Institute of Actuaries of India

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

July 2022 Examination

INDICATIVE SOLUTION

Introduction
The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

i) IV [2]
ii) I [2]
iii) I [2]
iv) II [2]
v) III [2]
[10 Marks]

Solution 2:

i) III [2]
ii) II [2]
iii) III [2]
iv) III [2]
v) II [2]
[10 Marks]

Solution 3:

i) III [3]

ii) Since the real rate is constant, short term nominal rates would reduce in future as and when inflation is expected to reduce from current levels [1]
Forward rates are expected to be lower than current short term nominal rate [1]
Since Spot rates are akin to geometric mean of forward rates, [1]
we expect the shape of the yield curve to be downward sloping. [1]
[4]
[7 Marks]

Solution 4:

i) Main Assumptions of Binomial Model are:

a. There are no trading costs
b. There are no taxes
c. There are no minimum or maximum units of trading
d. Stock and bond can only be bought and sold at discrete times 1,2,….
e. The principle of no arbitrage applies.

[Max 2]

ii) Under one-period Binomial Model, the value of the option is derived as

V_1 = { c_u if S1 = S0u
cd if S1 = S0d

then

V0 = e-r Eq[V1]
= e-r [qcu + (1-q) cd]

Here q denotes the risk-neutral basis of pricing the derivatives.

We can re-express this value in terms of real-world probability p where

V0 = e-r [p* (q/p) cu + (1-p) *((1-q)/(1-p))*cd]
= Ep[A1V1]
Where A1 is a random variable with

A1 = { e-r (q/p) if S1 = S0u
e-r ((1-q)/(1-p)) if S1 = S0d

In this case, instead of using risk-neutral basis, we are using real-world probabilities and a different discount factor. The discount factor A1, depends on whether the Share price foes up or down. This means that it is random and so we call it a Stochastic Discount Factor.

[3]

iii) b [3]

[8 Marks]

Solution 5:

i) I [2]

ii)

a) Let $S(t)$ denote the insurer's surplus at time t.

Then

$\psi(U)$ = Pr($S(t)$ < 0 for some value of t) i.e. the probability of ruin at some time

$\psi(U,t)$ = Pr($S(k)$ < 0) for some $k < t$ i.e. the probability that ruin occurs before time t. [2]

b) Immediately before the payment of any claims, the insurer has cash reserves of 10,00,000 + 10,000+5,000 =10,15,000.

The distribution of S(1) is given by

Deaths	S(1)	Prob	
None	1015000	=0.95*0.9 =	0.855
A Only	1015000-1700000=-685000	=0.9*0.05=	0.045
B only	1015000-400000=615000	=0.95*0.10=	0.095
A&B	1015000-1700000-400000= -1085000	=0.05*0.1=	0.005
Probability of Ruin is		=.045+.005	0.05

[4]

c) Assuming the surplus process ends if ruin occurs by time 1, then 2 possible values of $S(2)$ are -6,85,000 and -10,85,000.
If there are no deaths in year 1, possible values of $S(2)$ are
No deaths: 1015000 + 15000 = 10,30,000

Deaths	S(2) when no deaths in year 1	Prob	
None	10,30,000	=0.95*0.9 =	0.855
A Only	1030000-1700000=-670000	=0.9*0.05=	0.045
B only	1030000-400000=630000	=0.95*0.10=	0.095
A&B	1030000-1700000-400000=-1070000	=0.05*0.1=	0.005

If B dies in year 1, the possible values of $S(2)$ are:
A lives: 615000 + 10000 = 6,25,000
A dies: 615000 + 10000 – 17,00,000 = -10,75,000

The probability of ruin within 2 years is given by:
$0.05 + 0.855 \times (0.05 \times 0.9 + 0.05 \times 0.1) + 0.095 \times 0.05 = 0.0975$

[5]

[13 Marks]

Solution 6:

i)

1. Comparability

An investor can state a preference between all available certain outcomes.

2. Transitivity

If A is preferred to B and B is preferred to C, then A is preferred to C.

3. Independence

If an investor is indifferent between two certain outcomes, A and B, then he is also indifferent between the following two gambles:

(i) A with probability p and C with probability (1 - *p*); and

(ii) B with probability p and C with probability (1 - *p*).

4. Certainty equivalence

Suppose that A is preferred to B and B is preferred to C. Then there is a unique probability, p, such that the investor is indifferent between B and a gamble giving A with probability p and C with probability (1 - *p*). B is known as the certainty equivalent of the above gamble.

