Institute of Actuaries of India

Subject SP7 – General Insurance Reserving and Capital Modeling

March 2021 Examination

EXAMINERS' REPORT

Introduction

The Examiners' Report is written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

A. General comments on the *aims of this subject*

The aim of this General Insurance Reserving and Capital Modelling Specialist Principles subject is to instil, in successful candidates, the ability to apply, in simple reserving and capital modelling situations, the mathematical techniques and the principles of actuarial planning and control needed for the sound financial operation of general insurers.

B. Comments on candidate performance in this diet of the examination.

The aim of SP level exams is to test a candidate's ability to think out-of-box and move beyond standard bookwork questions. Candidates who demonstrated this ability were successful in the exam.

Most students performed poorly on the application/higher skills questions. Question 6 is one such example.

C. Pass Mark

The Pass Mark for this exam was 50.
29 candidates appeared and 4 passed.

Solution 1:

i) UPR for Erection All Risk (EAR) And Contractors' All Risk (CAR) Insurance

These policies have the following unique characteristics which makes it difficult to calculate UPR using 365th method.

- The risk exposure is not uniform throughout the period of insurance but builds up from zero to 100% during the period of insurance.
- Period of insurance is usually longer than one year, but can be shorter than a year for small projects.
- The value at risk builds up till erection is completed followed by 2-3 months of pre-commissioning, testing and commissioning period during which risk exposure is uniform at 100%
- Each project will have its peculiar risk development pattern depending upon several factors including:
  - The nature of machinery and civil work involved
  - Availability of material and labour
  - Experience of the contractor in such kinds of projects
  - Problems faced in the process of erection, e.g. labour, financial, nature related, political etc

[0.5 for each point, 3 Marks]

ii)

(a)	(b)	(c)	(d)=(b)-(a)	(e) = max(31 dec 2020,(b))-max (31 dec ember 2020 , (a)-1))	(f) = (d) - (e)	(g) = (e)/(d)* (c)	(h) = 1-(f3/d3)^2)*c3
Effective Date	**Pol Exp Date**	**Premium**	**TOT DAYS**	**UPR DAYS**	**Days Earned**	**UPR using Uniform approach**	**UPR - Linear Approach**
02-Jul-19	01-Jul-21	50,000	731	182	549	12,449	21,798
23-Nov-19	22-Nov-21	20,000	731	326	405	8,919	13,861
01-Jul-19	30-Jun-22	75,000	1096	546	550	37,363	56,113
15-Aug-19	14-Aug-21	25,000	731	226	505	7,729	13,069
01-Apr-19	28-Feb-20	30,000	334	-	334	-	-
Total		**200,000**				**66,460**	**104,840**

[3]

iii) Unexpired Risk Reserve (URR)

Unexpired Risk Reserve is different from UPR in that it is a measure of likely amount of claims liabilities on the unexpired portion of the policies at the end of an accounting year.

(0.5 marks)

The amount of unexpired risk reserve can be substantially different from UPR in the situations discussed below:

1. If the insurance policies have been under-priced, it is very likely that the expected claims on these policies plus loss adjustment expenses (including allocated and unallocated expenses) shall be more than the premium for these policies. This indicates that the expected claims plus loss adjustment expenses on the unexpired portion of the policies (which is URR) is also likely to be more than UPR.
2. This situation can also be visualized when an insurer anticipates a worsening claims trend in the coming accounting year due to some recent developments affecting claims which was not factored in the premiums when writing the policies in the earlier accounting period (say, a recent regulatory or judicial decision adversely affecting the liabilities on claims happening from a particular date, which was not factored in pricing)

3. On the other hand, if the insurance policies had been overpriced (which may be possible e. g, in a free pricing regime), or the insurer anticipates a favourable claims trend in the coming accounting year (although, such anticipation may not be advisable), URR may be lower than UPR

(1.5 marks, 0.5 for each correct point)

[2]

iv) Adverse development cover pays for the deterioration in claims over a specified period in excess of a pre-agreed amount.

