

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

25th March 2022

Subject CM1B – Actuarial Mathematics
(Paper B)

Time allowed: 2 Hours (14.30 - 16.30 Hours)

Total Marks: 100

Q. 1) Use the mortality data given on tab 'Q1 data'.

An endowment assurance policy with a policy term of 23 years has a sum assured of Rs.1,00,000 which is payable at the end of term or at the end of year of death if earlier. The policyholder is aged exactly 37 years at entry. Level annual premium of Rs. 2,844 is payable in advance.

The Insurance company uses Retrospective formula and the following basis for the calculation of reserves:

Mortality	: AM92 Ultimate
Interest	: 4% pa effective
Initial expense	: Rs.1,000
Regular expenses	: Rs.50 paid at the start of each year excluding the first
Claim expense	: Rs.500 paid at the time of claim either by death or maturity

- i) Calculate the retrospective reserve for this policy at each expired policy duration from year 1 to 23 inclusive before payment of any maturity claim due at the end of term. (21)
- ii) Calculate the profits, if any, that the company expects to make at the maturity date for each maturing policy and explain how the retrospective reserve is useful for the insurance company. (4)
- iii) The company intends to make a profit at maturity of 20% of total premium income received under the policy. Calculate how much premium should the company charge to achieve its profit target. (5)

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- Q. 2)** A life insurance company issues a 20-years unit-linked endowment policy to a life aged 40 years exact under which level premiums of ₹75,000 are payable yearly in advance throughout the term of the policy or until earlier death. If the policyholder dies during the term of the policy, death benefit of ₹20,00,000 or the bid value of the units, whichever is higher, is payable at the end of the policy year of death. The policyholder may surrender the policy only at the end of each policy year. On surrender, the bid value of the units is payable at the end of the policy year of exit. On maturity, 105% of the bid value of the units is payable. The company uses the following assumptions in carrying out profit tests of this contract:

Growth Rate on Assets in the Unit Funds	6.50% p.a.
Growth Rate on Assets in the Non-Unit Funds	5.50% p.a
Mortality	AM92 Select, Provided in Data Sheet
Initial Expenses	₹3,000
Renewal Expenses	₹650 Payable from beginning of second policy year
Initial Commission	10% of annual premium in the first year
Renewal Commission	2.5% of annual premium from second policy year
Risk Discount Rate	8.50% p.a.
Surrender Rate	2% of In-force policies at the end of each year

In the first policy year, 25% of the premium is allocated to units, 102.5% in the 2nd year up to year 10th and 100% thereafter. The units are subject to a bid-offer spread of 5%. An annual management charge of 0.5% of the bid value of units is deducted at the end of each policy year.

Management charges are deducted from the unit fund before death, surrender and maturity benefits are paid.

- i) Determine the Unit Fund cashflow for each year of the policy. (8)
 - ii) Calculate the profit margin of this policy. When developing your solution, clearly show the cash flows of the non-unit fund which are considered in determination of profit. (28)
 - iii) The company is looking to increase the renewal commission in light of competition in the market and want to test the level of renewal commission it can afford to offer while keeping the profit margins at 5%. What is the maximum level of renewal commission it can offer? (5)
 - iv) The company currently does not levy a mortality charge. How will the components of profit vector change if a mortality charge is levied? (2)
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- Q. 3)** i) Assuming an effective interest rate of 5% p.a. and a term of 10 years, prove the following identity showing detailed derivation of each of the terms which form part of this identity. Your solution should clearly show the timesteps, cash flows and discount factors.

$$\ddot{a}_n^{(p)} = \frac{i}{d^{(p)}} a_n \quad (9)$$

- ii) An investor purchases a financial instrument to hedge his exposure to change in interest rates.

The investor will make payments to the bank at a fixed rate of interest. These payments will be made annually in arrears at a rate agreed between the investor and the bank at the outset.

In return, the bank will make payments to the investor at a floating rate of interest annually in arrears. The floating rate of interest is based on the one-year risk-free forward rate curve less 0.2%. The current risk-free interest rate has been provided in the spreadsheet.

Assuming a tenure of 10 years for this financial instrument and a nominal amount of Rs 10,000 (to be used for calculation of interest payments which will be swapped), calculate the fixed rate of interest that the investor should agree with the bank which results in a “no arbitrage” position. The annual payment by each party is determined as the nominal amount multiplied by the interest rate applicable to the party as described above. (9)

- iii) Shortly after the investor signed the agreement with the bank to ensure no-arbitrage at the outset (as per (ii) above), the risk-free interest rates increased by 0.5% across all durations. Determine the profit or loss to the investor from changes in the market environment. (5)
 - iv) Using the risk-free interest rate curve provided to you, calculate the spot rate at each duration. (4)
- [27]**
