INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

7th December 2022

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

Time allowed: 3 Hours 15 Minutes (14.45 – 18.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.

Q. 1)

i) What is the result of widespread usage of the Internet with regards to efficient markets? (4)

ii) Explain what is meant by an excessive volatile market? (2)

[6]

Q. 2) A teenaged cricket fan, fond of betting on HPL cricket game assesses his utility of wealth using the utility function U (w) = 2(w^0.5–1).

i) Prove algebraically that the cricket fan is:

a) Non-satiated. (1)

b) Risk averse. (1)

ii) Prove that the cricket fan exhibits constant relative risk aversion. (2)

The cricket fan is intended to bet on two teams. The table below shows the pay-out per Rs. 1 bet on each of these teams if it wins the match, and the investor's estimated probabilities of each team winning the match. The pay-out is the total paid and is not in addition to the bet being returned.

Team	Winning payout per Rs. 1 bet	Probability of winning
A	1.69	60%
B	6.25	10%

The cricket fan has total wealth of Rs. 1,000 and he will bet all of his wealth on this match. Negative bets are not allowed.

iii) Calculate the amount he should bet on each team to maximize his expected utility of wealth. (7)

iv) Calculate the expected wealth resulting from the bets in part (iii). (2)

[13]

Q. 3) Define the following:

i) Anti-selection (1)

ii) Moral Hazard (1)

iii) Value at Risk (VaR) (1)

iv) TailVaR (1)

[4]

Q. 4) An investor has Rs. 8,00,000 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest.

The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean $\mu = 7\%$ p.a. and standard deviation $\sigma = 5.5\%$ p.a.

i) Calculate the following in respect of the investment at the end of 1 year:

a) The shortfall probability below a value of Rs. 7,20,000. (2)

b) The 99.5% value at risk. (3)

The second opportunity is a derivative that offers the following payoff in 1 years' time based on the performance of the index during the year.

Payoff (Rs) Scenario

7,30,000 when $X \leq -7.1\%$
7,50,000 when $-7.1\% < X \leq 7\%$
9,62,000 when $X > 7\%$

ii) Calculate the expected payoff from the derivative at the end of the year. (4)

iii) Calculate the following in respect of the payoff from the derivative:

a) The shortfall probability below a value of Rs. 7,20,000. (1)

b) The 99.5% value at risk. (1)

iv) Comment on how the investor may choose between the two investments. (3)
[14]

Q. 5)

i) Differentiate between fixed and varying rate model of investment return. (3)

ii) The annual rates of return from a particular investment, Investment A, is independently and identically distributed. Each year the distribution of $(1+ i(t))$, where $i(t)$ is the return earned on Investment A in year t, is log-normal with parameters μ and σ^2. The mean and standard deviation of $i(t)$ are 0.04 and 0.03, respectively.

a) Calculate the values of μ and σ^2. (4)

An insurance company has liabilities of Rs. 200 to meet in 3 years' time. It currently has assets of Rs. 175, which are invested in Investment A.

b) Determine the probability that the insurance company will be unable to meet its liabilities. (5)
[12]

Q. 6) The current value of a company's share price is Rs. 325. Yearly movements of a Company's share price follows log normal distribution.

The following details are given with respect to Mean Variance portfolio theory:

- beta between company's share price and market = 0.5
- correlation between company's share price and market = 0.7
- Expected market return = 6% p.a.
- Market σ = 20% p.a.
- market price of risk = 4%

i) The risk free rate of return is:

A. 5.00%
B. 5.10%
C. 5.20%
D. 5.30% (1)

ii) Expected return (E_p) on the share p.a.

A. 5.60%
B. 5.70%
C. 5.80%
D. 5.90% (1)

iii) Standard deviation (σ_p) of the return on the share p.a.

A. 10%
B. 12%
C. 14%
D. 20% (2)

iv) Derive the 95% confidence interval for share price in 6 months' time if share prices are modelled using Lognormal distribution with yearly mean equal to e^{Ep} and yearly standard deviation equal to σ_p; where E_p and σ_p were estimated in parts (ii) and (iii). (10)
[14]

Q. 7) Consider a two period recombining binomial model for St the price of a non-dividend paying security at times t = 0, 1 and 2, with real world dynamics:

St+1 = St u with probability p
= St d with probability 1–p

u >d>0

There also exists a risk-free instrument that offers a continuously compounded rate of return of 7% per period. The state price deflator in this model after one period is:

A1 = 0.7610 when S1 = S0 u
= 1.5220 when S1 = S0 d

i) Calculate the value of 'p' and the risk-neutral probability measure 'q'. (4)

ii) Calculate 'u' and 'd'. (7)

iii) Calculate the price at time 0 of a derivative that pays 1 at time 2 if S2 greater than or equal to S0 using the risk-neutral probability measure derived in part (i). (3)

[14]

Q. 8) The delay triangles given below relate to a portfolio of general insurance policies.

The cost of claims settled during each year is given in the table below:

(Figures in Rs.000s)

Accident Year	*Development year 0*	*1*	*2*
2019	4,244	794	283
2020	4,867	932	
2021	6,003		

The corresponding number of settled claims is as follows:

Accident Year	*Development year 0*	*1*	*2*
2019	606	100	53
2020	651	96	
2021	699		

i) Calculate the outstanding claims reserve for this portfolio using average cost per claim method. (10)

ii) State the assumptions underlying your result. (3)

[13]

Q. 9) On 1 January 2017 an insurer sells 100 policies to a certain region, each with a five-year term, to householders wishing to insure against damage caused by fireworks. The insurer charges annual premiums of Rs. 840 payable continuously over the life of the policy. The insurer knows that the only likely period a claim will be made is on 1st June each year, when it is traditional to have an enormous firework in that region. The annual probability of a claim on each policy is 40%. Claim amounts follow a Pareto distribution with parameters $\alpha = 10$ and $\lambda = 9{,}000$.

i) Calculate the mean and standard deviation of the annual aggregate claims. (4)

ii) Denote by $\Psi(U, t)$ the probability of ruin before time t given initial surplus U.

a) Explain why for this portfolio $\Psi(U, t1) = \Psi(U, t2)$ if $5/12 < t1, t2 < 17/12$. (2)

b) Estimate $\Psi(15000, 1)$ assuming annual claims are approximately normally distributed. (4)

[10]

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