INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

6th December 2022

Subject CS2A – Risk Modelling and Survival Analysis (Paper A)

Time allowed: 3 Hours 15 Minutes (14.45 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

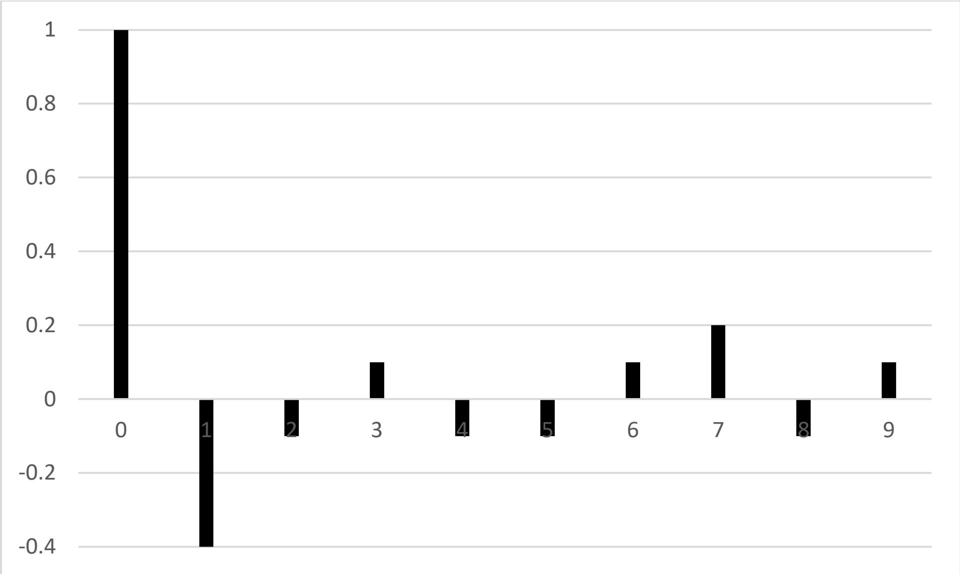
AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1)

i) Define a Moving Average (MA) process along with its complete notations. (2)

ii) The correlogram for a time series is:



A non-invertible MA model is fitted to this time-series. Find the equation of the model. (2)
[4]

Q. 2)

i) Differentiate between complete future life time and curtate future life time random variables. (2)

ii) State the formula for curtate expectation of life and explain all the notations involved in it. (2)
[4]

Q. 3) A participating policy by an insurer 'ABC' pays reversionary bonus yearly per 1000 sum assured as follows:

State	Yearly bonus/1000
Business Flourish (BF)	9
Business De-grown (BD)	5
Business Stagnant (BS)	0
Business Expansion (BE)	3

The transition of the business states is as per the below matrix:

P=		BF	BD	BS	BE
	BF	0.2	0.8	0	0
	BD	0	0.7	0.3	0
	BS	0	0	0.4	0.6
	BE	0.4	0	0	0.6

Annual interest rate = 0% and transition happens after bonus is paid.

i) Determine the period of each of the states of the chain. (1)

ii) Calculate the long-term yearly bonus. (4)

[5]

Q. 4) An annual subscription of a cloud-storage company follows a Markov chain with following states:

State A: subscription continued
State B: subscription cancelled by customer
State C: subscription not renewed by the company

At t = 0, the number of subscribed customers = 500

Transition between states occur at end of year

T_{xy} indicates the transition matrix from x^{th} year to the y^{th} year.

T_{01} =

0.86	0.10	0.04
0	0	1
0	0	1

T_{12} =

0.84	0.11	0.05
0	0	1
0	0	1

Calculate the number of expected subscriptions cancelled by customers in next 2 years. **[3]**

Q. 5)

i) A Naïve-Bayes classifier is required to classify a plant species (with feature: 'C & H') into class of X: 'Endangered' and Y: 'Critically Endangered'.
Apply a maximum likelihood estimation technique to predict whether the above plant belongs to X or Y.

Species No.	Features	Class
1	C M	X
2	D L H	Y
3	C K	X
4	D H P	X
5	L B C	Y

(6)

ii) An insurer while claim underwriting a line of business designates 10% customer as Moderate Risk and 90% as Low Risk. Number of claims follows Poisson (θ) and is independent of the number of claims made by a policyholder in previous year. For a moderate Risk policyholder, $\theta = 0.5$ and for a low Risk policy holder, $\theta = 0.2$. Calculate expected no. of claims made in a year by a policyholder who made one claim in 2021. (4)

[10]

Q. 6) A sports analytics firm is working on the form of sportspersons for an upcoming world cup tournament. A sportsman could be in either out-of-form ("O") or in-form ("I"). Probability of going in-form while being out-of-form in $(t, t+dt)$ is $0.25dt + o(dt)$. If a sportsman is in-form, going out-of-form has probability $0.75\, dt + o(dt)$ in $(t, t+ dt)$.