[4]

ii)

a) Wealth after the uncertain event will be either:

$1,00,000 \times (1.3a + (1 - a)) = 100000 + 30000a$ with probability 0.75

or:

$100,000 \times (0.4a + (1 - a)) = 100000 - 60000a$ with probability 0.25.

Thus expected utility of wealth is:

$0.75 \times \ln(100000 + 30000a) + 0.25 \times \ln(100000 - 60000a)$.

[2]

b) Differentiate with respect to a:

$30000 \times 0.75/(100000 + 30000a) - 60000 \times 0.25/(100000 - 60000a)$.

Set equal to zero:

$30000 \times 0.75 / (100000 + 30000a) - 60000 \times 0.25 / (100000 - 60000a) = 0$

$30000 \times 0.75 \times (100000 - 60000a) = 60000 \times 0.25 \times (100000 + 30000a)$

$30-18a= 20+6a$

$10=24a$

$a = 0.4167$

Check for maximum:

Differentiate with respect to a again:

$-(30000)^2 \times 0.75/(100000 + 30000a)^2 - (60000)^2 \times 0.25/(100000 - 60000a)^2$.

This must be negative because of the square terms, hence this is a local maximum.

[5]

[11 Marks]

Solution 7:

i) the major drawback of Black Scholes Option Pricing formula is that it requires knowledge of σ, the volatility of the price of the underlying asset. Unlike the other parameters in the formula, the volatility cannot be observed directly in the market. Hence, it must be estimated in order to be used in practice. [1]

ii)

a) Using put-call parity relationship
550.1 + 5900 * exp (-0.06 *1) = Value of put + 6000
Value of put = 106.51 [3]

b) First, we need to estimate the implied volatility of the stock using the information given in the question.
This is done using trial and error.

Using Black Scholes formula:
Ct = 6000 * Φ (d1) – 5900 * (exp(-0.06*1))* Φ (d2)

Where
d1 = (log[6000/5900) + (0.06 + 0.5 * σ^2) *1) / σ * sqrt(1)
d2 = d1 – σ *sqrt(1)

Using trial and error method to estimate σ, starting with a reasonable value of 0.12 for σ and substituting the parameters in the black scholes model above, we get,

d1 = 0.70006 and d2 = 0.58006
hence,

ct = 6000 * Φ (0.70006) – 5900 * (exp(-0.06*1))* Φ (0.58006)
substituting the values from the Actuarial tables after interpolation we get;

ct = 6000 * 0.758 – 5900 * (exp(-0.06*1))* 0.719 = 552.92

this is higher than the actual price of 550.1, hence, we should try a lower value for σ.

Trying for σ =0.10
we get
d1 = 0.818 and d2 = 0.718

hence,

ct = 6000 * Φ (0.818) – 5900 * (exp(-0.06*1))* Φ (0.718)
substituting the values from the Actuarial tables after interpolation we get;

ct = 6000 * 0.677727 – 5900 * (exp(-0.06*1))* 0.595358 = 516.93
as the two call options straddle the actual price of 550.1, we can interpolate between the two values of σ to obtain an estimate of the implied volatility:

σ ~ 0.1184

We can now use this value of σ to estimate the price of the **put** option with strike price of 5500

d1 = (log[6000/5500) + (0.06 + 0.5 * 0.1184^2) *1) / 0.1184 * sqrt(1) = 1.3005

d2 = d1 – 0.1184* sqrt(1) = 1.182
hence,

pt = 5500 * (exp(-0.06*1))* Φ (-1.182)- 6000 * Φ (-1.3005)

pt = 33.967 [6]

[10 Marks]

Solution 8:

i) Solution : IV – 7% [1]

ii) Solution : II [2]

iii) Let a = lower bound = 3%
Let b = upper bound = 11%

Variance for 1 year = (b- a)^ 2 / 12 = 0.053%

Variance for 10 years = (I ^ 2 + j^2 + 2 * I + 1) ^ 10 - (1+ I) ^ (2 *10) = 1.81% where I = yearly mean & j = yearly standard deviation

[3]

iv) 10 year Expected mean = 96.7151 %
(1+ 96.712%) *50 lakh = 98.328 lakhs [3]

v) X = amount to be invested

Working in lakhs ,

p (60%* X *product $(1+i_t)$ + 40% * x * (1.07)^10 > 98.328)

=>p (product $(1+i_t)$ > (98.328 - 40% x * (1.07)^10) /60%* x)

