Institute of Actuaries of India

Subject CM1A – Actuarial Mathematics (Paper A)

December 2022 Examination

INDICATIVE SOLUTION

Introduction
The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

The expected present value of maturity benefit is:

$$EPV = 50{,}000 \times \frac{D_{65}}{D_{[45]}} = 50{,}000 \times \frac{689.23}{1677.42}$$ (1)

$= 20544.35$ (0.5)

The expected present value of death benefit is:

(1)

$$EPV = 1577(IA)^{1}_{[45]:\overline{20|}} = 1577 \times \left[(IA)_{[45]} - \frac{D_{65}}{D_{[45]}}[(IA)_{65} + 20A_{65}]\right]$$

$$= 1577 \times \left[8.33865 - \frac{689.23}{1677.42}(7.89442 + 20 \times 0.52786)\right]$$ (0.5)

$$= 1193.98$$ (0.5)

Total value of benefits:

(0.5)

$$20544.35 + 1193.98 = 21738.33$$

[4 Marks]

Solution 2: Profit or loss expected to emerge =

Premium + investment income – expenses – claims – increase in reserves (1)

= 3000 + (3000 + 25130 - 90) x 0.04 - 90 – 0.03 x 3,000 x 12 – (28,950 x (1 – 0.03) – 25,130) (1.5)

= 0.1 (0.5)

[3 Marks]

Solution 3:

i) A reserve represents the amount of money that an insurer sets aside in respect of a policy that is currently in force, in order to meet future payments on that policy. (1)

In many life insurance contracts, the expected cost of paying benefits increases over the term of contract: as mortality increases with age increasing likelihood of death claims and maturity / survival based claims draw closer. (1)

The premiums which pay for these benefits are usually level. This means that premiums received in the early years of a contract are more than enough to pay the expected benefits that fall due in those early years, but in the later years the premium are too small to pay for the expected benefits (1)

It is, therefore, prudent for the premiums received in excess in the early years to be set aside, or reserved, to fund the shortfall in the later years. The fund so reserved are invested so that interest also contributes to the cost of benefits. (1)

[4]

ii) **Gross prospective reserve is defined as:**

Expected present value of future outgo less expected present value of future income (1)

Gross Retrospective reserve is defined as:

The accumulated value allowing for interest and survivorship of premium received to date less

the accumulated value allowing for interest and survivorship of benefits and expenses paid to date (1)

Conditions for equality:

1. The retrospective and prospective reserves are calculated on the same basis (1)

2. This basis is the same as the basis used to calculate the premium used in the reserve calculations (1)

[4]

[8 Marks]

Solution 4:

i)

	Outcome	Cash flow
(1)	HHH	25000,0,0
(2)	HHS	25000,0, -35000
(3)	HHD	25000,0,-50000
(4)	HSH	25000,-35000,0
(5)	HSS	25000,-35000,-35000
(6)	HSD	25000,-35000,-50000
(7)	HD	25000,-50000

[3]

ii) Completing the set of transition probabilities:

$$p_{50+t}^{HH} + p_{50+t}^{HD} + p_{50+t}^{HS} = 1$$

$\therefore p_{50+t}^{HH} = 0.65$ (0.5)

$p_{50+t}^{SS} + p_{50+t}^{SH} + p_{50+t}^{SD} = 1$

(0.5)

$\therefore p_{50+t}^{SS} = 0.25$

For each outcome

	Outcome	Cash flow	Probability
(1)	HHH	25000,0,0	0.4225
(2)	HHS	25000,0, -35000	0.1300
(3)	HHD	25000,0,-50000	0.0975
(4)	HSH	25000,-35000,0	0.1100
(5)	HSS	25000,-35000,-35000	0.0500
(6)	HSD	25000,-35000,-50000	0.0400
(7)	HD	25000,-50000	0.1500

(4)

[5]

iii) The associated outcomes may be considered as sum of two random variables as mentioned below:

a) Random Variable X – Discounted value at time 0 in respect of claim payable at time 1
b) Random Variable Y – Discounted value at time 0 in respect of claim payable at time 2

At time 1		Random Variable X
Status	Cash Flow	Discounted Value at Time 0
Health	0	0
Sick	35000	32407.40741
Died	50000	46296.2963

(0.5)

At time 2		Random Variable Y
Status	Cash Flow	Discounted Value at Time 0
Health	0	0
Sick	35000	30006.85871
Died	50000	42866.94102

(0.5)

Probability for occurrence

At time 1	Random Variable X	
Status	Discounted Value at Time 0	Probability for cash flow
Health	0	0.65
Sick	32407.40741	0.20
Died	46296.2963	0.15

