

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

28th November 2020

**Subject CM2B – Financial Engineering and Loss
Reserving (Paper B)**

Time allowed: 2 Hour (14.30 – 16.30 Hours)

Total Marks: 100

Q. 1) In the country of Actuarial, gambling with dice has recently been made legal. [Each die is six-faced and has numbers 1, 2, 3, 4, 5, 6 written on each of the faces.] Its most famous casino, the Modern Actuarial Casino House (MACH) has offered the following game to its patrons: on an initial payment of \$12, you roll three fair (i.e. all sides / numbers are equally likely) dice and receive an amount equal to the sum of all the three numbers rolled.

i) Since there are three dice each with six possibilities, there are 216 possible outcomes. List all of them and compute the amount received (sum of three numbers rolled) for each of them. (3)

ii) Also make a list of all possible net payoffs (amount received less amount paid) and compute the corresponding (relative) returns from a gambler's perspective and their respective probabilities. (2)

iii) Find the mean, variance and standard deviation of the returns using these probabilities. In addition, please compute the following measures of risk:

- Downside semi-variance
- Shortfall probability assuming benchmark return levels of -40% and -60%
- Value-at-risk (VaR) return levels at 95% and 99% confidence
- Expected shortfall below -35% and -50% return levels (12)

MACH's shares are listed on the Actuarial Stock Exchange (ASE) and currently trade at \$158. Usually, MACH doesn't pay any dividend to its shareholders. The continuously compounded risk-free rate of interest in Actuarial is 4% per annum.

iv) If a 6-month European put option on MACH's share with strike = \$155 is currently valued at \$7, find the implied annual volatility (σ). (5)

MACH has instituted for its key managerial personnel a stock option programme within their overall compensation framework. In particular, the CEO is entitled to a fixed salary of \$500,000 (in cash) and 10,000 European call options with strike price \$160 expiring one year from now.

v) Find the overall fair value of the options that form part of CEO's compensation using the Garman-Kohlhagen formula. (3)

vi) Also find the value of the following greeks by bumping the parameter slightly and revaluing the option:

- a. Delta (with a positive bump – increase in parameter value)
- b. Delta (with a negative bump – decrease in parameter value)
- c. Gamma (using the above two deltas)
- d. Vega
- e. Rho (8)

vii) The CEO has the option to get more call options in lieu of an increase in their strike price (must be an integer not less than 160). If a strike price of K is chosen, the number of call options (n) vested will be determined by the following formula:

$$n = 10,000 + 600 \cdot (K - 160)$$

What should the CEO choose in order to maximize the current fair value of her compensation? (5)
[38]

Q. 2) An investor wants to invest Rs. 1,00,000 for a period of 15 years. The investor has an option to invest in the following assets.

- a) 15 year Government bond providing a fixed rate of 6%
- b) **Asset A** with the expected investment returns following uniform distribution with a range of -14% to 30% over the 15 year period
- c) **Asset B** with the return of 4% and 10% with equal probability

100 Simulated returns for asset A using a stochastic model where return in each year is independent of the returns in previous years have been provided.

i) Calculate the Expected value and standard deviation of the accumulated fund at time 15 assuming the entire fund is invested in Asset A (Using the simulated returns provided for Asset A). (7)

ii) Calculate the Expected Fund value and Standard deviation of the fund at time 15 using the mean and standard deviation of expected returns of Asset A. (8)

iii) Compare and comments on the values calculated in part (i) and (ii) above. (5)

iv) Assuming CAPM model holds, calculate the market price of risk assuming the current market capitalization of Asset A to be 1/3rd of Asset B. (7)

Note: The correlation of returns between asset A and asset B is 0.5.

v) Calculate the expected return for **Asset C** with a standard deviation of investment returns of 5% and correlation of 0.6 with the market. (3)
[30]

Q. 3) You have been given cumulative claims incurred from a small property account. It gives the amounts for each of the last nine accident years, and the related earned premium. The cumulative paid claims for all accident years since, and including 2015 is Rs. 22,000.

i) Estimate the reserve required at the end of 2019 using the Bornhuetter-Ferguson method, stating the assumptions you make. (16)

ii) You have now been given annual claim payments inflation rates for the 12 months up to the middle of the year for each of the years 2015 to 2019. You can assume all payments are made in the middle of each calendar year. Calculate inflation adjusted total liability. You should use the same method as you used in parts (i). You do not need to allow for any inflation beyond 2019. (16)
[32]