Taking log on both sides

p (summation log $(1+i_t)$ > log (113.05 - 40% x * (1.07)^10) /60%* x)

log $(1+i_t)$ follows normal with mu = 9% and variance = 20% p.a
summation log $(1+i_t)$ normal with mu = 90% and variance = 2

p (Z > (log (113.05 - 40% x * (1.07)^10) /60%* x) - .9)/ sqrt(2))

x value of standard normal distribution cdf at 50 % is 0

(log (113.05 - 40% x * (1.07)^10) /60%* x) - .9)/ sqrt(2) = 0

Solving the above equation gives
2.263 * x = 98.358
X = 43.46 lakhs

[4]

[13 Marks]

Solution 9:

i) Policyholder Behaviour affects the risks and price charged in two ways:

a. Adverse Selection

Refers to the fact that people know more about their condition due to information assymtery. Thus, bad risks are more inclined to take out insurance compared to good risks.

Companies take care of it by trying to find out more about the potential policyholder through a series of questions at the time of policy issuance. This helps the company to categorise the policyholders into homogeneous groups of risks and charge the price accordingly.

b. Moral Hazard

This happens when the policyholder, due to the fact that they have insurance, act in a way which makes the insured event more likely to occur.

Moral hazard is more expensive for companies to handle and it may even push the price of insurance above the maximum price a person is ready to pay.

[3]

ii) The considerations which will be taken into account while assessing the effect of risk is:

a. Its likelihood
b. Severity

The four possible events for frequency-severity dynamics are:

- Low frequency-low severity
- High frequency – low severity
- High frequency – high severity
- Low frequency – high severity

[2]

iii)

a) Maximum Expected Loss = 0.9* 5000000 = 4500000

P(X = 0) = 0.99
P (X = -450000) = 0.01

VaR = -t where t = max { x: P(X<x) <=p)

But P (X <-4500000) = 0 and P(X<0) = 0.01.
Hence,
t = max { x: P(X<x) <=0.005) = -4500000

i.e. VaR = -4500000

b) we can assume that the blockchain contracts are expected to be independent in nature. Hence, we can assume a Binomial distribution for the losses.

Hence, Let X the total impact suffered by the company on these contracts, where X ~ Binomial (10, 0.01) * -4500000
Then as per the definition of Var for discrete random variable we have:
Var (X) = -t where t = max { x: P(X<x) <=p)

If no contracts suffer losses then X = 0
Then probability of this is

P(X = 0) = 10C0 * 0.01^(0) * (1-0.01)^(10) = 0.99^100 = 0.9044

And
P (X <0) = 1- 0.9044 = 0.0956

If exactly 1 contract suffers loss then X = -450000. The probability of this is:

P(X = -4500000) = 10C1 * 0.01^(1) * (1-0.01)^(9) = 0.0914

P (X <-4500000) = 1- 0.9044- 0.0914 = 0.0042

Thus,
t = max { x: P(X< x) <=0.005) = -4500000

hence, var is 4500000. [Max 5 marks]

[10 Marks]

Solution 10:

i)

a) If an investor prefers more to less (is non-satiated) then they will always prefer an investment that first order stochastically dominates another. [1]

b) If an investor is non-satiated and risk averse then they will prefer an investment that second-order stochastically dominates another. [1]

ii)

PDF	*P*(*ri* = –5%)	*P*(*ri* = –3%)	*P*(*ri* = 0%)	*P*(*ri* = +3%)	*P*(*ri* = +5%)
Asset 1	0.20	0.20	0.20	0.20	0.20
Asset 2	0.30	0.20	0.10	0.20	0.20
Asset 3	0.10	0.30	0.20	0.30	0.10

CDF	*P*(*ri* = –5%)	*P*(*ri* = –3%)	*P*(*ri* = 0%)	*P*(*ri* = +3%)	*P*(*ri* = +5%)
Asset 1	0.20	0.40	0.60	0.80	1.00
Asset 2	0.30	0.50	0.60	0.80	1.00
Asset 3	0.10	0.40	0.60	0.90	1.00

∫CDF	*P*(*ri* = –5%)	*P*(*ri* = –3%)	*P*(*ri* = 0%)	*P*(*ri* = +3%)	*P*(*ri* = +5%)
Asset 1	0.20	0.60	1.20	2.00	3.00
Asset 2	0.30	0.80	1.40	2.20	3.20
Asset 3	0.10	0.50	1.10	2.00	3.00

a) None
b) Second Order
c) First Order

[8 Marks]

****