The deterioration can be measured using paid claims or reported claims. The specified period would normally be more than one year.

It is usual for the ceding company to retain a share of the claims i.e. less than 100% is ceded.

[2]

v) Risks to ceding company:

- Claims experience is better than expected and the company pays the claims and the reinsurance premium
- Claims could deteriorate outside of the period of cover and therefore not be reinsured.
- Claims could deteriorate badly and the company's retained share could still be significant.
- The reinsurer could fail

(2 marks, 0.5 for each point)

Mitigated:

- Negotiating the lowest premium possible
- Retaining minimum possible share
- Spreading the ceded risk amongst several reinsurers
- Using only top credit rated reinsurers

(2 marks, 0.5 for each point)

[4]

vi) Risks to the reinsurer

- Underwriting risk — takes on a risk which is worse than anticipated at too low
- a premium
- Timing of claims within the coverage period
- Legislation/judicial decisions/interpretation mean that the original policies have wider coverage than expected, and hence more claims than expected.
- There may be significant latent claims in the portfolio.
- Currency fluctuations could make the claims larger than expected
- Invested returns may be lower than assumed in premium calculation.
- Moral hazard from the cedant, manipulating claim payments/reporting to fall within the policy period.

(2 marks, 0.5 for each point)

Mitigated by:

- Thorough analysis and investigation of the policy and claims files
- Purchasing reinsurance
- Careful monitoring and limiting of aggregates
- Hedging of currency
- Diversification of assets
- Participation clause
- Increase profit margin
- Claim audits to verify no changes in claims procedures.

(2 marks, 0.5 for each point)

[4]

vii) The company's decision does not seem to be appropriate:

- The claims experience has deteriorated only recently.

- The company should do more scrutiny/investigation of adverse claims trends and take suitable actions to remediate the portfolio such as charge a higher premium, change the business mix etc.
- As the portfolio is smaller is size, any adverse development is likely to be much smaller.
- As the company is large, it must be having good solvency. Unless solvency is very tight this does not look like a good option as the company will be ceding profit and paying brokerage when the company should be well able to pay the claims without reinsurance.

(2 marks, 0.5 for each relevant point)

[20 Marks]

Part (i) , (ii), (iii), (iv) Answered well by most

Part (v) Reasonable performance overall

Part (vi) and (vii) Most candidates performed poorly

Solution 2:

i) Curve Fitting method:

- A curve fitting method is a numerical analysis whereby a specific distribution curve is fit based on claims development till date and the curve is then used to extrapolate the tail development factors.
- The curve may be fit either on paid claims / incurred claims or on the link ratios at various development periods observed till date.
- The underlying principle of a curve fit method is fitting a curve that best represents the development decay in the claim development i.e. as time progresses, claims are expected to accumulate at a diminishing rate (monotonically decreasing rate).
- The parameters / coefficients are then estimated based on available data till date and the most common estimation technique is the least –squares method.
- Typically, the exponential family of curves is used in the curve fit method.

(3 marks, 0.5 for each point)

Uses:

- It is used in extrapolation of tail development especially where the insurer has adequate data but not fully run-off (i.e. incomplete history)
- Since paid claims data are typically used for fitting the curve, the method weeds out the volatility of outstanding claim reserves
- Where an insurer's data has anomalies or is distorted due to aberrations in claim experience, the method can be used for smoothing
- The method can be used to understand the underlying data by plotting residual plots to investigate deviation of actual observations from the fitted model
- Once the curve is fit, it is a straight forward application for estimating reserves

(2 marks, 0.5 for each point)

Limitations:

- The method is exposed to the possibility of over-parameterization and hence could lead to spurious accuracy. This typically arises where data is sparse.
- Like in any other model, the robustness of this method is as good as the assumptions underlying the method.
- Most curve fit methods assume decay in development pattern. Therefore, if the insurer's data does not represent the same (e.g., deliberate slow settlement procedures; hump shaped payment curves), the method may not be relevant.
- Also, most of these curves may fall through when negative incremental development is observed in claims paid data on a consistent basis since these curves typically produce development factors not less than 1.

