INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

23rd July 2022

Subject CM2A – Financial Engineering and Loss Reserving (Paper A)

Time allowed: 3 Hours 30 Minutes (09.30 – 13.00 Hours)

Total Marks: 100

Q. 1)   **i)** If W_t is a Wiener process, then which of the following statement(s) is/are correct.

A. W_t is a martingale

B. Covariance of W_{10} & W_{15}, is a martingale, and is 15

C. Covariance of W_{10} & W_{15}, is a martingale, and is 10

D. $X_t^2 - t$ is a martingale

   I.   A & B
   II. C & D
   III. A , B & D
   IV. A , C & D   (2)

ii) If securities markets are efficient, what is the NPV of any security, regardless of its risk?

   I.   NPV=0
   II.   NPV>0
   III. NPV<0
   IV.   Can't be determined   (2)

iii) A call option is described as out of money if:

   I. $S_t < K$
   II. $S_t =< K$
   III. $S_t > K$
   IV. $S_t = K$

Where S_t is the price of the underlying share price at time t and K is the exercise price.   (2)

iv) Which of the following (s) factors are ignored in the specification of Modern Portfolio Theory?

a. rationality of the investors
b. risk aversion of investors
c. skewness of the distribution of returns
d. kurtosis of the distribution of returns

   I.   a & b
   II.   c & d
   III. Only c
   IV.   a, b & d   (2)

v) There is an institution that has bought 1,00,000 put options on Share X, each of which has a delta of 0.75. In which of the following scenarios will the institution be delta-hedged?

   I.   The portfolio is already delta-hedged.
   II.   Selling 75,000 units of Share X
   III. Purchasing 75,000 units of Share X
   IV.   None of the above   (2)

[10]

Q. 2)   **i)**   If a market is inefficient and as a new information is received about a security then

    I.   nothing will happen.
    II. the stock price will fall at first and then later rise.
    III. there will be a lag in the adjustment of the stock price
    IV. there will be negative demand for the stock.   (2)

  ii)   The price of Share X is INR 120. Current price of a put option on Share X with an exercise price of INR 105 is INR 3.5. What is the intrinsic value and time value of this put option?

    I.   Intrinsic Value is -15 and time value is INR -11.5
    II. Intrinsic Value is 0 and time value is INR 3.5
    III. Intrinsic Value is -15 and time value is INR 18.5
    IV. Intrinsic Value is -15 and time value is 0   (2)

 iii)   As we move down the indifference curve from left to right, the slope of the indifference curve tends to which of the following?

    I.   Unity
    II. Zero
    III. Decline
    IV. Rise   (2)

 iv)   The following symbols denote: ρ , θ, λ

    I.   Change in derivative price w.r.t interest rate, share price, dividend rate respectively
    II. Change in derivative price w.r.t interest rate, time, volatility respectively
    III. Change in derivative price w.r.t interest rate, time, dividend rate respectively
    IV. Change in derivative price w.r.t volatility, share price, dividend rate respectively   (2)

  v)   Which of the following comment(s) is/are correct?

    a. $R_t > 0$ for all 't' under Vasicek's model
    b. Distribution of Cox-Ingersoll-Ross model is always Normal
    c. Mean reversion rate is time dependent under Hull white model

    I.   a & b
    II. c only
    III. a only
    IV. None of the above   (2)

                                                                       [10]

Q. 3)   **i)**   Choose the annual expected return using security market line given beta =1.2, market rate =0.06, market sigma = 0.2, market price of risk = 0.1

    I.   6.2%
    II. 6.3%
    III. 6.4%
    IV. 6.5%   (3)

  ii)   A country has a constant short term real rate of interest
       Real rate = nominal rate less inflation

Describe the shape of the yield curve if market participants believe that the current inflation is very high and future inflation would be much lower than current inflation? (4)

[7]

Q. 4) **i)** State the main assumptions under the Binomial Model for option pricing. (2)

ii) Explain the option pricing approach using a stochastic discount factor. (3)

iii) There is a one-period binomial model of a stock whose current price is 70. In a one-year period, it is expected that the stock price can move up to 100 or down to 30. The actual probability of up-movement is 70%. The continuously compounded risk-free rate of return is 7% per time period. The exercise price is 50.

Choose from the following downward payoff provided under a replicating call option.

a) 40
b) 0
c) -12
d) 12 (3)

[8]

Q. 5) **i)** Claims occur on a portfolio of insurance policies according to a Poisson process at a Rate λ. All claims are for a fixed amount d, and premiums are received continuously. The insurer's initial surplus is U ($< d$) and the annual premium income is $1.2\ \lambda\ d$. Pick the probability that ruin occurs at the first claim is:

  I. $1-e^{-1/1.2\ (1-U/d)}$
 II. $1-e^{-1/1.2\ (1-U/d\lambda)}$
III. $1-e^{-1/1.2\ \lambda\ (1-U/d)}$
 IV. $1-e^{-1/1.2\ d\ (1-U/d)}$ (2)

ii) An insurer has issued five-year term assurance policies to two individuals involved in a dangerous sport. Premiums are payable annually in advance, and claims are paid at the end of the year of death.

