

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

7th April 2021

Subject CM1B – Actuarial Mathematics (Paper B)

Time allowed: 2 Hour (15.30 – 17.30 Hours)

Total Marks: 100

- Q. 1)** Mr. Peter is looking to purchase a new flat to take advantage of the slump in property prices. He will need a loan of INR 60 lakhs and assessing loan offers from two banks Bank A and Bank B respectively. The terms of the loan of two banks are as under:

Bank A:

- Interest charged each month is based on the yield curve. Yield curve provided is the spot rates p.a. for each quarter.
- Term of loan is 20 years.
- The processing fee charged by the bank is 2% of the loan amount subject to a maximum of INR 1,00,000 to be paid by Mr. Peter upfront.

Bank B:

- Interest is charged @ 6.00% p.a. convertible monthly for first three years and @ 9.60% p.a. convertible monthly thereafter
- Term of loan offered is 15 years
- The processing fee charged by the bank is fixed at INR 15,000 to be paid by Mr. Peter upfront.

EMI under both Bank A & Bank B will remain same throughout the loan term and cannot be altered.

(Note: Round INR figures calculated to two decimal places)

- i)** Calculate the applicable term structure of interest rates on monthly intervals for the tenure of the loan for Bank A. (Hint: Derive monthly forward rates from the spot rate) (5)
- ii)** Calculate fixed EMI payable & develop loan schedule table under both Bank A and Bank B loan offers (using cash-flow projection throughout loan term & Goal Seek function). (17)
- iii)** From above developed loan schedule calculate following:
 - a)** Total amount paid in excess of loan amount under both loan offers. (2)
 - b)** Develop a loan schedule to determine if EMI determined for Bank B loan offer is used to payback loan as per Bank A terms (interest), in how many months will the loan be repaid. Also calculate the total amount paid in excess of the loan amount under this scenario.
(Hint: The repayment at any point of time should not exceed outstanding loan with interest.) (8)

- iv) Calculate annualized IRR for terms offered by banks A and B respectively (using Goal Seek function) under original arrangements and also if arrangement as per (iii) (b) is followed. (15)
- v) What considerations will Mr. Peter have when choosing one of the two loan offers? (3)
- [50]

Q.2) An insurance product, with a 13-year policy term provides benefits defined as below:

- Basic sum assured = 50,000
- Benefit payable upon death during the period $t-1$ to $t = 10,000 (1+0.5(t-1))$, where t reflects the annual time steps. This benefit however cannot exceed the basic sum assured. Death benefit is paid at the end of year of death.
- Benefit on maturity is equal to the 75% of the basic sum assured.

A single premium of INR20,000 is payable.

Mortality: q_{35} is 0.05% and q_{50} is 0.15%. Mortality rate increases linearly between q_{35} and q_{50}

Age of policyholder is 35 years exact.

- i) Prepare mortality rates for the policy term rounded to nearest 6 decimal places. (4)
- ii) Calculate the expected cash flows for premium, death benefit and maturity benefit at each annual time step starting 0 (inception), until 13 (maturity). Calculate the net cash flow at each time step. Net cash flow is defined as expected outgo less expected income. You do not need to consider any investment income. (10)
- iii) Assuming an interest rate of 6% p.a. (and ignoring any cost of holding reserves), determine whether this contract is expected to be profitable or loss making for the company. (4)
- iv) Calculate the reserves required to be held just before the 3rd policy anniversary. In doing so, you should make the assumptions below:
- Interest rate of 6% p.a.
 - Mortality rate is 115% of the q_{35} and q_{50} rates provided above.
- (6)

At this point (just before 3rd policy anniversary), the company wants to immunize its exposure to liabilities (calculated in part iv) by investing in two bonds:

- Bond A pays coupons of 8% p.a. annually in arrears and is redeemable at par in 20 years' time.
- Bond B is a zero-coupon bond that is redeemable at par in 6 years' time.

The gross redemption yield on both bonds is 6% p.a.

- v) Show the cashflows from both Bond A & Bond B.

Calculate the discounted mean term of both Bonds showing detailed calculations for both (i.e. you should use mathematical formulae as described in course notes instead of using MS Excel shortcut functions for calculation of duration). (7)

vi) Determine the amount the company should invest in each bond so that the following conditions are met:

- Present value of assets equal reserves (as calculated in part (iv) above)
- Discounted mean term of assets equals to discounted mean term of net cash flows from liabilities.

(6)

vii) There has been a change in interest rates (as provided in Excel). Calculate the percentage change in value of reserves and the value of the asset portfolio under the new interest rates. Comment on the results. (8)

viii) Calculate the below:

- Running yield of Bond A under both interest rate scenarios defined above.
- For the interest curve provided in part (vii) above, calculate $f_{3,3}$

(5)

[50]