Notation	Probability that sportsman is:	Given sportsperson is at t=0:
$P_{OI}(t)$	In-Form	Out-of-Form
$P_{OO}(t)$	Out-of-Form	Out-of-Form
$P_{IO}(t)$	Out-of-Form	In-Form
$P_{II}(t)$	In-Form	In-Form

i) Write down the generator matrix based on the above information. (2)

ii) Explain whether the sportsperson's form may or may not be considered as a Markov jump. (2)

iii) Solve the Kolmogorov Forward equation for $dP_{OO}(t)/dt$. (6)
[10]

Q. 7) A personal accident policy providing benefits based on two risk profiles of policy holders are as follows:

<u>Risk group-A</u>
No. of Policy holders = 880
Benefit amount = 10000
Claim probability = 0.0012

<u>Risk group-B</u>
No. of Policy holders = 765
Benefit amount = 8500
Claim probability = 0.008

Aggregate claim payable can be assumed to follow individual risk model.

i) If 'C' is the aggregate claim amount payable, then derive the following expressions from the first principles using the compound binomial distribution. State the appropriate notations, underlying assumptions and the model formula wherever applicable.

a) Expected claim pay out. (2)

b) Standard Deviation of the Claim. (2)

c) Skewness of the claim. (4)

ii) Calculate Mean, Variance & coefficient of Skewness of total claim amount for all the policyholders. (4)

iii) Calculate the probability that the aggregate claims payable are between 100000 and 200000 using an appropriate approximation. (3)
[15]

Q. 8) For a claim distribution 'C' with LogNormal ($\mu = 6$, $\sigma^2 = 4$), calculate reinsurer's expected payout per claim to insurer where the reinsurer pays:

$$P = \begin{cases} 0 \ \text{if } C < 900 \\ C - 900 \ \text{if } 900 < C < 1700 \\ 800 \ \text{if } C > 1700 \end{cases}$$

[6]

Q. 9)

i) For a Frank Copula with Generator Function: $\Psi(t) = -\ln \{(e^{-\alpha t} - 1) / (e^{-\alpha} - 1)\}$:

a) Determine the inverse generator Function. (2)

b) Derive the Copula function C[u,v]. (2)

ii) For two claims X and Y with claim amount and probabilities of $P(X<100)$ and $P(Y<100)$, find the joint probability with Frank Copula of $\alpha = 2.9$. (2)

[6]

Q. 10)

i) For a time-series model $(1-\alpha\beta)^3 X_t = e_t$,
Where,
β is the Backward Operator.
And e_t is Standard white noise.
If X_t is stationary, calculate ACF for 1st two lags: ρ_1 and ρ_2, using Yule-Walker equation. (6)

ii) Carry out (a) Ljung-Box test (b) turning points test, and (c) SACF inspection for the following SACF estimates on the residuals of ARMA(1,1), given the sample size of 278 observations and for 198 turning points.

Lag	Co-efficient
1	0.03
2	0.09
3	-0.11
4	-0.02
5	0.02

(9)

[15]

Q. 11)

i) Give four important reasons as to why the crude mortality rates needs to be graduated. (2)

ii) Graduation of mortality rates was done as per the following table. Find range of "p' with appropriate explanations, such that 99% confidence interval is reached as per Chi-square test.

Age x	Exposed to Risk	Observed deaths	Graduated mortality rates
35	5444	80	0.01658
36	5355	102	0.01787
37	5268	88	0.01894
38	5197	110	0.01988
39	4978	91	0.02022
40	4831	106	0.02154
41	4654	123	0.02365
42	4521	107	0.02811
43	4487	122	0.02957
44	4321	125	0.03069
45	4101	140	0.03081
46	4021	145	0.03166
47	3951	140	p

(8)

iii) State four other types of tests that may be used other than Chi-square test for goodness of fit. (2)

iv) State two disadvantages of Age-period-cohort model. (2)

[14]

Q. 12) An investigation has been carried out into the shortlisting rates of different movies which have undergone a critical evaluation process at a major awards function. For each movie, data relating to the language (English and Non English), genre (Drama, Action and Comedy), and Run time (Low and High). A Cox proportional hazards model was fitted to the data, and the results are given below.

Covariate	**Parameter Estimate**	**Standard Error of the estimate**
Language		
English	0	
Non English	-0.15	0.1
Genre		
Drama	0	
Action	-0.08	0.03
Comedy	-0.20	0.07
Runtime		
Low	0	
High	-0.13	0.04

i) Write down a formula for the force of elimination according to this model. You should define all the terms that you use. (3)

ii) Can this model be treated as a proportional hazards model? Explain. (1)

iii) State, in the context of this model, (a) the group of movies to which the baseline hazard refers to and (b) the group of movies which have the lowest force of elimination. (2)

iv) Calculate the proportion, according to the fitted model, by which the force of elimination for an English Action movie with a low runtime exceeds or fall short of a non-English comedy movie with a high run time. (2)

[8]

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