INSTITUTE OF ACTUARIES OF INDIA EXAMINATIONS

9th December 2022

Subject CM2B – Financial Engineering and Loss Reserving (Paper B)

Time allowed: 1 Hour 45 Minutes (14.45 – 16.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Mark allocations are shown in brackets.***
2. ***Attempt all questions beginning your answer to each question in the template provided.***
3. ***Attempt all sub-parts of the question in the template provided only, unless otherwise instructed to do so.***
4. ***The working of each part of the question should be on a separate tab (sheet). For example, question 1(i) should be worked out within the tab (sheet) with name 1(i) of worksheet.***
5. ***Where possible, summarize your data used & assumptions made (if any) in a separate tab.***
6. ***Do save your work in solution template on a regular basis.***
7. ***All the detailed guidelines are available on exam screen.***
8. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you. You are requested to save and submit the work before leaving the examination premises.

Q. 1) A working couple wants to invest a fund of Rs.1 Crore in fixed deposit and stock market in equal proportion for their daughter's education. The fixed deposit pays guaranteed interest of 4% per annum. You have been given a distribution of the 1-year return on the equity index in the 'Q.1 Data' worksheet.

The couple uses the utility function U (w) = Ln (w).

i) Calculate the expected utility of each investment at time $t = 1$. (5)

ii) Comment on absolute risk aversion and relative risk aversion in part (i). (3)

The husband chooses consultant A, who proposes to invest the amount in a stock market Index and promises good returns.

iii) Assume next 15 years stock return will be as given in the spreadsheet and there is no correlation between returns on fixed deposit and returns on stock market, tabulate the Surplus Process (i.e. movement of total fund) over the next 15 years. (3)

Consultant 'A' confirmed to the couple that the total return from the Stock market would follow a normal distribution ($\mu t, \sigma^2 t$); t = time period from initial investment, μ = 9% per annum and σ^2= 30% per annum.

The couple defines "Ruin" as: Fund being less than $25*(1.09)^{(t-1)}$ lakh at the end of year t, i.e. to avoid ruin, the fund should not be less than 25 lakhs at the end of 1st year, 25*1.09 at the end of 2nd year and so on. The couple is interested to understand the probability of ruin at the end of each of the year for the next 15 years.

iv) Find the Probability of Ruin on the investment at the end of each of year for the next 15 years, basis the strategy proposed by Consultant 'A'. (15)

The wife chooses Consultant 'B' who promises to manage the investment in a "better way". He would trade in equity derivatives. He guarantees to pay 5 lakhs at the end of every year for 15 years. This payment received by the couple does not earn any interest in future. The only issue is that every now and then there would be events with extreme losses. The loss count up to time 't' follows Poisson distribution with mean (λt) where, λ=0.75 per annum and loss amount is 8 lakhs whenever loss occurs.

v) Calculate the probability of ruin on the investment at the end of each year for 15 years, basis the strategy proposed by Consultant 'B'. (10)

vi) Plot a chart (one single chart) depicting calendar year on X axis and Probability of Ruin on Y axis for consultant A & B at the end of each year for 15 years. Comment on the decision made by the couple as per the chart. (3)

[39]

Q. 2) The annual yields, i, from a fund are independent and identically distributed. Each year, the distribution of (1 + i) is lognormal with parameters μ = 0.051 and $\sigma 2$ = 0.0039.

i) Comment on some key features of the lognormal distribution that makes it a relevant distribution of choice for (1+i). (2)

An amount of Rs. 4,500 is invested in the fund at the start of each of the next 20 years, i.e. at times t = 0, t = 1, t = 2, …, t = 19.

ii) Calculate, using the probability distribution above, the expected accumulated value of the fund at time t = 20. (5)

A student has performed a simulation of the annual returns from the distribution and the results are provided in the 'Q.2 Data' worksheet.

iii) Calculate the expected accumulated value of the fund at time t = 20, using the simulated returns. (3)

iv) Comment on the results in parts (i) and (ii). (3)

v) Using the simulated returns from part (iii):

a) Calculate, the cumulative probability that the fund value at time t = 20 reaches or exceeds each of the values from Rs. 100,000 to Rs. 300,000 in increments of Rs. 5,000. (3)

b) Plot a chart of the cumulative probabilities. (3)

c) Comment on the shape of the chart in part (v)(b) above. (3)

[22]

Q. 3) A large bank links its deposit rates to the forward rates available in the market. Spot rates are provided in 'Q.3 Data' worksheet.

The bank has recently come out with a recurring deposit scheme (RD account) in which:
A customer has to invest an amount equal to Rs. 1 lakh at the beginning of each year starting now for the next 5 years. The customer receives a lump sum at the end of the 5 years.

i) Calculate the amount of lump sum received by the customer at the end of the 5 years. (12)

The Bank allows for premature closure of the RD account. RD account has a penalty of 'x'% on the lump sum payment if the customer chooses to prematurely close the account. Lump sum amount at any point is the addition of principal and accrued interest till the date of premature closure.

ii) Find 'x' such that the IRR after continuing with the account till the end of 5 years and the IRR from pre-mature closure at the end of 2 years, just before paying the investment due, are equal. (10)

Five variables from a Uniform distribution (0,1) are given. Using the following:

- spot rate at time 0 given in part (i)
- mean level of 5%
- speed of reversion of 0.2
- standard deviation of the error term as 3% per annum

iii) Create five Monte-Carlo simulated paths of the one–year short rates (t) (where t = interval of 1 year) for the next 5 years using the Cox-Ingersoll-Ross interest rate model showing the paths in a line chart. (17)

[39]

****