

# **INSTITUTE OF ACTUARIES OF INDIA**

## **EXAMINATIONS**

**7<sup>th</sup> December 2022**

**Subject CM1A – Actuarial Mathematics (Paper A)**

**Time allowed: 3 Hours 15 Minutes (10.15 – 13.30 Hours)**

**Total Marks: 100**

### **INSTRUCTIONS TO THE CANDIDATES**

1. *Please read the instructions inside the cover page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions beginning your answer to each question on a separate sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

#### **AT THE END OF THE EXAMINATION**

**Please return your answer book and this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you.**

- Q. 1)** A man aged 45 years exact buys a 20-year endowment assurance policy that pays Rs. 50,000 on maturity. The man pays a premium of Rs. 1,577 at the beginning of each year throughout the policy term, or until death if that happens first. On death during the policy term, all premiums paid till the date of death are returned to the nominee without any interest at the end of the year of death.

Calculate the expected present value of the benefits payable under this policy assuming mortality rate of AM92 Select and interest rate of 4% p.a.

[4]

- Q. 2)** In a life insurance policy, the death benefit payable at the end of the year of death is a return of premium paid without interest. A level premium of Rs. 3,000 is payable annually in advance throughout the term of the policy. Using the information given below calculate the profit or loss expected to emerge at the end of 12<sup>th</sup> policy year per policy in force at the start of that year:

Reserve per in-force policy at the start of the year of Rs. 25,130.

Reserve per in-force policy at the end of policy year of Rs. 28,950.

Probability of death during the policy year of 0.03.

Expenses incurred at the start of the policy year of Rs. 90.

Rate of Interest earned of 4% per annum.

The reserves provided above are immediately before the payment of premium due.

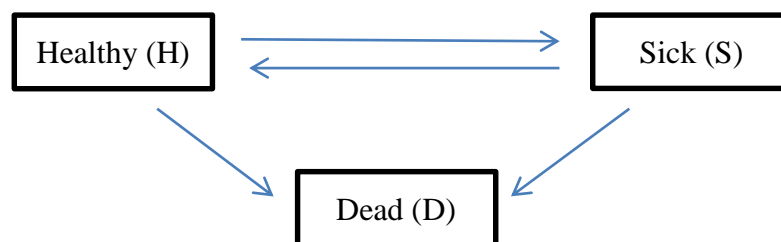
[3]

**Q. 3)**

- i) Define the term “reserves” and explain why a life insurance company will need to set up a reserve for an insurance policy. (4)
- ii) Define “gross prospective reserves” and “gross retrospective reserve”; and state the conditions under which the two may be equal. (4)

[8]

- Q. 4)** A life insurance company uses the following 3 state model, to estimate the profit in respect of a 2 year combined death benefit and sickness policy issued to a healthy policyholder aged exact 50 at inception.



In return for a Single premium of Rs. 25,000 payable at the outset the company will pay the following benefits:

- Rs. 50,000 if the policyholder dies within 2 years, payable at the end of the year of death.
- Rs. 35,000 at the end of each of the 2 years if the policyholder is in a sick state.

Let  $S_t$  represent the state of the policyholder at age  $50+t$ , so that  $S_0 = H$  and for  $t = 1$  and  $2$ ,  $S_t = H, S$  or  $D$ .

The company uses transition probabilities defined as follows:

$$p_{50+t}^{ij} = P(S_{t+1} = j | S_t = i)$$

For  $t=0$  and  $1$  the transition probabilities are:

$$p_{50+t}^{HD} = 0.15, \quad p_{50+t}^{SD} = 0.2, \quad p_{50+t}^{SH} = 0.55 \quad \& \quad p_{50+t}^{HS} = 0.2$$

- i) One possible outcome for this policy is that the policyholder is healthy at times 0, 1 and 2. List all possible outcomes and the associated cash flows. (3)
  - ii) Calculate the probability of occurrence for each of the possible outcomes. (5)
  - iii) Assuming a rate of interest of 8% per annum, calculate the mean and standard deviation of the net present value at time 0 of the profit for the policy. (7)
- [15]

