

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

20th November 2019

Subject CM1B – Actuarial Mathematics (Paper B)

Time allowed: 1 Hour 45 Minutes (10.15 – 12.00 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Mark allocations are shown in brackets.*
- 2. Attempt all questions, beginning your answer to each question in the template provided.*
- 3. Attempt all sub-parts of the question in the template provided only, unless otherwise instructed to do so.*
- 4. The working of each part of the question should be on a separate tab (sheet). For example, question 1(i) should be worked out within the tab (sheet) with name 1(i) of worksheet named CM1B.*
- 5. Where possible, summarize your data used & assumptions made (if any) in a separate tab.*
- 6. Do save your work in Excel files on a regular basis.*
- 7. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*
- 8. Ensure that you click the “Submit” button only when you have completed the question paper and final submission has to be made.*

AT THE END OF THE EXAMINATION

Please return this question paper to the supervisor separately. You are not allowed to carry the question paper in any form with you. You are requested to save and submit the work before leaving the examination premises.

- Q. 1)** A life insurer operates in a market for which an extract from a commonly used life table by the market has been provided (in Excel).
- i)** Using this data and assuming the number of lives aged 40 years exact to be 1500, calculate the l_x and q_x at each of the ages provided in the extract. (4)
 - ii)** Plot the mortality rates derived in part (i) above on a suitable graph and comment on same. (4)
 - iii)** Assuming an effective interest rate of 6% p.a. and life table as provided for part (i) above, calculate the expectation and standard deviation of the random variables below:
 - a)** X is the present value of endowment assurance with term of 5 years for a sum assured of INR 1 for a policyholder aged 45 years. Benefits are payable end of year. (5)
 - b)** X is the present value of temporary annuity due to a life aged 45 years exact, payable for a period of 5 years. Annual annuity amount is INR 1 is paid in advance. (5)

A life insurer has decided to issue unitized with-profits endowment policy with a policy term of 10 years to a policyholder aged exactly 50 years. Given below are policy features:

- Allocation rate is 90% in year 1, 95% in year 2 and 100% thereafter.
 - Premiums of INR 4,000 paid at the start of each year and are payable annually.
 - Bonus is added at the end of each policy year by means of increasing fund value.
 - Policy fee of 1% of the unit fund value (derived after regular bonus has been added to it) is deducted at the end of each year by cancelling units.
 - Death benefit payable at the end of year of death: End of year unit fund value (after deduction of all applicable charges and addition of any regular bonus during the year) plus terminal bonus or INR 20,000 whichever is higher.
 - Maturity benefit: End of year unit fund value (after deduction of all applicable charges and addition of any regular bonus during the year) plus terminal bonus.
 - Surrender benefit: Unit fund value less a surrender penalty as described below.
- iv)** Ignoring any reserving requirements, calculate the present value of profit by preparing a cashflow model in excel for this contract, based on the assumptions given below. Please present all calculations clearly in separate columns with relevant headers:
- Initial expenses: INR 400.
 - Renewal expenses: INR 30 every year assuming nil inflation and are assumed to occur at start of the year from the first policy year itself.
 - Claim expenses: INR 50 per death / surrender / maturity.

- Surrender penalty: 2% of unit fund value if the surrender happens from end of year 1 to end of year 4; 1% of unit fund value if the surrender happens from end of year 5 to end of year 9. For avoidance of doubt, this refers to unit fund value after application of all charges and regular bonus rates during the year.
- Terminal bonus rate: 2% of the unit fund value at the end of each year (1 to 3 years); 5% of the unit fund value at the end of each year (4 to 6 years); 8% of the unit fund value at the end of each year thereafter.
- Mortality rate: Provided as part of (i) above.
- Surrender probability: 6% of all policies in force at the end of each year.
- Bonus interest rate: 4% p.a.
- Initial commission: 3% of the first year's premium.
- Renewal commission: 1% of annual premium from second policy year onwards.
- Non unit interest rate: 2% p.a. which would be calculated on non-unit cashflows after expenses and commission.
- Risk discount rate: 7% p.a.

(30)

v) The policyholder wants to know the return he will earn on his investments if he lives till the end of the policy term. Calculate the return to the policyholder? (5)

vi) Alternatively, the policy holder is considering investing in a 10 year fixed interest bond which has a market value of INR 98 per 100 nominal and pays semi-annual coupons of 6% p.a. Compare the benefits of this investment vs the product mentioned above. (8)

vii) On analysing the surrender rates for a few more years, it was seen that the rates were higher at 10% in year 1, 5% in year 2, 1% till year 7 and no surrenders thereafter. All other assumptions remain unchanged. Calculate the expected present value of profits for this contract, based on the change in surrender rates. (7)

viii) Explain your results for change in part (vii). (2)

ix) In addition to change in part (vii), the company has decided to increase the policy fee to 1.15% of unit fund value, to be deducted at the end of each year by cancelling units. At the same time, the company has also decided to allow for expense inflation on renewal expenses and claim expenses. An expense inflation of 5% p.a. applies from end of first policy year itself. Determine a revised regular bonus rate assumption, which would result in the same present value of profit as was calculated in part (vii) above. (10)

[80]

Q. 2) i) You have been provided with a curve which contains the spot rates varying by duration. Calculate the annualised forward rate for each half year period as provided in the data and plot it on a graph. (5)

- ii)** A man invests INR 400,000 in a project. Once he makes this investment, he expects to start incurring costs which are accounted for at the end of each year, beginning with INR 20,000 at the end of the first year, increasing by INR 5,000 thereafter. He expects to start earning an annual income of INR 100,000, receivable half yearly, from the third year after his initial investment (to be modelled as end of half year periods). In subsequent years, the annual income is expected to grow at a rate of 20% p.a. for the first 5 years over which it is received and thereafter by 6% p.a. In all years, the annual income will be received each half-year and the growth rate will also apply on a half yearly basis. The project will close at the end of 12 years.

Using the forward rates calculated above, calculate the present value of this project for the investor. Your solution should demonstrate calculations for each of the cash flows above. (8)

- iii)** For the project above, calculate the payback period and the discounted payback period. (7)
[20]
