on this measure. [½]
The 99.5% VaR is higher (i.e. a greater loss) for the derivative, so based on this measure the investor may prefer to invest in the stock index. [1]
The pay off on the derivative is significantly higher than the index when the return is slightly above the mean, so the investor may prefer this. [1]
[Max 3]
[14 Marks]

Solution 5:

i) Under fixed rate model, the effective annual rate of return is a single rate, which will apply in each future years. [1.5]

Under varying rate model, the effective annual rate of return can be different in each years. [1.5]

[3]

ii)

a)

1.04=exp (μ +σ^2/2) [0.5]

0.32 =exp(2 μ +σ^2)[exp(σ^2)−1] [0.5]

σ^2 = 0.00083175 [1.5]

μ =0.038805 [1.5]

[4]

b)

$$(1+i_t) \sim Lognormal(\mu, \sigma^2)$$
$$ln(1+i_t) \sim N(\mu, \sigma^2)$$
$$ln\left(\prod_{t=1}^{t=n}(1+i_t)\right) = \sum_{t=1}^{t=n} ln(1+i_t)$$

[1]

$$\sum_{t=1}^{t=n} ln(1+i_t) \sim N(n\mu, n\sigma^2)$$
$$\prod_{t=1}^{t=n}(1+i_t) \sim Lognormal(n\mu, n\sigma^2)$$

[1]

P(S_3<200/175)= P(Z<ln(1.1429) [1]

= .63407 [1]

where ln(S_3)~N(3*μ, 3*σ^2) = N(3 × 0.038805 ,3 × 0.00083175) = N(0.11641,0.0024953) [1]

[5]

[12 Marks]

Solution 6:

i) C [1]

ii) A [1]

iii) C [2]

iv) So = 325

Yearly Mean of the returns= exp (.056) [½]

Yearly Variance of the returns = .14^2 [½]

Mean = exp (mu + .5 * sigma ^ 2) [½]

Variance of the share = exp (2 mu + sigma ^ 2) * (exp (sigma ^2) -1) [½]

Solving both the equations:

SQRT(Variance of the share)/Mean = sqrt (exp (sigma ^2) -1)) [½]

LN (1 + s.d of the share/Mean)^2) = sigma ^2) [½]

LN(1 + 0.135077^2) = sigma ^2) [½]
sigma ^2= 1.80813 [½]
sigma= 13.45% [½]

And

mu = ln(Mean) - .5 * sigma ^2 [½]
=.056 - .5 * 1.80813 [½]
= 4.6959% [½]

95% Confidence Interval (CI):
Log Su = log So + mu * 0.5 +/ - 1.96 * sigma * sqrt (0.5) [1]
Giving
Upper limit = 5.9937, lower limit = 5.6209 [2]
CI for the share price
[exp (5.9937), exp (5.6209)] [½]
= [400 .90, 276.14] [½]

[Max 10]
[14 Marks]

Solution 7:

i)

$0.7610 = e^{-r} q/p$ if S1 = S0u [½]

$1.5220 = e^{-r} (1 - q)/(1 - p)$ if S1 = S0d [½]

Solving the 2 equations with 2 unknown gives

$e^{r} * .7610 * p = (1-1.5220 (1 – p) * e^{r})$ [½]

$= 1 - 1.5220 * e^{r} + 1.5220 * p * e^{r}$ [½]

$1.5220 * e^{r} -1 = p * e^{r} * (1.5220 - .7610)$ [1]

This gives p = 0.7748 and q = 0.63236. [1]

[4]

ii)

$(e^{r} – d)/(u-d) = q$ [½]

Using the definition for a recombining model ; u x d = 1 [½]

Rearranging the terms gives;

$(e^{r} – d)/(1/d-d) = q$ [½]

$(e^{r} – d)d/(1-d^{2}) = q$ [½]

$(e^{r} d – d^{2}) = q - d^{2}*q$ [½]

d^2 x (1 –q) – d x e^r + q = 0 [½]

Solving the quadratic equation; Roots are:

= - b +/- sqrt(b2 – 4ac) / 4a [½]

= e^r +/- sqrt(e^{2r} - 4 * (1- q) * q) / 4 * (1-q) [½]

Substituting values of r and q gives

= (1.072508 + 0.469412)/ 0.74 or (1.072508 - 0.469412)/ 0.74 [1]

d = 2.0970 or 0.8202 [½]

since ‘d’ has to be less than 1 [½]

d = 0.8202 [½]

u = 1.2192 [½]

[7]

iii) In this case price = 0.63^ 2 x exp (-0.07×2) + 2 × 0.63 × 0.37 x exp (-0.07×2) [2]

= 0.3476 + 0.4042 [½]

= 0.7519 [½]

[3]

[14 Marks]

Solution 8:

i) Cumulative cost of claims paid

	Dev Year			
Accidental year	0	1	2	Ult
2019	4244.0	5038.0	5321.0	5321.0
2020	4867.0	5799.0		
2021	6003.0			

[1]

Cumulative settled claims

	Dev Year			Ult
Accidental year	0	1	2	
2019	606.0	706.0	759.0	759
2020	651.0	747.0		803
2021	699.0			869

[1]

Average cost per settled claims

	Dev Year			Ult
Accidental year	0	1	2	

2019	7.003	7.136	7.011	7.011
2020	7.476	7.763		7.627
2021	8.588			8.678

[2]

Grossing up factors for claim numbers

	Dev Year			Ult
Accidental year	0	1	2	
2019	0.798	0.930	1.000	1.000
2020	0.811	0.930		
2021	0.805			

[2]

Grossing up factors for average claim amounts

	Dev Year			Ult
Accidental year	0	1	2	
2019	0.999	1.018	1.000	
2020	0.980	1.018		
2021	0.990			

[2]

Total Ultimate loss	ACPC	claim number	Projected loss
2019	7.011	759	5321
2020	7.627	803	6125
2021	8.678	869	7540
Total loss			18986

Claims paid to date 17123
Outstanding claim 1863

[2]

[10]

ii)

- Claims fully run-off by end of development year 3.
- Projections based on simple average of grossing up factors.
- Number of claims relating to each development year are a constant proportion of total claim numbers from the origin year.
- Similarly for claim amounts i.e. same proportion of total claim amount for origin year.

[0.75 marks for each, **Max 3]**

[13 Marks]

Solution 9:

i) The number of annual claims N follows a binomial distribution: N ~ B(100, 0.4) then E(N) = 100 *0.4 = 40 and Var(N) = 100 *0.4 *0.6 = 24.
[1]
Let X denote the distribution of the individual claim amounts, so that X ~ Pareto(10, 9000). Then E(X) = 9000 /(10 -1) = 1,000
Var(X) = (9000^2*10) /(9^2*8)= 1250000.
[1]
The annual aggregate claim amount S has E(S) = E(N)E(X)
= 40 *1,000 = 40,000
[1]
Var(S) = E(N)Var(X) + Var(N)E(X)2
= 40 *1250000 + 24* 1000^2 = 74000000 = (8602.33)^2
[1]

[4]

ii) a) Since claims can only fall on one day of the year, there is effectively only one day of the year on which ruin can occur, namely 1 June (or strictly shortly thereafter). For a year after 1 June, the insurer will be receiving premiums but paying no claims, and hence solvency will be improving.
Hence (U, t1) = (U, t2) if 5/12 < t1, t2 < 17/12 [2]

b) We must find Ψ (15000, 1). But ruin will have occurred before time 1 only if it occurs at t = 5/12.

[1]

Just before the claims occur, the insurer's assets will be
5/12* 100* 840 + 15000 = 50000

[1]

and ruin will occur if the aggregate claims in the first year exceed this level. Assuming that S is approximately normally distributed,
we have P(Ruin) = P(N(40000, (8602.33)^2) > 50000)
[1] P (N (0, 1)> (50000-40000)/8602.33)
= 1 - ϕ (1.162) = 0.123 [1]

[4]

[10 Marks]

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