(2 marks, 0.5 for each point)

[7]

ii) The **Cape Cod (CC) method** is used to calculate **loss reserves**, which use weights proportional to loss exposure and inversely proportional to loss development.

This method operates under the assumption that premiums or other volume measures are known for historical accident years, and that ultimate loss ratios are identical for all accident years.

(1 mark, 0.5 for each point)

The primary difference between the Cape Cod and Bornhuetter-Ferguson methods is that the Cape Cod method creates ultimate loss estimates using both internal and external information.
In the Cape Cod method, loss reserves are calculated as the loss-to-date divided by the exposure and then divided by the ultimate loss development factor.

(1 mark, 0.5 for each point)

A key drawback of the Cape Cod method is that it does not take into account variability in both historical loss estimates and loss development factors and the loss exposure is assumed to be constant over time.
The method also provides greater weight to historical experience over recent experience, since more mature accident years are closer to the ultimate loss.

(1 mark, 0.5 for each point)

[3]

iii)

Year	Earned premium (in INR 000's)	Earned Policy Years	Incurred claims (in INR 000's)	Incurred cumulative development factor	Selected Ultimate Loss Ratio	Selected Ultimate Loss
2015	11,750	1,150	8765	1.000	75%	8,765
2016	13,000	1,275	10,350	0.960	76%	9,936
2017	12,500	1,125	9,235	0.940	69%	8,681
2018	13,250	1,050	9,500	0.920	66%	8,740
2019	15,250	1,125	11,250	0.975	72%	10,969
2020	17,650	1,265	9,575	1.520		

Year	Premium rate increases for 2020 level	Claims cost Increase for 2020 level	2020 On-level Loss Ratio	Selected Ultimate's losses (in INR 000's)
2015	1.366	1.28	70.1%	8,236
2016	1.368	1.22	67.5%	8,776
2017	1.256	1.16	63.6%	7,951
2018	1.106	1.10	65.8%	8,720
2019	1.029	1.05	73.4%	11,201
2020			68.3%	13,697

Where premium rate factor for 2015 is (17,650/1265)/(11750/1150) = 1.366, etc.
Claims cost increase for 2015 is 1.05 ^5 = 1.276 etc.
2020 on level loss ratio is 75% × (1.276/1.366) = 70.1%, etc.
2020 ELR = (11750 × 70.1% + 13000 × 67.5% + … + 15250 × 73.4%) / (11750 + 13000 + …. + 15250)
= 68.3%
2020 Selected BF Ultimate = 9575 + (1-(1/1.520))×(68.3%×17650) = 13,697.

[8]

iv)

a) Underwriters view

- Underwriter has knowledge of terms and conditions, market conditions etc.
- Information from pricing review

- Based on Subjective judgement
- Potentially not independent
- View may be optimistic
- Good when there are material changes to business

(0.5 marks for each point, maximum 2, both positives and negative to be mentioned)

b) Market loss ratio benchmark

- Simple to use
- Provides independent estimates
- May reflect market rate changes and inflation effects as well as trends in claim frequency and average cost
- Mix of business likely to be different for every company
- Need to check treatment of premiums is consistent (i.e. gross or net of commission) and of reserving and other inconsistencies
- Need to check consistency of reinsurance cover
- Market info may not be available or may be out of date
- If the benchmarks are up-to-date, they themselves are estimates and hence have similar drawbacks to those we are facing in providing estimates.