(0.5)

At time 2	Random Variable Y	
Status	Discounted Value at Time 0	Probability for cash flow
Health	0	0.6825
Sick	30006.85871	0.18
Died	42866.94102	0.1375

(0.5)

<u>Mean Calculation</u>

At time 1	Random Variable X		
Status	Discounted Value at Time 0	Probability for cash flow	Probability * Discounted value
Health	0	0.65	0
Sick	32407.40741	0.20	6481.48148
Died	46296.2963	0.15	6944.44445

<u>Mean of Random variable X = 13425.92593</u>

(1)

At time 2	Random Variable Y		
Status	Discounted Value at Time 0	Probability for cash flow	Probability * Discounted value
Health	0	0.6825	0
Sick	30006.85871	0.18	5401.23457
Died	42866.94102	0.1375	5894.20439

<u>Mean of Random variable Y = 11295.43896</u> (1)

Mean of NPV = Premium – E [Random variable X] – E[Random variable Y]
= 25000 – 13425.59259 - 11295.43896
= 278.63511

<u>Variance Calculation:</u>

The random variables can be considered as independent,

At time 1	Random Variable X			
Status	Discounted Value at Time 0	X^2	Probability for cash flow	Probability * X^2
Health	0	0	0.65	0
Sick	32407.40741	1020240055	0.20	210048011
Died	46296.2963	2143347051	0.15	321502057

(1)

Variance of Random variable X = E[(X^2)] - { E[X]}^2
= 531550068 – 13425.92593^2
= 351294581

At time 2	Random Variable Y			
Status	Discounted Value at Time 0	Y^2	Probability for cash flow	Probability * Y^2
Health	0	0	0.6825	0
Sick	30006.85871	900411569	0.18	162074082
Died	42866.94102	1837574632	0.1375	252666512

(1)

Variance of Random variable Y = E[(Y^2)] - { E[Y]}^2
= 414740594 – 11295.43896^2
= 287153652

Variance of NPV = Variance [Random variable X] + Variance [Random variable Y]
= 638448233 (1)

[7]

[15 Marks]

Solution 5: A unit-linked policy has the following profit vector:

Policy Year	Net Cashflow
1	-37
2	-15
3	-8
4	38
5	49

i) The provisions required at the end of year 2 and year 1 are:

$_2V = \frac{8}{1.065} = 7.512$

(1)

$_1V = \frac{1}{1.065}\{\, 15 + (1 - 0.025)\frac{8}{1.065} \} = 20.961$ (1)

[2]

ii) Before zeroisation, the net present value (based on a risk discount rate of 10%) is

$$NPV = \frac{-37}{1.1} + \frac{-15 \times 0.975}{1.1^2} + \frac{-8 \times 0.975^2}{1.1^3} + \frac{38 \times 0.975^3}{1.1^4} + \frac{49 \times 0.975^4}{1.1^5}$$ (1)

=0.114 (0.5)

After zeroisation the profit in year 1 will become:

Profit in Year 1 = $-37 + \frac{-15 \times 0.975}{1.065} + \frac{-8 \times 0.975^2}{1.065^2}$
= -57.434

(1)

NPV after zeroisation =

$$\frac{-57.434}{1.1} + \frac{38 \times 0.975^3}{1.1^4} + \frac{49 \times 0.975^4}{1.1^5}$$ (1)

= -0.67 (0.5)

NPV is reduced after zeroisation as the funds are set aside earlier to accumulate at an interest rate that is lower risk discount rate (1)

[5]

[7 Marks]

<u>Solution 6</u>:

i) There will be a series of negative cash flows (1)
throughout the specified term or until death, if earlier (0.5)
followed by a large positive cash flow on death, if death occurs before the end of the term (1)
If policyholder survives to end of term, there will be no positive cash flow (0.5)

[3]

ii) Possible decrements to model may be:
Death (1)
Lapse (1)

[2]

iii) To develop the assumptions, following steps may be followed

1. Identify the purpose of the modelling exercise. Assumptions may be affected by objective
2. Collect the necessary data – in this case this would be the experience data
3. Design the structure of the assumptions, e.g. term dependant vs point estimate
4. Involve experts to get feedback on the assumptions
5. Write the program to perform the backing analysis
6. Review the analysis for reasonableness
7. Perform sensitivity analysis to understand implications of the assumptions, therefore, level of accuracy needed
8. Ensure adhered to any professional standards or regulations