Individual	Annual Premium (in Rs)	Sum Assured	Prob. of death (annual)
A	10000	17,00,000	0.05
B	5000	4,00,000	0.10

Assume that the probability of death is constant over each of the five years of the policy. Suppose that the insurer has an initial surplus of U.

a) Define what is meant by $\psi(U)$ and $\psi(U,t)$. (2)

b) Assuming $U = 10,00,000$, determine the distribution of $S(1)$, the surplus at the end of the first year, and hence calculate $\psi(U,1)$. (4)

c) Assuming $U = 10,00,000$, determine the possible values of $S(2)$ and hence calculate $\psi(U,2)$. (5)

[13]

Q. 6) **i)** State in words the four axioms of the Expected Utility Theorem. (4)

ii) An investor measures the utility of his wealth using the utility function $U(w) = \ln(w)$ for $w > 0$. He has Rs.100,000 available to invest in two possible assets, Asset A and Asset B.

The future value of Asset A depends on an uncertain future event.

- Every Rs.1 invested in Asset A will be worth Rs.1.30 with probability 0.75 and Rs. 0.40 with probability 0.25.
- Asset B is risk-free, so every Rs.1 invested in Asset B will always be worth Rs.1.

The investor does not discount future asset values when making investment decisions. He decides to invest a proportion a of his wealth in Asset A and the remaining proportion $1 - a$ in Asset B.

a) Express his expected utility of wealth in terms of a. (2)

b) Determine the amount that he should invest in each of Asset A and B to maximise his expected utility, using your result from part (ii). (5)

[11]

Q. 7) **i)** Explain one drawback of the Black-Scholes Option Pricing Formula. (1)

ii) An investor buys, for a premium of 550.1, a call option on a non-dividend paying stock whose current price is 6,000. The strike price of the call is 5,900 and the time to expiry is 1 year. The risk-free rate of return is 6% pa continuously compounded.

The Black Scholes formula for the price of the call option on a non-dividend paying share is assumed to hold.

a) Calculate the price of a put option with the same time to maturity and strike price as the call. (3)

b) The investor buys a put option with strike price 5,500 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (a). (6)

[10]

Q. 8) An investor plans to retire 10 years from now. She believes that in 10 years' time an amount equal to 50 lakhs is sufficient to keep a planned standard of living.

However, the above amount ignores the impact of inflation.

Yearly expense Inflation over the next 10 years is expected to follow uniform distribution between 3% to 11%

Estimate:

i) Mean inflation rate over 1 year

   I.   3%
  II.   4%
 III.   6.5%
 IV.   7% (1)

ii) Mean inflation rate over 10 years assuming independent yearly inflation over the next 10 years.

    I.   7%
    II.   96.72%
   III. 6. 5%
   IV. 94.32%   (2)

iii) Variance of the accumulated inflation over the next 10 years.   (3)

iv) Expected Amount to be invested today to retain the purchasing power after 10 years.   (3)

The investor decides to invest 60% in equities and 40% in fixed deposit giving a risk free return of 7% p.a.

Yearly growth rate of equities are iids and log of the growth rate follows normal distribution with mean of 9% and variance of 20%.

v) Find the amount to be invested to keep the planned standard of living after 10 years with 50 % probability if mean inflation is considered.   (4)

                                                                           [13]

Q. 9)   **i)** Explain how policyholder behaviour affects the risks and price charged by an insurance company.   (3)

An insurance company wants to start insuring Blockchain Technology risks.

   ii) State the considerations the company will take while assessing the effects of the above risk.   (2)

   iii) The company conducted a consumer survey to understand the marketability of the Blockchain Technology Insurance product. In the consumer response, it assessed that the expected business from the product would be 10 policies in the first year. Each policy will have a sum insured of Rs. 50,00,000 and the company is expected to pay 90% of the sum insured in case there is a loss event. The company expects that there is one hundredth chance of suffering the loss.

   a) Calculate the 99.5% Value at risk from one policy.

   b) Calculate the 99.5% Value at risk from 10 policies.   (5)

                                                                           [10]

Q .10) **i)** Explain how an investor's economic characteristics will affect his choice of an investment such that:

   a) first order stochastically dominates another   (1)

   b) second order stochastically dominates another   (1)

   ii) Following three assets will deliver a one-year return r_i on asset i with probabilities as set out below:

	$P(r_i = -5\%)$	$P(r_i = -3\%)$	$P(r_i = 0\%)$	$P(r_i = +3\%)$	$P(r_i = +5\%)$
Asset 1	0.2	0.2	0.2	0.2	0.2
Asset 2	0.3	0.2	0.1	0.2	0.2
Asset 3	0.1	0.3	0.2	0.3	0.1

Determine which type of dominance, if any, is exerted by:

a) asset 2 over asset 3. (2)

b) asset 3 over asset 1. (2)

c) asset 1 over asset 2. (2)

[8]

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