(0.25 marks for each point, maximum 2, both positives and negative to be mentioned)

c) Adjusted average over 5 years

- Not independent although expected loss ratio would be independent from claims of that year.
- Can make correct allowance for rate changes and inflation
- Reflects trends in own data
- Five-year average loss ratio may not reflect long term average especially for lines which have cat coverage
- May not take account of changes in terms and conditions, mix of business, underwriting cycle
- Own data may be lacking or not credible but is large so should be credible

(0.25 marks for each point, maximum 2, both positives and negative to be mentioned)

[24 Marks]

Part (i) , (ii) Reasonable performance overall

Part (iii) Most candidates performed poorly

Part iv(a), (b), (c) Reasonable performance overall

Solution 3:

- The prime objective in the investment of assets is to maximise investment return, subject to meeting all contractual obligations and recognising the uncertainty involved.
- This company, although solvent, is unlikely to have large amounts of free assets, since otherwise it would be unlikely to have been closed.
- This means that there is not likely to be a great deal of freedom to invest,
- The company will need to make a projection of its liability outgo to extinction and ideally it ought to invest in assets that match it reasonably closely.
- This matching will need to be done by term and by currency,
- Although it is possible that a small company may have assets and liabilities all in a single currency.
- However, the company has only recently closed to business; for some months at least, it may have exposure to property catastrophes, for which a degree of liquidity is required.

- There will be no premium income going forward so a greater need for liquidity.
- Although as has just gone into run-off there will be some premium income in the short term
- There may be some regulatory constraints on a company in run-off which are likely to be more stringent than for an on-going company
- The constraints will likely be exacerbated as this is a small company
- Expenses will be more significant in run-off including redundancy costs increasing the need for liquidity.
- There may be a need for more cash to be held for liquidity
- Although possibly reduced as in run-off less expectation of spikes from catastrophes
- The amount held in non-investible assets should be considered
- Also, any credit facilities being used should be considered as will reduce need for liquidity
- In addition, a company in this situation is usually looking for opportunities to commute its liabilities favourably, and to take advantage of any such opportunities it may need to be able to liquidate assets quickly.
- The desirability of liquidity suggests that it may be appropriate to invest at a shorter term than matching would imply.
- The company's liabilities are almost all real in nature/affected by inflation, even if all the payments are affected by a number of different inflation rates.
- Liability claims may take more than ten years to settle, and possibly much longer if there are likely to be latent claims.
- This highlights the need to hedge against inflation in its investment selection.
- Depending on the local valuation rules there may be a need to avoid excessive volatility. For example, if reserves are valued on an undiscounted basis and assets at market value and if interest rates rise then there is a possibility that the company might become insolvent even if its projected asset income fully covered its liability outgo at all times.
- The need to avoid volatility in the value of assets means that the company will need to avoid investing in equities.
- It will also be impossible to match the term of assets to liabilities using equities.This is despite the fact that equities provide a hedge against inflation.
- If the liability account does include some extremely long-tailed claims, such as latent disease, the need for a long-term hedge might outweigh these considerations.
- The desire to maximise investment income argues for the inclusion of some high-grade corporate bonds in the portfolio.
- The need for liquidity argues for the use of government securities to match the projected outgo.
- It also argues for the use of earlier maturities that would match the projected outgo.
- The need to remain solvent under particular accounting rules may also argue for investing shorter than the term of the expected liability outgo.
- Index-linked stocks may suit all purposes, including hedging against inflation, but they are not available in all jurisdictions and often only for government bonds, which is likely to reduce yield.
- The price of these may also be prohibitive particularly if pushed up by pension-fund investors.
- The assets currently held are important.
- If the company has substantial equity holdings as currently matched assuming future new business and/or because the company was growing then a disinvestment strategy will need to be considered.
- Timing will need to be considered as to how long it should take to get to a desirable portfolio.
- It may need to consider trigger points for when to disinvest from equities.
- Outwards reinsurance is important, this will reduce volatility, with stop-loss cover if available being particularly effective in reducing volatility and the need for liquidity
- Tax will need to be considered.e.g., as just gone into run-off likely to have tax losses brought forward

[0.5 mark for each point, Total 9 marks]

Most candidates provided generic responses with only few addressing the implications on the investment strategy of a portfolio being in run-off.