(1/2 mark per valid reason) **[4]**

iv) We should assess the sensitivity of the underlying calculations to the proposed accuracy for the assumption input. If the impact is immaterial, then the cost of implementing the model development may be more than the benefit. [2]

[11 Marks]

<u>Solution 7</u>:

Fund value after 25 years:

For the first 15 years, i(12) = 6%
Therefore, effective interest rate i = (1 + 6%/12)^12-1 = 6.168%
Effective interest rate half-yearly = 3.038%
Corresponding value for d = (0.5)

For the next 10 years, j(2) = 6%
Therefore, effective interest rate j = (1 + 6%/2)^2-1 = 6.090%
Effective interest rate half-yearly = 3.000% (0.5)

500 $\ddot{S}_{\overline{30|}}^{(3.038*\%)}$ x$(1.03)^{20}$ +500x$\ddot{S}_{\overline{20|}}^{(3\%)}$ (1)
= 500 x 49.3215 x 1.8061 + 500 x 27.6765 (1)
(1)
= 58,378

[4 Marks]

Solution 8:

i) APR is the effective annual rate of interest…. (0.5)
… rounded to the nearest 1/10th of 1% (0.5)

In the UK, lenders are required to provide the APR. As lenders provide the information on the interest rate in consistent format, the consumer is able to draw comparisons and make informed decisions. (1)
[2]

ii) Prospective method involves finding the present value of future payments (1)
Retrospective method involves calculating the accumulated value of the initial loan less the accumulated value of payments till date (1)

A student takes out a student mortgage loan of Rs. 2,000,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective **[2]**

iii) Let X be the monthly instalment

2000000 = 12X a15(12) (1)

So, X = 16,706 (1)

Capital outstanding at the start of the seventh year is calculated prospectively as

12X a9 = 1,400,642 (1)

Capital outstanding at the end of the seventh year

12X a8 = 1,278,755 (1)

Capital repaid = 1400642 – 1278755 = 121,887 (1)

Interest component of the 85th instalment = 1278755 X (1.06^1/12 -1) = 6,224 (1)
[6]

iv) Loan outstanding at the end of 10 years = 12X a5 = 867,433 (0.5)

Monthly instalment = 150% x 16,706 = 25,059 (0.5)

Therefore the equation of value = 867,433 = 12 x 25,059 x an (0.5)

Solving for n, we get 3.17, i.e. 13 years and 2 months (2)

So loan reduces by 1 year and 10 months. (0.5)

Total interest payments over term of loan as per original loan:
12 x 16,706 x 15 – 2000000 = 1,007,058 (0.5)

Total interest payments over term of loan as per revised schedule:
12 x 16,706 x 10 + 12 x 25,059 x (3 + 2/12) – 2000000 = 956,962 (1)

Interest saved = 1007058 – 956962 = 50,097 (0.5)

[6]

[16 Marks]

Solution 9: $\bar{A}^1_{x:\overline{n}|} = \sum_{t=0}^{n-1} t|\bar{A}^1_{x:\overline{1}|}$ (0.5)

$$\bar{A}^1_{x:\overline{n}|} = \sum_{x=0}^{n-1} v^t\, tp_x\, \bar{A}^1_{x+t:\overline{1}|}$$ (0.5)

$\bar{A}^1_{x+t:\overline{1}|} = \int_0^1 v^s\, sp_{x+t}\, \mu_{x+t+s}\, ds$ (1)

Assuming a uniform distribution of deaths, then $sp_{x+t}\, \mu_{x+t+s} = q_{x+t}$ (0.5)

$$\bar{A}^1_{x+t:\overline{1}|} = \int_0^1 v^s\, q_{x+t}\, ds \quad = q_{x+t} \int_0^1 v^s ds$$

$= q_{x+t}\frac{iv}{\delta}$ (1)

$$\bar{A}^1_{x:\overline{n}|} = \sum_{x=0}^{n-1} v^t\, tp_x\, q_{x+t}\frac{iv}{\delta}$$

(0.5)

$$\bar{A}^1_{x:\overline{n}|} = \frac{i}{\delta}\sum_{x=0}^{n-1} v^{t+1}\, tp_x\, q_{x+t}$$ (0.5)

$\bar{A}^1_{x:\overline{n}|} = \frac{i}{\delta}\, A^1_{x:\overline{n}|}$ (0.5)

[5 Marks]

Solution 10:

i) Let P be the annual premium
Expected present value of premiums

$P\ddot{a}_{[40]} = 15.494\,P$ (0.5)

EPV of benefits = 150000 x $A_{[40]}$ = 150000 x 0.12296 (0.5)
= 18,444 (0.5)

