

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

8th April 2021

**Subject CM2B – Financial Engineering and Loss
Reserving (Paper B)**

Time allowed: 2 Hour (15.30 – 17.30 Hours)

Total Marks: 100

Q. 1) You have been provided with data relating to securities 1, 2 and 3. The returns for each security can be modelled using an appropriate single-index model. The expected return and standard deviation of the market is 8% and 4%.

i) Calculate:

- a)** The expected return and standard deviation of return for each security (4)
- b)** Covariance of returns between each pair of securities (2)
- c)** Create the variance/covariance matrix for the 3 securities (2)

Consider a portfolio A with the following mix (50% in security1, 30% in security2, 20% in security3)

ii) Calculate:

- a)** Expected return and variance of portfolio A (3)
- b)** Systematic risk of the portfolio A (3)
- c)** Specific risk of the portfolio A (1)

Consider a portfolio B with the following mix (30% in security 1, 40% in security 2, 30% in security 3) and Portfolio C with the following mix (10% in Security 1, 49.4725% in security 2 and 40.5275% in security 3)

iii) Calculate:

- a)** Expected return and variance of portfolios B and C (2)
- b)** Comment on the comparative performances of the three portfolios A, B and C. (3)

You have been given the historical information of annual returns for security X and Security Y operating in an overseas market for last 30 years.

iv)

- a)** Calculate the expected return and standard deviation for Securities X and Y. (3)
- b)** Assuming CAPM applies, calculate beta for security X and Security Y. (6)
- c)** Make comparative comments on the two securities in the context of CAPM. (3)

[32]

Q. 2) 10 friends decide to create a Trust in which all of them are equal beneficiaries. Each person would invest an amount of 10 lakh into the Trust. The main goal of the Trust is to manage the fund.

30% of the fund is invested in fixed deposit in a trust-worthy bank which earns a guaranteed return of 3% per annum. The trust intends to appoint a Money manager to manage the remaining 70% of the fund.

Manager 'A' proposes to invest the amount in a stock market Index and promises good returns. The historical annual total returns of one of the stock market indices for last 10 years are provided in a table.

- i) Assuming that the next 10 year repeats the above historical return chronologically and that there is no correlation between returns on bank deposit and returns on stock market, tabulate the Surplus Process (i.e. movement of total fund) over the next 10 years.

(3)

The Trust defines "Ruin" as: *Fund being less than $50 * (1.08)^{(t-1)}$ lakh at the end of year t , i.e. to avoid ruin, the fund should not be less than 50 lakh at the end of 1st year, $50 * 1.08$ at the end of 2nd year and so on.* In case of a ruin, the Fund is liquidated and amount divided between the beneficiaries.

Manager 'A' has done some research and concluded that the total return from the Stock market would follow a normal distribution $(\mu, \sigma^2 t)$; t = time period from initial investment, $\mu = 8\%$ per annum and $\sigma^2 = 45\%$ per annum.

ii)

- a) Find the Probability of Ruin at the end of each of years for the next 15 years.

(15)

- b) Repeat the above process of calculation of ruin probabilities with an initial investment of 1.5 crores (instead of 1 crore). Plot a chart (one single chart) depicting calendar year on X axis and Probability of Ruin on Y axis with two different initial investment levels at the end of each year for 15 years. Also comment on the chart.

(4)

Money manager 'B' comes forward and promises to manage the investment in a "better way". He would trade in equity derivatives. He guarantees to pay 10 lakhs at the end of every year for 15 years. This payment received by the Trust does not earn any interest in future.

The only issue is that every now and then there would be events with extreme losses. The loss count up to time ' t ' follows poisson distribution with mean (λt) where, $\lambda = 0.75$ per annum and loss amount is 8 lakh whenever loss occurs.

- iii) Calculate the probability of ruin of the Fund under Manager 'B' the end of each year for 15 years.

(9)

[31]

- Q. 3)** An investor holds an asset that produces a random rate of return R , over the course of a year. The distribution of this rate of return is a mixture of normal distributions, i.e. R has a normal distribution with a mean of 10% and standard deviation of 15% with probability 0.7 and a normal distribution with a mean of 25% and a standard deviation of 30% with a probability of 0.3.

S is the normally distributed random rate of return on another asset that has the same mean and variance as R .

- i) Calculate the mean and variance of R .

(5)

- ii) Calculate the shortfall probabilities for R and S using:

a) a benchmark rate of return of 0%

b) a benchmark rate of return of -10%

(5)

[10]

- Q. 4)** In a zero interest rate country, the current share price of the company XYZ is Rs. 500. The share price after one year can take the following values with the corresponding probabilities:

Probability of share price reaching to that price	Share price
5%	400
10%	425
15%	450
25%	470
30%	540
10%	550
5%	600

If an investor A buys 100 shares today, compute the mean, variance and downside semi-variance for her profit.

[6]

- Q. 5)** Calculate using Black-Scholes formula the price of a 6-month call option on a non-dividend paying stock currently priced at Rs. 420, if the exercise price of the option is Rs. 400, the risk free interest rate is 10% p.a. and volatility is 20% p.a. Using this call option price and the put-call parity, compute the price of a put option with same exercise price and time to expiry.

[7]

- Q. 6)** A stock is currently priced at Rs. 400. Over a 3 month period, it is expected to either go up by 15% (with a 50% probability) or down by 10% (again 50% probability). The continuously compounded risk-free rate is 2% per quarter.

i) Construct a binomial tree for the next year and calculate the risk-neutral probability of an up-jump.

(4)

ii) At each of the terminal nodes (at time 1 year), calculate the option payoff value of 1-year European call option with strike price of Rs. 450. Where the payoff exceeds zero, also calculate the risk-neutral probability of reaching that node. Hence calculate the current price of that simple European call option.

(6)

iii) Consider a special European call option (called Barrier option) with same strike price and maturity. However, the holder of this option has the right to exercise his/ her option at maturity only if the stock price has been above the barrier level (Rs. 600) at some point during the year. Calculate the current price of this Barrier option.

(3)

iv) Comment on the relationship between the price of the Barrier option with that of the simple European call option.

(1)

[14]