Solution 4:

1. Number of claims

    a. In general insurance, as distinct from life assurance, there is often no limit to the number of claims that can arise from a policy while it is in force.
    b. MOD may have a relatively high claim frequency, with sometimes 15% or more of policies having a claim each year.
    c. There may be BI claims as well which may be fewer in number compared to the above.
    d. We may use Poisson distribution to model the number of claims.

2. Claim cost

    a. The cost of a claim from any given policy cannot be predetermined. While there will often be a maximum sum insured stated (or implied) in the policy, relatively few claims will be settled for that maximum sum. Most claims will therefore be for only a portion of the maximum cover, depending on the circumstances of the incident.
    b. Mostly, a large proportion of claims will be for small amounts and there will be only a small number of large claims. The precise distribution of claim amounts will, however, vary greatly by class, in particular between property and bodily injury claims and also year by year.
    c. When a probability distribution is used to represent the distribution of the sizes of claims in a class of business it is conventional to use highly skewed distributions with no theoretical upper limit, such as lognormal or Pareto.
    d. Exposures are not all identical, even within the same class of business, and each exposure has its own claim-cost distribution. Exposures are often too small to yield useful information individually, so their experience is aggregated to derive parameters of the claim-cost distributions when the risks are homogenous enough and attritional.
    e. The shape of these claim-cost distributions depends on risk characteristics demonstrated by different classes of business and the insured's risk profile, among other factors
    f. Large losses [marks available if under separate head] – can be modelled using frequency severity approach. (frequency discussed in Section 1). For severity, usually family of Generalised Pareto distribution maybe used.

3. Accumulations

    a. Some classes of business are subject to accumulations of risk. ODP are prone to catastrophes: external events that affect a large number of policies at the same time, possibly causing mainly small claims, but in aggregate giving rise to a very large total loss.
    b. The most important examples are extreme weather, earthquakes and civil disturbance. Motor insurance is less susceptible to large single-incident accumulation losses, however, at times there may be Motorway pile-ups leading to large TPL losses
    c. Extreme weather risks may be modelled using proprietary CAT models.

4. Delay Pattern

    a. Claim delays can arise for various reasons, for example:
    b. A delay between the incident occurring and the policyholder becoming aware of it.
    c. A delay between the insured becoming aware of the loss and reporting it.
    d. A delay before sufficient details of the incident can be gathered to assess the value of the claim.
    e. A delay until an injured party's condition stabilises to the extent that assessment of damages is appropriate.
    f. A delay in agreeing the actual value at which the claim is to be settled, and the payment of this amount to the insured.
    g. The typical extent of such delays will differ according to the class of business.

h. Bodily injury cases tend to have the longest delay tails, owing to the contentious issues of many of the claims involved, often with the need for legal proceedings. This may be worsened by the greater likelihood of latent claims/injuries where the delay from event to reporting can be considerable.
i. By contrast, property damage claims have a much shorter delay tail, and hence in this respect a lower degree of risk, since the losses are more immediately apparent and can usually be valued reasonably accurately by a competent assessor

5. Variability of experience

a. Environment: The number and cost of claims can vary according to such features as unusually bad weather, the economic situation and catastrophes.
b. Inflation: Where a class is exposed significantly to the risk of inflation, any unexpected change in inflation, for whatever reason, will affect the risk profile of that business. Different classes of business are affected by inflation in different ways.
c. Property damage claims responds mostly to the cost of property, and claims will tend to increase in line with general inflation, although repair costs can be linked to earnings. A large proportion of motor claims cost is for the repair of vehicles and will be affected by the level of earnings since these determine labour costs.
d. BI claims are often subject to higher levels of inflation, especially on personal injury claims, as there is a trend to more generous compensation in many markets. However, this may arise in steps rather than as a continuous process of inflation, as landmark legal judgements are handed down or legal reform comes into effect.
e. Fraudulent claims: Often, it will be difficult or uneconomic for the insurer to check whether the claims are genuine or not. At the extreme, such false claims could include arson and embezzlement. The rate of fraudulent claims has been observed to increase in times of economic stringency. We may allow for these through "Shocks".