**Q. 5)** A unit-linked policy has the following profit vector:

Policy Year	Net Cashflow
1	-37
2	-15
3	-8
4	38
5	49

- i) Calculate the reserves required to zeroise losses at the end of year 2 and year 3, assuming a rate of accumulation of 6.5% and that mortality rate is 0.025 at each age. (2)
  - ii) Assuming a risk discount rate of 10% p.a., determine the net present value of the profits before and after zeroization. State with reasons which of these figures you would expect to be greater. (5)
- [7]

**Q. 6)** An insurance company is launching a regular-premium term assurance product, with no surrender benefit. The policy terminates on completion of policy term or on earlier death. The company has developed an actuarial model to model the expected profits from this product.

- i) Describe the cash flows from the perspective of a policy holder. (3)
- ii) List the possible decrements that the insurance company should consider while modelling. (2)
- iii) For each of the decrements, the insurance company is to provide an assumption to the model. Briefly describe key modelling steps applicable in developing such an input for the model. (4)
- iv) The model currently has the capability of accepting a discount rate assumption with up to 4 decimal places (e.g. 0.0418). Cost will be incurred to upgrade the model to accept an assumption with up to 7 decimal places (e.g. 0.0418213), considered ideal by the management of the company. List the key considerations when upgrading the model. (2)

[11]

- Q. 7)** Ram invests into a 25-year savings plan by paying Rs. 500 every half year in advance. The interest rate is 6% per annum convertible monthly for the first 15 years and 6% per annum convertible half yearly for the final 10 years. Determine the fund value at the end of policy term. [4]

- Q. 8)**
- i) Define Annual Percentage Rate (APR). What is the purpose of the APR? (2)
  - ii) Describe the prospective and retrospective methods of determining the loan outstanding. (2)

A student takes out a student mortgage loan of Rs. 20,00,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective.

- iii) For the loan above, calculate the capital repaid in the seventh year and the interest component of the 85<sup>th</sup> loan instalment. (6)
  - iv) 10 years after taking out this loan, the student decides to repay back the loan by paying 150% of original loan instalment as the monthly instalment for the remainder of the term. Calculate the length of time by which the term of the loan will reduce if repayments are altered and determine the savings on interest payments. (6)
- [16]

- Q. 9)** By considering a term assurance policy as a series of one year deferred term assurance policies, show that

$$\bar{A}_{x:n|}^1 = \frac{i}{\delta} A_{x:n|}^1 \quad [5]$$

- Q. 10)** A life insurance company issues whole of life insurance policies to lives age 40 years exact for a sum assured of Rs. 1,50,000 payable at the end of year of death. Premiums are payable annually in advance.

- i) Calculate the annual premium for each policy using the basis below. (4)

Basis:

Mortality	AM92 Select
Interest	6% per annum
Initial Commission	75% of annual premium
Initial Expense	Rs.500
Renewal Commission	3% of each annual premium excluding the first
Renewal expense	Rs.100 per annum at the start of second and subsequent policy years

- ii) Calculate the minimum annual premium the company should charge in order that the probability of making a loss on any policy would be 5% or less. (7)
- [11]

- Q. 11)** A life insurance company issues an annuity policy to two lives, a male and a female, aged 55 years exact and 50 years exact respectively. A single premium of Rs. 1,00,000 is payable at the outset. An annuity is payable annually in advance while at least one of the lives is alive.

- i) Write down an expression for the net future loss random variable at the outset for this policy. (3)
- ii) Derive the annuity in respect of the premium payable using the basis below.

Basis:

Mortality rate of PMA92C20 for the first life, PFA92C20 for the second life  
Interest rate of 4% p.a. (3)

- iii) Explain the following terms: “death strain at risk”, “expected death strain” and “actual death strain”. (3)  
[9]

**Q. 12)**

- i) A student develops a line graph plotting the net present value of a stream cash flows. The x-axis represents the discount rate and the y-axis represents amounts. What do the points at which this graph intersects with the x-axis and y-axis represent? (2)
- ii) Consider a project where external borrowing is involved. List the two key project appraisal metrics which should be considered. Further, provide a list of other factors for the company to consider when deciding between projects. (5)  
[7]

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