INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

7th December 2022

Subject SP6 – Financial Derivatives

Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***
3. ***Attempt all questions beginning your answer to each question on a separate sheet.***
4. ***Mark allocations are shown in brackets.***
5. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1) An investor holds a portfolio of Rs. 50 million. This portfolio consists of A-rated bonds (Rs. 20 million) and BBB-rated bonds (Rs. 30 million). Assume that the one-year probabilities of default for A-rated and BBB-rated bonds are 0.02 and 0.04, respectively, and that they are independent. If the recovery value for A-rated bonds in the event of default is 60 percent and the recovery value for BBB-rated bonds is 40 percent, what is the one-year expected credit loss from this portfolio? **[3]**

Q. 2)

i) Explain the use and significance of Ito's lemma for the valuation of derivatives based on stochastic processes. (1)

ii) Explain what it means for a stochastic process X_t to be a martingale under a probability measure **P**. (2)

iii) Let

$$Y_t = W_t^2 - t,$$

Where Y_t is a standard ***P-Brownian*** motion W_t. Obtain the differential equation for the stochastic process that is sufficiently bounded and state with reasons whether it is a martingale. (3)

iv) What value(s) of the constant 'a', if any, would a sufficiently bounded stochastic process Z_t defined by

$$Z_t = W_t^4 - 6W_t^2 t + at^2$$

be a martingale. (3)

[9]

Q. 3) A stock is currently priced at $320 and pays no dividends. The price at time "i+1" is given by S_{i+1} and can be written as $S_{i+1} = S_i + 0.5\ S_i$ for the up move or $S_{i+1} = S_i - 0.5\ S_i$ for the down move. The risk-free interest rate of 7.5% per period and the strike price is $150. For a European Call option on this asset with a strike price of $300 with three periods to expiration:

i) Draw a three-period binomial tree. (3)

ii) Calculate the value of the down probability. (2)

iii) Using the tree drawn in part (i), price a European Call Option using the information given in the question (rounded to the nearest integer). (4)

iv) Repeat part (ii) but now price the corresponding American Call Option. (4)

v) Finally, using the tree drawn in part (i) and the information given, what will be the price of the American Put Option? (4)

[17]

Q. 4) Explain the following terms:

i) A Credit Default Swap (CDS) (2)

ii) A Third – to – Default Basket Swap (2)

Let "CDS-bond basis" for a particular corporate bond be the difference between the corporate bond is defined as the CDS spread and the excess of the bond yield over the risk-free rate.

iii) In a theoretical environment, what arbitrage opportunity should exist when the CDS bond basis is positive, and the bond is trading at par. (4)

iv) However, such arbitrage as highlighted in (iii) may not exist in practice. Explain. (4)

You are managing a fund comprising of ten different corporate bonds in similar proportions and a small amount of cash. The fund has no other assets. You wish to partially reduce the credit risk in the portfolio for a limited cost and to that end, you want to introduce credit derivatives into the portfolio. The alternates before you are:

- Purchasing credit default swaps for five of the corporate bonds.
- Purchasing a third-to-default basket swap, where the basket comprises the ten holdings.

v) Compare the likely effectiveness of each of these alternatives towards meeting your objective as stated above. (6)

vi) Which of the strategies would you recommend and why? (3)
[21]

Q. 5)

i) What are options on futures? Describe the payoff to a holder when a call option on a future expires. (3)

ii) Explain the difference between the payoff of a call option on future of stock and a call option on a stock. Calculate payoff for a call option with the strike price of 100, expiry of 3 months that allows the holder to go long on a 1-month stock future upon expiry. Compare this with payoff from 3 months call option on a stock with same strike price. The following information is given at the time of expiry of the option - Stock price of 110 and 1-month future price of 112. (3)

The spot and forward exchange rates in US$ per INR are quoted below:

Days	USD/INR
30	0.012987013
60	0.012658228
90	0.012345679

Company has to sell $1 million in 60 days and buys back $1 million in 90 days and is contemplating either to do that in spot market or forward market.

iii) Calculate how much the company would pay and receive in 60 and 90 days' if company uses forwards. (2)

iv) Estimate loss by the company if the company decides to do transaction in Spot Market. Spot rates on 60 days and 90 days are 74 and 83 respectively. Explain why company might consider forward transaction in future irrespective of the loss. (3)

As an alternative to the above contract Investment bank offers forward contract at 90 days period to sell $1 million at below rate

a. FX rate X1 = 74 for any spot rate below 74 and

b. FX rate of X2 = 83 for any spot rate of greater than 83

v) Construct the pay-off function and estimate profit or loss for spot rate of INR/USD – 70,78 and 85. Why do many companies prefer this style of contract compared with vanilla forwards? (4)

vi) Show that interest rate parity holds for call and put options on Currency where interest rates are r_{ind} and r_{usd} for India and US respectively. (3)

Option implied volatilities have been at historic lows but exchange rate has become more volatile.

The following table of one-year option implied volatilities has just been supplied by a currency options broker

Relative Strike	Strike	volatility
ATM	75	8.00%
5	80	8.50%
10	85	9.00%
-5	70	8.30%
-10	65	8.80%

vii) Explain why the implied volatilities of the options shown in the table above are not all the same. (3)

[21]

Q. 6) The price X of a non-dividend paying asset follows geometric Brownian motion process:

$$dx = \mu x dt + \sigma x dW_t$$

where μ *and* σ, are positive constants and W_t is a standard Brownian motion.

i) Using Ito's Lemma, find the processes followed by:

- $Z_1 = \log_e x$
- $Z_2 = x^2$

(5)

With reference to the Black Scholes formula,

ii) Derive an expression for N′(x) and what is it significance as part of representing standardised normal variable distribution. (2)

iii) Show that $\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$ and that $\frac{\partial c}{\partial S} = N(d_1)$. (3)

iv) Show that $\frac{\partial c}{\partial t} = -rXe^{-r(T-t)}N(d_2) - SN'(d_1)\frac{\sigma}{2\sqrt{T-t}}$ where c is price of a European call option. You may assume that $SN'(d_1) = X\, e^{-r(T-t)}N'(d_2)$. (4)

[14]

Q. 7)

i) Define the following terms in relation to bonds and options on bonds:

- Duration (*D*),
- Forward price volatility (σ_p),
- Forward yield volatility (σ_y) and

Prove approximately

$$\sigma_p = Dy\sigma_y$$

Where *y* is the forward yield on the bond. (5)

ii) Describe callable bond and state the relationship between price of callable bond with similar bond without call feature. (2)

iii) Continuous GRY of non-callable bond is 8% (continuous compounding), forward yield volatility is 15% and duration of 3.6 years now and 2.8 in 1 year. Risk free interest rate 6% continuously compounded. Calculate the price of 4-year callable bond with 7% coupon, callable in 1 year. (8)

[15]

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