[0.5 Marks for identification of each characteristic, max 2.5 marks. 1 mark for each major point of discussion]

[Max 10 Marks]

Reasonable performance overall

Solution 5:

i)

Define:

1. Operational risk includes the risk of loss due to fraud or mismanagement within the insurer itself.

[1 mark]

Categories:

1. Administration risk
   a. The risk of error or failure associated with the administrative aspects of the firm's operations.
   b. This risk will depend on the extent to which the firm's administration is outsourced, the extent of centralised and decentralised functions, and the level of staff expertise.
   c. Eg: single-person dependency risk
2. Compliance risk
   a. The risk of non-adherence to legislative and internal firm requirements. We might also consider the cost of implementing future regulatory reforms.
   b. e.g.: regulatory fines due to delay in IRDAI submission
3. Event risk
   a. The risk associated with the potential impact of significant events on the operations of the firm, such as a financial system crisis, a major change in the fiscal system or a natural disaster.

   b. Event risk is not intended to cover the firm's underwriting losses from such events as this would normally be covered within insurance risk.
   c. E.g.: Repair cost of one of the branch offices after an earthquake.
4. Fraud risk
   a. The risk associated with intentional misappropriation of funds, undertaken with the objective of personal benefit at the expense of the firm.
   b. Considerations include: the possibility of fraudulent acts occurring within the firm, and the extent of controls which management have established to mitigate this risk.
   c. E.g.: Employee embezzlement and any insurance against such acts
5. Governance risk
   a. The risk associated with the board and/or senior management of the firm not effectively performing their respective roles.
6. Strategic risk
   a. The risk arising from the inability to implement appropriate business plans and strategies, make decisions, allocate resources or adapt to changes in the business environment.
   b. E.g.: Overestimation of synergy benefit during an acquisition.
7. Technological risk
   a. The risk of error or failure associated with the technological aspects of its operations.
   b. This risk refers to both the hardware systems and the software utilised to run those systems.
   c. Cyber risk (any risk of financial loss, disruption or damage to the reputation of an organisation from some sort of failure of its information technology systems) might be included here or separately.
8. Employee Defined Benefit scheme risk
   a. The risk that the firm is required to make good any shortfall in pension or Gratuity scheme assets relative to its liabilities.
   b. Considerations include: the extent to which extreme events might impact upon the pension scheme funding position; the financial consequences for the firm in such cases; any commitment to fund or eliminate any existing pension scheme deficit.
   c. For example, we may consider holding capital to the value of five years' worth of contributions in excess of the standard contributions required to fund the deficit of the pension plan.

[0.5 mark for listing each category, 1 mark each for describing the categories,0.5 mark each for relevant example]

[Max 10 Marks]

ii)

Modelling:

1. We should identify all material operational risk scenarios specific to the firm's business. We may do this by compiling and maintaining a list of all the risks that may affect the operations of the firm, referred to as a 'risk register'. We should consider these risks when we derive an operational risk capital charge. A brainstorming session may also be advisable.
2. We may use stochastic techniques in assessing the capital impact of operational risk. While we may have limited data to determine the parameters, we can still model it stochastically using simulations of operational risk losses. It will usually be modelled as frequency/severity.
3. Alternatively, risks may be considered separately by personnel with the skills to appraise such risks. This inevitably requires them to make judgments about the degree of loss that each risk may give rise to, the type of event that may cause the loss, and the frequency of such a loss occurring. In addition, we may consider each loss gross and net of any mitigating controls. For example, a firm may have Cyber insurance cover against risk of hacking / data theft / etc. A firm might also have rigorous internal processes that reduce the likelihood of an event occurring or reduce the loss to the firm if the event did occur.