EPV of expenses
$0.75P + 500 + (100 + 0.03P)a_{[40]}$ (1)
= 0.75P + 500 + (100 + 0.03P) x 14.494
= 1949.4 + 1.18482 x P (0.5)

So,

15.494P = 18444 + 1949.4 + 1.18482 P (0.5)

P = 1425 (0.5)

[4]

ii) Let P' be the required minimum office premium

Then the insurer's loss random variable for this policy is given by:

$$L = 150000\ v^{K_{[40]}+1} + 500 + 0.75P' + (0.03P' + 100)a_{\overline{K_{[40\}}}|} - P'\ddot{a}_{\overline{K_{[40]}+1}|} \quad (2)$$

We need to find the value of t such that

$$P\ (L > 0) = P(T < t) = \ 0.05 \Rightarrow P(T > t) = 0.95 \quad (1)$$

Using AM92 Select, we require:

$$\frac{l_{[40]+t}}{l_{[40]}} \geq 0.95 \Rightarrow l_{[40]+t} \geq 0.95 l_{[40]} = 0.95 \times 9854.3036 = 9361.5884 \quad (1)$$

As $l_{58} = 9413.8004\ and\ l_{59} = 9354.004\ then\ t\ lies\ between\ 18\ \&\ 19$

$$so\ K_{[40]} = 18 \quad (1)$$

We therefore, need the minimum premium such that L=0

$$L = 0 = 150000v^{19} + 500 + 0.75P' + (.03P' + 100)a_{\overline{18}|} - P'\ddot{a}_{\overline{19}|} \quad (1)$$

$$\Rightarrow 0 = 150000 \times 0.33051 + 500 + 0.75P' + (.03P' + 100) \times 10.8276 - 11.8276P'$$

$$\Rightarrow P' = \frac{51159.71}{10.75277} = 4757.82 \quad (1)$$

[7]

[11 Marks]

Solution 11:

i) Let x be the annual annuity payment amount.

The net future loss random variable at the outset for this policy is

L = x ($\ddot{a}_{\max(k_{55}m, k_{50}f)}$ + 1) – P

(2)

P is the single premium i.e. 1,00,000

$K55$ is the curtate future lifetime of a male life aged 55

$K50$ is the curtate future lifetime of a female life aged 50 (1)

[3]

ii)

$$P = x\ \ddot{a}_{\overline{55^m:50^f}}\ @4\%$$

(1)

$$P = x\left[\ddot{a}_{55}m + \ddot{a}_{50}f - \ddot{a}_{55^m:50^f}\right]$$

(0.5)

$$100000 = x\ [17.364 + 19.539 - 16.602]$$

(0.5)

$$x = 100000 \div 20.301$$

(0.5)

$$x = \text{Rs. } 4925.87$$

(0.5)

[3]

iii) Death Strain at Risk in a policy year is defined as the benefit payable on death during the year in excess of the amount required to set up the reserve, if any, at the end of the year. (1)

Expected Death Strain is the expected amount of the death strain. This is the amount that the life insurance company expects to pay in addition to the year-end reserve for the policy. (1)

Actual Death Strain is calculated in respect of the policies where the death has occurred during the year. It is the actual amount of the benefit paid on death less the reserve required to be set up at the end of the year in respect of such policies had death not happened for such policies. (1)

[3]

[9 Marks]

Solution 12:

i) The x-axis represents the discount rate and the y-axis represents amounts.

The point at which graph intersects x-axis represent the internal rate of return (1)

The point at which graph intersects y-axis represents the sum of all cash flows (1)

[2]

ii) Two key project appraisal metrics where borrowing is allowed:

1. Accumulated profit, taking into account conditions of the loan (0.5)
2. Discounted payback period (0.5)

Other factors to consider

Cash flows profile such as are the cash flow requirements consistent with other business needs, over what period will profits be produced. (1)

Borrowing requirement: can the business raise the necessary cash at times required, what rate of interest will the business have to pay on borrowed funds, are the time limits or other restrictions imposed on borrowings (1)

Resources: Are the other resources required for the project available, does the business have necessary staff, technical expertise, equipment (1)

Risk: financial risks of the project, appropriateness of risk discount rate, can suppliers be relied on, can timelines be met

Investment conditions: What is the economic climate, are interest rates expected to rise or fall? (1)

Indirect benefits: Are there any additional benefits associated with the project, will it provide value in future. (1)

(maximum 4 marks to be awarded for list of factors… brief description for each factor is expected) **[Max 5]**

[7 Marks]

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