Parameterisation:

4. The actuary may identify mean / median losses from an identified risk category based on past experience.
5. He may ask relevant experts for their judgment or rely on historical data to identify another data point(/s), say a 1 in 10 year event. For instance, CTO will be able to identify various technological risks.
6. Then the actuary can fit an appropriate distribution curve to these data points.
7. The actuary might then want to allow for correlation among the major categories of the operational risk and with other risk types, wherever applicable

Difficulties to judge which risks we have already allowed for appropriately so we can avoid double counting, and to identify which risks have been omitted. The operational risk capital charge will therefore contain a capital charge in respect of the balance of operational risks not dealt with elsewhere. Where elements of operational risk have been captured within other risk categories, it is good practice to identify and quantify these.

Operational risk can be difficult to assess and requires judgment and experience. However, broad-brush measures are generally **not** accepted in determining operational risk; for example, setting an operational risk capital charge to be a defined percentage of other risk charges. This is because this does not demonstrate a thorough assessment of operational risk.

[1 mark for each point, marks available for other suitable points, Max 10]

[20 Marks]

Part (i) Most candidates performed well

Part (ii) Most candidates struggled

Solution 6:

i)

A copula is a mathematical relationship between the individual distributions of random variables and the joint distribution of those variables.

In general terms, suppose we have two random variables with distribution functions $F_X(x)$ and $F_Y(y)$. We can express the joint distribution function as a function of the individual distributions using a third function C: $F(x,y) = C(F_X(x), F_Y(y))$. We can define C as:

$C(u,v) = F(F^{-1}(u), F^{-1}(v))$, which gives $C(F_X(x), F_Y(y)) = F(F^{-1}(F_X(x)), F^{-1}(F_Y(y))) = F(x,y)$.

The function C(u,v) is known as a copula.

[1 mark for definition, 1 mark for formula]

Advantage:

Simple correlation factors give rise to symmetric dependency structures. For some risks, the correlation between them may vary in a more complex way (for example, having a greater level of dependency in the tail of the distribution than around the mean).

Copulas allow us greater flexibility when we model multiple dependencies than single correlation factors allow us.

[1 mark for each point, max 2]

[4]

ii) Let T = Aggregate Total Losses, I = payout of insurer, R = payout of reinsurer.

The distributions of T, I and R is given in the tables below:

	P
P(T = 0)	60.0%
P(T=10000)	16.0%
P(T=20000)	12.8%
P(T=100000)	4.0%
P(T=110000)	3.2%
P(T=200000)	4.0%

Insurer	P
P(I = 0)	60.0%
P(I=10000)	16.0%
P(I=20000)	12.8%
P(I=50000)	11.2%

Reinsurer	P
P(R = 0)	88.8%
P(R=50000)	4.0%
P(R=60000)	3.2%
P(R=150000)	4.0%

The joint distribution of I and R is given by:

Joint	P	F(I)	F(R)
P(I=0,R=0)	60.0%	60.0%	88.8%
P(I=10000,R=0)	16.0%	16.0%	88.8%
P(I=20000,R=0)	12.8%	12.8%	88.8%
P(I=50000,R=50000)	4.0%	11.2%	4.0%
P(I=50000,R=60000)	3.2%	11.2%	3.2%
P(I=50000,R=150000)	4.0%	11.2%	4.0%

P = F(I,R) and the marginals are F(I) and F(R).
One can see that F(I, R) = C(F(I),F(R)) = min(F(I), F(R)).

[1 mark each for the distribution of the aggregate total losses, insurer and reinsurer payouts. 3 marks for the joint distribution and 1 mark for identifying min(u,v) as the function]

[7]

iii) Types:

a) Gaussian copula: Not a strong correlation between the two. And even if there were, the correlation is expected to be linearly distributed. Gaussian copula is the copula embedded in the standard multinormal distribution and there is no upper tail dependency. [3]

b) Gumbel copula: The property claims are expected to have a strong correlation with Business Interruption especially for larger claims. For instance, a large fire in a factory resulting in loss of production for a few weeks. The Gumbel copula gives a strong tail correlation, and is also non-symmetric, giving more weight to the right of the distributions.

[3]

[17 Marks]

Part (i), (ii), (iii)(a), (iii)(b) Almost all candidates performed poorly

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